Displacement: x
Aerodynamic Force: X
Aircraft Velocity: U
(steady)
Aircraft Velocity: u
(perturbation)
Angular Displ.: \( \phi \) (roll)
Angular Velocity: p
Moment (rolling): L

Displacement: y
Aerodynamic Force
Aircraft Velocity: V
(steady)
Aircraft Velocity: v
(perturbation)
Angular Displ.: \( \theta \) (pitch)
Angular Velocity: q
Moment (pitching): N

\[ q = \frac{1}{2} c \rho V^2 \]
\[ q = \frac{1}{2} c U^2 \] — This is used now (instead of \( c U \) for forward speed)

**Fig. 5** Notation and sign conventions for body axes.
The diagram shows four reference frames:

1. Earth axes frame $F_e$
2. Intermediate frame $F_{ie}$
3. " " frame $F_d$
4. Body axes frame $F_b$

The sequence of rotations is:

1. Rotation $\psi$ about $z_e$ to intermediate frame $F_{ie}$
2. Rotation $\theta$ about $y_e$ to intermediate frame $F_d$
3. Rotation $\phi$ about $x_e$ to body axes frame $F_b$

**Fig. 6. The Euler Angles**
\( \alpha \) is the angle between the \( x_B \) axis and the plane containing the \( y_B \) axis and the velocity vector.

\( \beta \) is the angle between the velocity vector and the plane of symmetry (\( x_B z_B \)).

**Fig. 7** Wind axes and the aerodynamic angles.
Fig. 10  Block Diagram of the Flight Equations Solved in Body Axes
3.0 The Inertial Terms

Another possible source of confusion is that moments of inertia about tilted axes are widely quoted as \( I_{xx}, I_{xy}, I_{yy} \), etc. whereas it has long been the British aeronautical tradition to use:

\[
\begin{align*}
A &= \sum m (y^2 + z^2) \rightarrow I_x \\
B &= \sum m (x^2 + z^2) \rightarrow I_y \\
C &= \sum m (x^2 + y^2) \rightarrow I_z \\
D &= \sum m y z \quad \rightarrow I_{xy} \text{ or } I_{yx} \\
E &= \sum m x z \quad \rightarrow I_{xz} \text{ or } I_{zx} \\
F &= \sum m x y \quad \rightarrow I_{yz} \text{ or } I_{zy}
\end{align*}
\]

This could be a negative sign.

Here \( x, y, z \) are positions of masses \( m \) relative to the coordinate origin.

We shall use the \( I \)-form initially, but the British notation later on. You should at least recognise both forms.

Two other small matters deserve attention.

a. The "gravity terms" are few and it can be helpful to show them explic. Therefore the three general forces \( X, Y, Z \) in the directions of the co-ordinate axes are taken to be forces other than of gravitational origin. In general there are no gravity-induced moments so the three moments \( L, M, N \) also have no gravity term present, but neither are there such terms elsewhere in the rotational equations.

b. For virtually all our work the propulsive force will be constant so that \( F \) will hardly appear at all; it will generally be in equilibrium with dra which does not usually appear either.

4.0 The Formal Equations

The translational equations can be reduced to the following, even allowing for non-zero values of \( V, W, \) and a climb angle \( \theta \):

\[
\begin{align*}
\text{Fore/ Aft:} & \quad m(\dot{U} - rV + qW) = X - mg \sin \theta \\
\text{Lateral:} & \quad m(\dot{V} - pW + rU) = Y + mg \cos \theta \cos \phi \\
\text{Transverse:} & \quad m(\dot{W} - qU + pV) = Z + mg \cos \theta \sin \phi
\end{align*}
\]

Similarly, the rotational equations are expressed in a relatively complete form:

\[
\begin{align*}
\text{Roll:} & \quad I_x \ddot{r} - (I_y - I_z)qr - I_{yx} (q^2 - r^2) - I_{xz} (r + pq) - I_{xy} (\dot{q} - rp) = L \\
\text{Pitch:} & \quad I_y \ddot{q} - (I_z - I_x)rp - I_{yx} (r^2 - p^2) - I_{yz} (\dot{r} + qr) - I_{zx} (p + q) = M \\
\text{Yaw:} & \quad I_z \ddot{p} - (I_x - I_y)pq - I_{xy} (p^2 - q^2) - I_{xz} (q + rp) - I_{yz} (\dot{r} - qr) = N
\end{align*}
\]

(You are not expected to remember these!)
5.0 The Cross-Inertias

In practice a simpler form of these equations is used, if only because we have a body with a plane of symmetry which, in our case, leads to \( I_{xy} \) and \( I_{yz} \) being zero. An interpretation of this is that for any dimension \( x \) away from the origin (fore/aft) there are equal masses at \( \pm y \) which cancel in the summations of Eqn.(1). Similarly, for a choice of some fixed position \( z \) away from the origin (vertically) there will be equal masses at \( \pm y \). An obvious candidate that prevents \( I_{zx} \) being zero is the tail (fin), for at aft positions \( x \) we do not have equal masses at \( \pm z \) (there is no fin below the fuselage).

6.0 The Six Equations Simplified

A version of the equations conforming to the above simplifications or assumptions is:

\[
\begin{align*}
\text{Fore/aft: } m(v + qW) &= X - mg \sin \theta \\
\text{Lateral: } m(v - pW + rU) &= Y + mg \cos \theta \sin \phi \\
\text{Transverse: } m(w - qU) &= Z + mg \cos \theta \cos \phi
\end{align*}
\]

which retains the transverse velocity \( W \) to allow for steady flight where the body axes are not aligned with the direction of flight:

\[
\begin{array}{c}
\text{U} \\
\text{flight} \quad \text{dir.} \quad \text{W}
\end{array}
\]

and in the rotational senses:

\[
\begin{align*}
\text{Roll: } I_x \dot{p} - (I_y - I_z)q \dot{r} - I_{xz} (\dot{q} + pg) &= L \\
\text{Pitch: } I_y \dot{q} - (I_z - I_x) \dot{p} - I_{yz} (\dot{r} - p \dot{q}) &= M \\
\text{Yaw: } I_z \dot{r} - (I_x - I_y) pg - I_{zx} (\dot{p} - q \dot{r}) &= N
\end{align*}
\]

These retain the single cross-inertia or "product of inertia" \( I_{xz} \).

7.0 Coupled Motion or Independence?

In general, the motions are coupled and, for example, an impulsive force (moment) applied in roll will eventually disturb the motions in all 6 directions and there will be non-zero values for all 6 motion variables. However, for many applications it is standard practice, and sufficient, to separate the equations into two de-coupled sets of three freedoms each to provide what is called

- the **longitudinal** equations in \( x, z, \theta \)
- the **lateral** equations in \( y, \phi, \psi \).

In the form quoted above, the equations are not yet linearised and they show