AMME3500: System Dynamics and Control

Lab 2- Proportional and PD Control - Linear Plant

**Note:** This lab contributes 25% towards your mark for assignment 2. This lab will be undertaken during Week 7 or 8 in the Control Systems lab in the EIE522. Marking will take place during the lab session. The objective of this lab is to familiarize you with techniques for identifying system parameters. **This lab should take an average student 3 hours to complete plus 1 hour of preparation and 1 hour to complete the associated questions.**

This document outlines experiments which identify the plant parameters using the ECP Systems Model 210a Linear Plant. This material is reproduced in large part from the appropriate lab manual, which is available in the lab and on-line for you to prepare yourself for the lab. **The safety portion of this manual, Section 2.4, must be read and understood by any user prior to operating this equipment.** To become more familiar with these operations, it is strongly recommended that the user read Chapter 2 in its entirety prior to undertaking the operations described here. Remember here, as always, it is recommended to save data and control configuration files regularly to avoid undue work loss should a system fault occur.

This experiment demonstrates some key concepts associated with proportional plus derivative (PD) control and subsequently the effects of adding integral action (PID). This control scheme, acting on plants modeled as rigid bodies finds broader application in industry than any other. It is employed in such diverse areas as machine tools, automobiles (cruise control), and spacecraft (attitude and gimbal control). The block diagram for forward path PID control of a rigid body is shown in Figure 6.2-1a where friction is neglected.\(^1\) Figure 6.2-1b shows the case where the derivative term is in the return path. Both implementations are found commonly in application and, as the student should verify, both have the identical characteristic roots. They therefore have identical stability properties and vary only in their response to dynamic inputs.

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\(^1\)The student may want to later verify that for the relatively high amount of control damping in the scheme that follows – induced via the parameter \(k_d\) – that the plant damping is very small.
The closed loop transfer functions for the respective cases are:

$$c(s) = \frac{x(s)}{r(s)} = \frac{(k_{hw}/m)(k_d^2+k_p^2+k_i)}{s^3+(k_{hw}/m)(k_d^2+k_p^2+k_i)}$$  \hspace{1cm} (6.2-1a)

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For the first portion of this exercise we shall consider PD control only ($k_i=0$). For the case of $k_d$ in the return path the transfer function reduces to:

$$c(s) = \frac{k_{hw}k_p}{s^2+(k_{hw}/m)(k_d+s_k)}$$  \hspace{1cm} (6.2-2)

By defining:

$$\omega_n = \sqrt{\frac{k_p k_{hw}}{m}}$$  \hspace{1cm} (6.2-3)

$$\zeta = \frac{k_d k_{hw}}{2m \omega_n} = \frac{k_d}{2 \sqrt{mk_p k_{hw}}}$$  \hspace{1cm} (6.2-4)

we may express:

$$c(s) = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2}$$  \hspace{1cm} (6.2-5)
The effect of $k_p$ and $k_d$ on the roots of the denominator (damped second order oscillator) of Eq (6.2-2) is studied in the work that follows.

**Preparation**

a. Show that the two system diagrams in Figure 6.2-1 have identical characteristic equations and that the diagram of the system with the damping in the feedback path reduces to the transfer function shown in Equation 6.2-2 if the $k_i$ term is set to zero.

b. Derive the transfer function for the linear inertia/spring/damper system shown in Figure 6.2-2. How do the viscous damping constant, $c$, and the spring constant $k$ correspond to the control gains $k_d$ and $k_p$ in the PD controlled rigid body of Figure 6.2-1b?

c. Using the results of Lab 1, construct a model of the plant with four 500g mass pieces on the first mass carriage with no springs or damper attached. You may neglect friction.

**Proportional & Derivative Control Actions**

1. Show your tutor your answers to the preparation questions and discuss your understanding of the objectives of the lab with them.

2. Set-up the plant in the configuration described in Preparation Step c (i.e. four mass pieces on the first carriage). There should be no springs or damper connected to the first carriage and the other carriages should be secured away from the range of motion of the first carriage.

3. From Eq (6.2-3) determine the value of $k_p$ ($k_d=0$) so that the system behaves like a $\sqrt{2}$ Hz spring-mass oscillator.

4. Set-up to collect Encoder #1 and Commanded Position information via the Set-up Data Acquisition box in the Data menu. Set up a closed-loop step of 0 (zero) counts, dwell time $= 3000 \text{ ms}$, and 1 (one) rep (via Trajectory in the Command menu).

5. Enter the Control Algorithm box under Set-up and set $T_s=0.0042 \text{ s}$ and select Continuous Time Control. Select PID and Set-up Algorithm.
Enter the k_p value determined above for √2 Hz oscillation (k_d & k_i = 0, do not input values greater than k_p = 0.08^2) and select OK.

Place the first mass carriage at approximately the -0.5 cm (negative is toward the motor) mark.

In this and all future work, be sure to stay clear of the mechanism before doing the next step. Selecting Implement Algorithm immediately implements the specified controller; if there is an instability or large control signal^3, the plant may react violently. If the system appears stable after implementing the controller, first displace it with a light, non sharp object (e.g. a plastic ruler) to verify stability prior to touching plant

Select Implement Algorithm, then OK.

6. Enter Execute under Command. Prepare to manually displace the mass carriage roughly 2 cm. Select Run, displace the mass approximately 3 cm and release it. Do not hold the mass position for longer than about 1 second as this may cause the motor drive thermal protection to open the control loop.

7. Plot encoder #1 output. Determine the frequency of oscillation. What will happen when proportional gain, k_p, is doubled? Repeat Steps 5 & 6 and verify your prediction. (Again, for system stability, do not input values greater than k_p = 0.06). Show your tutor your result for natural frequency and discuss the impact of proportional gain on system performance.

8. Determine the value of the derivative gain, k_d, to achieve k_dk_hw = 50 N/(m/s).^4 Repeat Step 5, except input the above value for k_d and set k_p & k_i = 0. (Do not input values greater than k_d = 0.04).

9. After checking the system for stability by displacing it with a ruler, manually move the mass back and forth to feel the effect of viscous damping provided by k_d. Do not excessively coerce the mass as this will again cause the motor drive thermal protection to open the control loop.

10. Repeat Steps 8 & 9 for a value of k_d five times as large (again, k_d < 0.04). Can you feel the increased damping? Show your tutor your result.

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^2Here due to friction the system, which is ideally quasi-stable (characteristic roots on the jω axis), remains stable for small k_p. For larger values, the time delay associated with sampling may cause instability.

^3E.g. a large following error at the time of implementation.

^4For the discrete implementation you must divide the resulting value by T_s for the controller input value. Here, since the PD controller is improper, the backwards difference transformation: s = (1−z^−1)/T_s is used.
PD Control Design

11. From Eq's (6.2-3, -4) design controllers (i.e. find \( k_p \) & \( k_d \)) for a system natural frequencies \( \omega_n = 4 \text{ Hz} \), and three damping cases: 1) \( \zeta = 0.2 \) (under-damped), 2) \( \zeta = 1.0 \) (critically damped), 3) \( \zeta = 2.0 \) (over-damped).\(^5\) Show your tutor your resulting designs and discuss their impact on system performance.

Step Response

12. Implement the underdamped controller (via PI + Velocity Feedback) and set up a trajectory for a 2500 count closed-loop Step with 1000 ms dwell time and 1 rep.

13. Execute this trajectory and plot the commanded position and encoder position (Plot them both on the same vertical axis so that there is no graphical bias.)

14. Repeat Steps 12 & 13 for the critically damped and over-damped cases. Show your tutor the resulting graphs and explain the differences you notice in the response of the system. Save your plots for later comparison.

Adding Integral Action

15. Now compute \( k_i \) such that \( k_i k_{hw} = 7500 \text{ N/(m-sec)} \).\(^6\) Implement a controller with this value of \( k_i \) and the critically damped \( k_p \) & \( k_d \) parameters from Step 11. (Do not input \( k_i > 3.0 \))\(^7\). Be certain that the following error seen in the background window is within 20 counts prior to implementing.). Execute a 2500 count closed-loop step of 2000 ms duration (1 rep). Plot the encoder #1 response and commanded position.

16. Increase \( k_i \) by a factor of two, implement your controller (do not input \( k_i > 3.0 \)) and plot its step response. Manually displace the mass by roughly 5 mm. Can you feel the integral action increasing the restoring control force with time? (Do not hold for more than about 2 seconds to avoid excessive force build-up and hence triggering the motor thermal protection.) What happens when you let go? Show your tutor your final results and discuss the impact of the integral term on system performance.

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\(^5\)Recall that for discrete implementation, you must divide the \( k_d \) values by \( T_s \) for controller input.

\(^6\)For discrete implementation you must multiply the resulting value of \( k_i \) by \( T_s \) before inputting into the controller.

\(^7\)For discrete implementation, do not input \( k_i > 0.3 \times T_s \).
**Report**

Prepare a report outlining the findings of your lab. This should be prepared as a group and submitted along with your first assignment. Only one report needs to be submitted as long as you identify the apparatus you are using and all group members by name and SID. The report should describe the steps taken as outlined above showing the results from each step. Include plots of the output where appropriate. In addition, address the following questions as part of your report.

**Questions (to be answered as part of the Assignment 2):**

a. Provide your answers to the lab preparation questions included above.

b. What is the effect of the system hardware gain, $k_{hw}$, the mass, $m$, and the control gains, $k_p$ and $k_d$, on the natural frequency and damping ratio?

c. Describe the effects of natural frequency and damping ratio on the characteristic roots of Eq’s 6.2-1. Use an S-plane diagram in your answer to show the effect of changing $\zeta$ from 0 to $\infty$ for a given $\omega_n$.

d. Review the two step response plots obtained by adding integral action (Steps 15 & 16) with the previous critically damped and the critically damped plot ($k_i = 0$) of Step 14. What is the effect of the integral action on steady state error?

e. Static or Coulomb friction may be modeled as some disturbance force acting on the output as shown in Figure 6.2-3. Assume that this force to be step function\(^8\), and use the final value theorem to explain the effect of such a step on the PD controlled system with and without the addition of integral action.

f. How does integral action effect overshoot (again, compare with the critically damped plot of Step 14). Why?

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\(^8\) In practice, friction and its effect on system response are much more complex. This assumption however is valid in discussing the effect of the integrating term.