FRAMES & STRUCTURES

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Frame lect 015.doc

1. A safe prediction. It is quite common for an engineer from time to time to unexpectedly have to design a frame. That is a frame that is bolted or welded together. The challenge and fun of designing frames is fortunately not just reserved for the chosen few, but it can be trusted upon the unworthy new graduate just when he/she (or she/he) has found some comfort zone. Unfortunately designing frames can have its pitfalls, once we become aware of the features of a well designed frame we find that badly designed ones are very annoying. Worst of all, the most annoying frames will be the faulty ones that we have designed ourselves. Such frames are usually difficult to hide, they can stand out like bad works of art for everyone to cast aspersions upon. So be warned and take care.

2. What is meant by ‘frames design’ here. We will deal with just one side of such the design task, we will consider only the qualitative aspects of frame design. We will deal here with the layout, shape and relative sizes of the various parts of a frame. Quantitative aspects such as force analysis and the selection of the size and type of the elements to be used, together with how to join them together, will be left for later. After all a frame, like everything else in engineering, has to be invented before it can be analysed and improved (some simple minded people call this step ‘optimised’). Frame design must begin with good application of spatial principles, so here we will try to begin with that.

We will assume that our frames are pin jointed, although in reality there are very few pin jointed frames, so why bother? The assemblies that we will call frames are made up of relatively thin long beams, struts and columns, usually welded or firmly bolted at their joints. In our analysis we simplify their connections as being pin (2D) or ball (3D) joints, which hypothetically allows free rotation of the elements about the centre of their joints. Forces and reactions applied at nodes can only be transmitted through these joints as forces aligned with the long axis of the members, as moments cannot be transmitted. The fact that the joints are in practice welded or bolted solid, means that any deformation in the shape of a frame will be resisted, in part by the aligned forces in the members, and in part by bending moments from the rigid joints. Thus applying pin jointed analysis to a rigid jointed frame would predict higher tensile stresses than actually would be present, but some unquantified moments would also exit. In section 5 below we will look at ways of estimating the balance between aligned and bending loads.

This simple modelling provides a great deal of insight and leads to an understanding of more elaborate load bearing components. Components such as machine frames where there may be multiple loads and load paths, generating moments, shear and normal loads acting concurrently. We should also note that the frames discussed here are also kinematically speaking structures, which is of course they are rigid, if Kuztbach criterion is applied to such pin or ball jointed assembly a mobility of 0 or less would result. The removal of one or more elements may leave us with a mechanism, of mobility of 1 or more.

3. Triangulation in 2D. In planar applications such as a simple cantilever beam, a means of 'removing' the moment and its associated bending stresses is to ‘triangulate’ the beam. That is, we add a strut or brace either above or below the beam creating a triangle, which will transform the moment into two forces. Fig1 a) shows a dreamy cantilever beam with its reactions at the wall, while b)
shows a less dreamy horizontal beam with a vertical and angled brace. Elements in b) can be either in tension or compression depending on the direction of the load and the means of resisting it. The displacement at the end of the horizontal beam in a) is typically much larger than in b), consequently the latter is a more efficient means of providing a rigid structure.

4. Frames members with moments. Nearly always there will be intentionally some significant bending moments imposed on members of real frames. For practical reasons, even in space crafts, many loads are not applied just to the nodes of frames. Such frames are not necessarily tragedies or errors but may be just very practical solutions to a need. Fig 1 c) & d) show 2 possible variations of the basic simple triangulated frame of Fig 1 b). Equipment may have to be fitted within or very close to an element of a frame making it impractical to load a frame ideally at its nodes. Consequently the bending moment at the rigid joint of the upper horizontal beam in Fig 1 c) will be $f \times d$. In d) the bending moment in the upper horizontal beam, between its 2 pin joints will be the sum of the 2 moments created by the bodies shown. The simple consequence of a frame member being subjected to bending as well as aligned loads is that the member has to be of a size and section that will be able to carry the combined stresses.

5. Truss system versus cantilever beams. P Orlov has provided us with a number of illuminating examples of the advantages of frame systems over cantilever beams. Fig 2 shows a frame and a beam with an overhang of 1000 units, all from solid circular section. In a) the diameters of all components is 20. In b) the diameter of the beam is increased to give equal stress in the beam as in the frame members, and in c) the beam diameter is again increased to produce the same deflection in the beam as in the frame. In b) the diameter had to be increases by a factor of about 8, in c) by 11. The increase in the mass of beams, that give equal stress and deflection has to be the square of those ratios. This is the sort of penalty that is incurred if a component is subjected to a bending as opposed to purely tensile loads. On the other hand, a triangulated structure like that shown on Fig 2 occupies a great deal more space than the beams shown below it, and by necessity in many practical situations one may have to choose a beam over a frame because of space limitations.
Fig 2 d) shows the effect of varying the angle \( \alpha \) on the relative deflection of the beam to the frame \( (\delta_b/\delta_f) \). The upper curve relates to the diameters and frame lengths shown on figs 2 a), that is \( l/d = 1000/20 = 50 \). For angles somewhere between 45\(^\circ\) to 60\(^\circ\) the ratio of deflection of the cantilever beam to the frame is approaches 10\(^4\). Even for relatively stout members, of a proportions of 10/1 length to diameter, the ratio of deflections is about 300 times in favour of the frame. Fig 2 e) gives the ratios of stresses for the same variables. In more general circumstances where the forces may come from any direction designers seem to prefer a total included angle of about 60\(^\circ\) (ie \( \alpha \approx 30\(^\circ\) \)) building frames up with almost isosceles or equal sided triangles.

Stresses due to bending are unevenly distributed across the section of a component, whereas for aligned loads the stresses are uniform. Bending stresses increases proportionally with the length of the component whereas for tensile loads the stresses remain constant. In bending deflection increases with the third power of that length, for tensile loads the deflection is just proportional to the length. It can be said that structures subjected to aligned tension or compression are more rigid than comparable components in bending because better use is made of the material within them, or that the stresses are simply more evenly distributed.

6. Free standing self-supporting frames. Unless allowances have been made in the design and construction of a building, a frame should not rely on walls, floors or columns for reinforcement to its strength or rigidity. In nearly all situations any sizeable frame must itself provide the strength and stiffness required to maintain its shape and size, either while under load or while being transported. In modern buildings anchor points in walls and floors typically can only be relied upon to locate a frame, while it is subjected to its working loads, preventing the frame and its equipment from being moved inadvertently or possibly by vibrations.

There is a subset of frames that may not be required to be free-standing. Such frames may in their function rely on some other components to contribute to their 3D stability. They may function while being part of a chassis, building structure or another frame and rely on those other components to make a complete frame. In principle they are no different to our free-standing frames, except that some members may be a heavy casting, or weldment. For simplicity, we will ignore such partial frames.
7 Overall dimensions of a frame. Figure 3 a) below shows a load, represented by the arrow, and 2 locations where the reaction to that load are to take place. The dotted lines indicate some hypothetical rational limits within which a 2D frame can extend. At the outset we have to decide how big to make the frame within these limits. The frame may be looked upon as a single 2D component subjected to normal, shear and bending loads. We know that to reduce the stresses due to bending it is advantageous to use the maximum available height to create the largest second moment of area, about the axis about which the bending is to take place. That is an axis perpendicular to the paper. In 3D, frames often extend to the limits available to them to generate the largest second moment and polar moment of area to best deal with moments and torques. This principle also applies to structure made of panels and shells such as motor car bodies and airplane fuselages, where to provide the lightest and stiffest structure the shells are placed at the limits of the available space. That is, the greater part of the strength of an aircraft structure is actually provided by the skin of the fuselage and a car’s strength is provided by the outer body shell itself.

Depending on the importance of the moments and torques transmitted, a frame may in some areas expand to the limits of its available envelope, only to contract to just relatively strong nodes where the forces generating the moments and torques enter and exit the frame.

Any incremental increase in the size of the frame in the direction of that load, will disproportionally increase the second moment of area resisting that load. This may allow us to use lighter if not more complex frames. The advantage of making ever larger, more complex but lighter frames, will be reduced by the penalties of increase in cost, construction time, fragility of the structure and loss of space. The urge to have a more ideal design will be have to be balanced by these more practical consideration, depending on the relative importance of cost, mass, robustness and space for each application. Thus we end up with frames like locomotive and tractor chassis where high density and large mass is an advantage, aircraft engine frames and mobile crane jibs where low mass is desirable and costs and sizes may be far less of a problem. Exactly how far one design philosophy is driven or replaced by another is a based mostly on judgement, what is normal and what the competition is doing, seldom on analysis. Except in aerospace few companies may analyse the frame to the point of knowing where the trade-off should be.
8 Continuity in the layout of frame. In the example shown on Fig 3 a) we will presume that we can use all or any part of the available area if it is of advantage. In 3 b) a tentative perimeter frame, with equally spaced diagonals, is laid down with the intention of making a practical frame with members of equal lengths. Unfortunately these elements do not generate the necessary strong points, that is the nodes are not at the reaction points. In c), using unequal spacing the nodes made by the diagonals can be placed where the reactions have to be taken in the frame. In general, the greater the load taken at a node, the stronger the members and or the greater their number that should meet at that node. It is advantageous to use equally sized isoscelean triangles, but where this conflicts with other requirements we should consider the use more than one set of triangles, of different sizes.

9 Economy of elements. If Fig 3 c) represents a pin jointed frame, then the lower right and left corners do not contribute to the strength and stiffness of the frame. Most likely such a frame would be welded and the bottom corner joints would resist some moment, but as we said above that contribution would not be worth the extra construction. In d) the lower corner elements are removed. In the main we require that frames like any other mechanical component, should approach an ideal held in design, that they be everywhere equally stressed. To achieve that condition we can rearrange or invent new layouts and we can systematically remove the elements that are subjected to the lowest stresses and reinforce the more heavily stressed ones.

10 Lighter more complex frames In Fig 3 e) we reduce the length of elements, by adding diagonals, increasing their resistance to buckling. In d) we added cross bracing, providing a new degree of safety by providing alternate if not completely redundant load paths. In Fig 4 a), as one may expect, the bending moment created by the applied force increases with the distance from the force towards the location of the reactions. Consequently we can vary the height of the frame, varying its second moment of area, to reflect the magnitude of the moment. This procedure would result in the lower profile of this frame to approximate the shape of a parabola.

In b) the process applied in 4 a) has been carried out further, resulting in a more complex but potentially lighter frame. Shaping the frame as shown here in a) & b) is done only if the frame is large, produced in quantity and if a lighter frame is a major advantage. An example of this sort of frame is electrical power transmission towers, where the lower mass and transportation costs make their analysis and manufacture worth while.

11 Triangulation in 3D. A tetrahedron is the simplest spatial structure that will transmit a force acting at one node to aligned or tensile loads to the adjoining members. In the absence of knowing the locations of the loads and reactions the most appropriate structure is a regular tetrahedron, ie made up of equal sided triangles as seen on Fig 5. It is often said that the ideal space frame is made up of such tetrahedrons, and you should deviate from that only if you have to. Although some architects employ them to good effect, machinery designers find them hard to use. Fortunately irregular or distorted tetrahedron can be readily used and can function very effectively.
12 **Cubism.** The cube is a useful building block in the engineering world and the cube can be distorted or generalised to any six-sided prism and even down to the shape of a wedge. If a frame made of steel sections were welded up as shown on Fig 6 a), and were it subjected to loads, it would readily lozenge and twist. Should such a frame have ball joints it would be a mechanism and of course capable of being completely flattened or collapsed.

A regular minimal way to brace the cube is shown on Fig 6 b). This is a 3D example of the principles mentioned in section 8 & 9 above, of continuity and economy. The diagonals are extended continuously from one node to the other and the last diagonal meets the first. Figures c) & d) show that in so doing we have formed 5 tetrahedron. Shown in c) is the regular tetrahedron in the middle of the cube (1-2-3-4). d) Shows 2 of the 4 right-angle tetrahedrons at the corners of the cube. The simply braced cube of Fig 6 b) provides us with an elegant frame but the reinforced nodes, that is the strong points, may not be conveniently located, to suit where the loads and reactions take place.

Fig 7 a) shows an N braced cube, which can have the tendency to twist as the diagonals elongate under load. While b) shows X bracing, this double bracing provides security as well as uniformity of deformation under load. It is implied that there is similar bracing on the top and bottom of these cubes, but that has not been shown in these figures for the sake of clarity.
13 Prisms as building blocks. Fig 8 a) demonstrates M bracing; this has obviously relatively more but shorter elements, providing also a larger number of strong points. This bracing is preferred if the cube is relatively large (2 m or more per side) and/or if the cube is required to be particularly stable. Cubes as 8 a) and 6 b) may be stacked up to form a long columns or beams, to be used as building blocks where it is convenient to vary the height or length in increments. They commonly are seen as part of crane bases and gibs, 8 a) for the heavier duty cranes and 8 b), or some variation of it, for lighter ones.

14 Long prisms and openings. Should a relatively long prism be required, then to each of its side we may lay out bracing applying the principle of continuity and economy discussed above. It may not be easy to ensure that the nodes formed on one side meet the required nodes on adjacent sides. Depending on the importance of the nodes the spacing can be varied and double bracing may be used to ensure that sufficiently strong points appear at critical locations. Fig 9 a) shows a long prism on which areas A and B have to left open, possibly to allow access. Fig 9 b) shows 2 ways by which the ‘weakness’ brought about by these openings can be dealt with. At A deeper members are used, with a relatively large second moment of area about axes perpendicular to the plane of A. At B webs or short braces have been added to the corners. Reinforcing the corners reduces an effect which is akin to the beams hinging at the corners. Combining the high stress concentration at the corners with the large displacement caused by moments, the members making up an un-diagonalised face like B, will appear to hinge or rotate about the corners. A combination of deeper members and corner reinforcement is most effective.
3D frames  The principles that apply to 2D frames also of course apply in 3 dimensions. It is just a little more difficult, using flat sheets of paper to demonstrate the applications of those principles to 3D. To try to deal with this shortcoming of paper and ink, we will make use of an example that hopefully the reader will have some familiarity with, namely the frame for a 2 seater sport car.

Shown on figure 10 a) is a hypothetical envelope within which a simplified car frame has to be contained. Like we saw in section 10 and Fig 3, when dealing with 2 D frames, using the maximum extents of an allowable envelope can give us the lightest frame, for any required strength and stiffness. But we must take care that we not try to use more of this envelope than will be effective. If our frame filled the envelope we would have the same situation as described in Fig 3 c), that is there would be many elements of the frame would be relatively under-stressed.

Fig 10 a)

Fig 10 b) shows a simple but primitive frame that can be made for such a vehicle. It is really a flat 2D frame known as a ladder frame. The history of car design shows that this is exactly the sort of frame used in early cars. For simplicity in these examples we will just show the location of the front and rear wheels centrelines. We will not deal with the requirements of suspension and steering. Such frames as b) are weak in torsion and in bending, if there is a difference in the wheel loading from side to side, or lateral acceleration due to cornering, torsion will transmitted through the frame. This frame will readily twist, and will be subjected to relatively large torsional stresses. A related undesirable feature of such a frame is that the suspension designer cannot be sure where the contact patch of the tires will be, during any manoeuvre. This is because any load, such as from the manoeuvre itself or from road irregularities, will move the reference coordinates that has to be used in the design of the suspension. Modern trucks still use ladder frames like these, with planar reinforcements where engines and loads are located. For rigid chassis trucks approximately 30% of the wheel movements can be the result of chassis flex and 70% provided by the suspension. These chassis have been known to develop cracks, but their practical advantages make them an economic solution.
Fig 11 a) represents some of the early attempts to make competition car frames with more substantial 3D rigidity. Here the vertical loads due to gravity create bending moments that are greatest near the centre of the vehicle. The side view of such a frame looks a little like a bridge. These frames are good in bending but not at all good in torsion.

16 Tetrahedrons in 3D frames. Fig 11 b) & c) represent a solution to our frame requirements, for a simplified frame, rigid in bending and in torsion. In c) some of the lines representing the frame elements are shown dashed to highlight the fact that this frame is made up of irregular tetrahedrons. If one examines all the smallest voids bound by lines, one can see that Fig 11 b) & c) is made up of just 6 tetrahedrons, with 3 being mirror images of the other 3. The vertical rectangle at the middle of the frame, acts as the plane of symmetry about which the mirroring takes place.

This frame is a marvel of economy. It provides torsional and bending stiffness by increasing the depth of the structure just where the loads would be the greatest, but it applies our initial simplifications of ignoring the requirements of occupants and machinery a little too excessively.
Fig 15 at left a 1958 Lister Jaguar, a very flexible and weak 2D ladder frame as shown on Fig 10 above.

Fig 16 above is a rigid and strong 1960 birdcage Maserati, but an example of under-stressed welded frame as discussed in Fig 3 c) above.

Fig 17 shown below is the major components of a 1955 Mercedes 300SL, it is an example of a tetrahedral space frame, as shown on Fig 13 above. Here it meets the difficult space constraints of accommodating occupants, engine, drive train and other bits.
18, above, a rollover cage for a tractor with corner reinforcements.

19, right, coach frame, space frame below floor, undiagonalised open frame above. Laminated front window is a structural member.

20, below, locomotive frame where mass and density is desirable.
Mobile cranes, telescopic pedestal, of square cross section and triangular section jib.

Note the design of the bolted joint at the segment connections or ends.