MTRX4700: Experimental Robotics

Assignment 1

Note: This assignment contributes 10% towards your final mark. This assignment is due on Wednesday, March 28th during Week 4 before 4pm or via email prior to that time. Submit your report to the assignment box on the 3rd floor of the Mechanical Engineering Building or via WebCT. Late assignments will not be marked unless accompanied by a valid special consideration form. Plagiarism will be dealt with in accordance with the University of Sydney plagiarism policy. You must complete and submit the compliance statement available on the course website.

The objective of this assignment is to explore manipulator kinematics and dynamics. The MATLAB robotics toolbox developed by Peter Corke might be a useful aid. It can be downloaded from [http://petercorke.com/Robotics_Toolbox.html](http://petercorke.com/Robotics_Toolbox.html). Please read “robot.pdf,” especially Chapter 3, and run “rtdemo” to get familiar with the toolbox.

Total Marks: 100.

This assignment should take an average student 8-10 hours to complete.

The front page of your report should include:
- Your name and SID
- Your tutorial group number

1. (5 points) Calculate the homogeneous transformation matrix ${}^A_T B$, given the translations ($A_P B$) and the roll-pitch-yaw rotations (as $\alpha-\beta-\gamma$) applied in the order yaw, pitch, roll. (hint: the Homogeneous Transformations functions in the toolbox are useful for this. Try the Transformations module of rtdemo for a demonstration of these functions).
   a. $\alpha=10^\circ$, $\beta=20^\circ$, $\gamma=30^\circ$, $A_P B = \{1\ 2\ 3\}^T$
   b. $\alpha=10^\circ$, $\beta=30^\circ$, $\gamma=30^\circ$, $A_P B = \{3\ 0\ 0\}^T$
   c. Compare the output of: $\alpha=90^\circ$, $\beta=180^\circ$, $\gamma=-90^\circ$, $A_P B = \{0\ 0\ 1\}^T$ and $\alpha=90^\circ$, $\beta=180^\circ$, $\gamma=270^\circ$, $A_P B = \{0\ 0\ 1\}^T$

2. (15 points) Given the following 3x3 rotation matrices:
   
   $R_1 = \begin{bmatrix} 0.7500 & -0.4330 & -0.5000 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6250 & 0.2165 & 0.7500 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.7725 & -0.4460 & -0.5150 \\ 0.2165 & 0.8750 & -0.4330 \\ 0.6000 & 0.2078 & 0.7200 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 0.7500 & -0.2165 & -0.6250 \\ 0.4330 & -0.8750 & -0.2165 \\ 0.5000 & 0.4330 & -0.7500 \end{bmatrix}$

   a. Are these (within practical numerical limits) valid rotation matrices? Why?
   b. Determine the Roll, Pitch, and Yaw that define each matrix. Do you believe their values?
   c. If not, is it still possible to estimate values of Roll, Pitch, and Yaw? (if it is possible, please explain and do so. If not, please explain why).
3. (15 points) (a) For the robot shown in the following figure, find the table of DH parameters according to “Standard” DH conventions. (b) Additionally describe this in the “modified” DH convention (note: you are allowed to move the initial frame to fit convention(s), the final joint rotates the arm out of the page)

4. (10 points) Sketch the 5R robot corresponding to the “modified” DH convention given in the table.

<table>
<thead>
<tr>
<th>Link</th>
<th>$a_i$</th>
<th>$a_i$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>$-\pi/2$</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$\pi/2$</td>
<td>10.5</td>
<td>$\theta_5$</td>
</tr>
</tbody>
</table>

5. (10 points) Define, Draw and Drive an arm using the Robot toolbox. Try the following examples in the toolbox:

Define the manipulator

```
>> puma560
```

Draw it

```
>> plot(p560, qz)
```

Drive it

```
>> drivebot(p560)
```

Try Stanford manipulator

```
>> stanford
```

```
>> plot(stanf, qz)
```

```
>> drivebot(stanf)
```

Now define, draw and drive the robot arms in Questions 3 using the toolbox (or any package of your choice).
6. (10 points) The workspace of a manipulator describes the points in space which it
   can reach. (1) Plot both the workspace and the configuration space of the 3-link
   planar arm in the following figure with the following specifications. (2) What are
   the singularities for this robot?
   
   **(HINT:** solve the forward kinematics for the manipulator for a series of values
   of the joint variables. Each solution corresponds to a point in the workspace).

   \[ L_1 = 4 \text{ feet}, \]
   \[ L_2 = 3 \text{ feet}, \]
   \[ L_3 = 2 \text{ feet}, \]
   \[ -\pi/3 \leq \theta_1 \leq \pi/3 \]
   \[ -2\pi/3 \leq \theta_2 \leq 2\pi/3 \]
   \[ -\pi/2 \leq \theta_3 \leq \pi/2 \]

7. (15 points) Derive the forward and inverse kinematics solutions for this robot
   (same as question 6, planar RRR with \( L_1 = 4 \text{}', L_2 = 3 \text{'}, L_3 = 2 \text{'} \)). Use this to calculate
   all possible inverse kinematic solutions \( \{\theta_1, \theta_2, \theta_3\} \) for the manipulator poses
   shown below. Analytic and geometric solutions are acceptable.

   a. \( ^0 \text{H}_{\theta} = \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)

   b. \( ^0 \text{H}_{\theta} = \begin{bmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)
8. (20 points) Consider the two link manipulator below in both a vertical and horizontal configuration (relative to gravity).

(a) vertical configuration
\[ l_1 = 1 \text{ m} \]
\[ l_2 = 0.75 \text{ m} \]
\[ m_1 = 5 \text{ kg} \]
\[ m_2 = 4 \text{ kg} \]
\[ I_1 = 0.4 \text{ kgm}^2 \]
\[ I_2 = 0.2 \text{ kgm}^2 \]
\[ T_1 = \text{torque at joint 1} \]
\[ T_2 = \text{torque at joint 2} \]
\[ \theta_1 = \text{joint angle 1} \]
\[ \theta_2 = \text{joint angle 2} \]

(b) horizontal configuration (gravity into page)
\[ l_2 = 0.75 \text{ m} \]
\[ m_1 = 5 \text{ kg} \]
\[ m_2 = 4 \text{ kg} \]
\[ I_2 = 0.2 \text{ kgm}^2 \]
\[ F = \text{force applied to cart} \]
\[ x_1 = \text{position of cart} \]
\[ \theta_2 = \text{joint angle 2} \]

(a) vertical configuration

a. Derive the equations of motion that describe the system dynamics (hint: the dynamics for very similar systems were shown in the handout distributed during the lecture).

b. Create a simulation of the system. Describe the implementation of the system and show how your system performs for a few select inputs. (hint: focuses on being able to calculate (or predict) the motion and torques required)

c. Describe how you might go about selecting appropriate actuators (it need not be a DC motor) for your system. (1) If the distal joints are actuated, either include the mass of the actuator or explain how motion will get transmitted. (2) Consider the torques or forces required to statically support the system. (3) What is required move a load with reasonable performance (i.e., travel time).

d. For a given design, what is the maximum load the machine can move in each (i.e., vertical and horizontal) configuration?

e. These systems will require some sort of controller to be useful. Augment your vertical arm simulator with a controller. Show the implementation of your controller and its performance in meeting the control objectives.