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THE UNIVERSITY OF SYDNEY

FAMILY NAME ........................................................................................................
GIVEN NAMES ........................................................................................................
STUDENT NUMBER ................................................................................................

FACULTY OF ENGINEERING
MTRX4700
Experimental Robotics
Semester 1, 2009
Time allowed: 3 hours

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No copy of this paper is to be removed from the examination room by candidates or supervisors, nor may any portion of the paper be copied. All copies must be returned to the examiner.

There are four (4) questions in this exam. All questions count towards your final mark. These questions are to be completed in the ANSWER BOOK PROVIDED. Plots for Question 4 are provided in this exam booklet and are to be returned with the answer book and exam booklet at the end of the exam period. Clearly label which question you are answering.

The total marks for this exam is 100. This exam is worth 30% of the final mark for this Unit of Study. Be sure to read through the exam in its entirety before starting to work on the solutions.

This is an open book exam.
This exam booklet is to be handed back with your answer book.
Non-Programmable calculators are allowed.
Section A – Kinematics [20 marks]

1. Rotation Matrices [10 marks]

In order to demonstrate your understanding of general rotations in 3D Cartesian space, please provide short answers to the following questions.

a. Given unit frames \( \{A\} (a_x, a_y, a_z) \) and \( \{B\} (b_x, b_y, b_z) \) the orientation between them can be described as a rotation matrix between them written in terms of the dot products of the elements. What are the entries of this matrix?

\[
{\mathbf{R}} = \begin{bmatrix}
   a_x \cdot b_x & & \\
   a_y \cdot b_y & & \\
   a_z \cdot b_z & & \\
\end{bmatrix}
\]

b. List 3 of the six possible lower-pair joints (for rigid-body motion)
1. ____________________________
2. ____________________________
3. ____________________________

c. If \( x = _b^a R \), then \( x^T \) is? __________

d. What is the rotation matrix for **counter-clockwise** rotation, \( \theta \), of a right-handed frame about the third principal axis (i.e., the \( a_z \) axis of the \( a_x, a_y, a_z \) frame given above)?

\[
R_z (\theta) = \begin{bmatrix}
   & & \\
   & & \\
   & & \\
\end{bmatrix}
\]

e. What is the rotation matrix for the above case for a **clockwise** rotation \( \alpha \)?

\[
R_z (\alpha) = \begin{bmatrix}
   & & \\
   & & \\
   & & \\
\end{bmatrix}
\]
2. 3R Planar Manipulator [10 marks]
   Team Omega-Cross-R is building a planar manipulator to pick up a **spherical marble**.

   Its dimensions are:
   \( L_1 = 1.5 \text{ m} \), \( L_2 = 1.0 \text{ m} \), \( L_3 = 0.5 \text{ m} \),

   a. Compute the homogeneous transformation matrix from the base to the hand (end effector) (i.e., \( ^0H^T \)) as a function of the joint angles \( \theta_1 \), \( \theta_2 \), and \( \theta_3 \)?

   b. What is the determinant of the Jacobian? (Hint: does this robot have redundancy?)

   c. For the simplified case of \( \theta_3 = 0 \), calculate the Jacobian

   d. For the simplified case of \( \theta_3 = 0 \), list the singular configurations of the robot.
Section B – Sensing [25 marks]

3. In order to demonstrate your understanding of the sensing systems we have studied in this course, provide short answers to the following questions.
   a. Line Fit: The following diagram shows a number of points collected by a laser scanner. Although finding the equations for the ‘lines’ in this diagram is simple by inspection, show how the line splitting algorithm we examined in Assignment 2 can be used to first identify the lines and then solve for the least squares error line fit for the two lines. [10 marks]

   ![Diagram showing line fit](image)

   b. Hough Transform: The followings diagram shows a number of points collected by a laser scanner. Although finding the equations for the ‘lines’ in this diagram is simple by inspection, use the Hough transform in the (m, b) space associated with the line equations \( y = mx + b \) to determine the equation for the lines represented by the black points. [10 marks]
Visual Image Processing: Matching features between images is important for many image processing tasks, including stereo reconstruction, data association and structure from motion. We examined a number of methods for matching feature templates in images. These are similar to performing an image convolution between image patches. Examples of these matching functions include Sum of Squared Difference (SSD) and Sum of Absolute Difference (SAD). These are defined as:

\[
SSD: d(u, v) = \sum_{x,y} (f(x,y) - t(x - y, y - v))^2
\]

\[
SAD: d(u, v) = \sum_{x,y} |f(x,y) - t(x - y, y - v)|
\]
where \( f \) is the image, \( t \) is the template and summation is over the positions \( x,y \) under the template positioned at \( u, v \).

For the image template in the following, compute the SSD and SAD when compared against the 3 image patches. Which patch is the most likely?

<table>
<thead>
<tr>
<th>Template</th>
<th>Candidates</th>
<th>SSD</th>
<th>SAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 255</td>
<td>0 0 255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 255</td>
<td>0 0 255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>255 255 255</td>
<td>0 0 255</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| | 0 0 255 | 0 0 255 | 255 255 255 |
| | 255 255 255 |

| | 0 0 0 | 0 0 0 | 255 255 255 |
| | 0 0 0 |

| | 255 255 255 |
Section C – Navigation and Data Fusion [25 marks]

4. A vehicle model describes the evolution of the pose of a vehicle as a function of time. For an Autonomous Underwater Vehicle, we can model the vehicle’s lateral motion using a simplified model as shown here. A vectored thruster exerting a thrust $T$ at a steer angle $\theta$ is used to control the vehicle.

\[
\dot{\psi} = -\frac{TD}{J}\sin(\theta)
\]

\[
\dot{V} = \frac{1}{m}\left(T\cos(\theta) - 0.5\rho C_d AV|V|\right)
\]

\[
\dot{x} = V\cos(\psi) + v_x
\]

\[
\dot{y} = V\sin(\psi) + v_y
\]

Assuming we have a vehicle with specifications outlined in Table 1 that behaves according to this vehicle model, answer the following questions.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>500kg</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>500 kg m²</td>
</tr>
<tr>
<td>Max. Thruster propulsion</td>
<td>100N</td>
</tr>
<tr>
<td>Max. Thruster Steering Angle</td>
<td>+/- 45deg</td>
</tr>
<tr>
<td>Water Density (sea water) ($\rho$)</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>Wetted Surface Area (A)</td>
<td>1.0m²</td>
</tr>
<tr>
<td>Drag coefficient ($C_d$)</td>
<td>0.025</td>
</tr>
<tr>
<td>Distance to centre of thrust (D)</td>
<td>1.0m</td>
</tr>
</tbody>
</table>

a. If the vehicle starts at the origin of some frame of reference, with initial heading of 0° (North in a North-East-Down coordinate frame) with zero initial velocity and turn rate, what will our estimate of its position be at time $T=100s$ if we are given the following series of control inputs $T$ and steer angle $\theta$? [10 marks]

<table>
<thead>
<tr>
<th>Timestamp (s)</th>
<th>Thrust T (N)</th>
<th>Steer angle $\theta$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20.0</td>
<td>50.0</td>
<td>0.0</td>
</tr>
<tr>
<td>30.0</td>
<td>50.0</td>
<td>0.0</td>
</tr>
<tr>
<td>40.0</td>
<td>50.0</td>
<td>$-\pi/80$</td>
</tr>
<tr>
<td>50.0</td>
<td>50.0</td>
<td>$-\pi/80$</td>
</tr>
<tr>
<td>60.0</td>
<td>50.0</td>
<td>$\pi/80$</td>
</tr>
<tr>
<td>70.0</td>
<td>50.0</td>
<td>$\pi/80$</td>
</tr>
<tr>
<td>80.0</td>
<td>50.0</td>
<td>0.0</td>
</tr>
<tr>
<td>90.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>100.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
b. At time $T=100s$ we receive two range observations from acoustic transponders located at positions $(0, 0)$ and $(500, 0)$ in the survey area [10 marks].

<table>
<thead>
<tr>
<th>Transponder</th>
<th>Position (m)</th>
<th>Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0, 0)</td>
<td>115</td>
</tr>
<tr>
<td>2</td>
<td>(500, 0)</td>
<td>415</td>
</tr>
</tbody>
</table>

What are the possible locations of the AUV if we assume that there is no error in these measurements? [10 marks]

c. Finally, given the observation from the two navigation beacons, we would like to fuse this with the current estimate of the vehicle position. Using the low pass filter fusion equations

$$
\begin{bmatrix}
  x(k | k) \\
  y(k | k)
\end{bmatrix} = (1 - \alpha_p) \begin{bmatrix}
  x(k | k - 1) \\
  y(k | k - 1)
\end{bmatrix} + \alpha_p \begin{bmatrix}
  x_{\text{obs}}(k) \\
  y_{\text{obs}}(k)
\end{bmatrix}
$$

with filter parameter $\alpha_p=0.1$, what is the resulting best estimate of the vehicle’s pose? Use the position observation closest to your current estimate of the vehicle’s location. [5 marks]
Section D – Planning [30 marks]

5. In order to demonstrate your understanding of planning methods we have studied in this course, provide answers to the following questions.

a. **Configuration Space.** The following diagram depicts a polygonal robot and polygonal obstacles in a bounded two-dimensional workspace. On the next page, draw the configuration space obstacles with respect to the designated robot reference point. [10 marks]
b. **Visibility Graph**: Given the following start point, goal point, and configuration space obstacles, draw the full visibility graph and show the shortest path. [10 marks]
c. **Trapezoidal Decomposition**: Now draw the trapezoidal decomposition for the same example. [10 marks]

There are no more questions.