**AREA MOMENT OF INERTIA**

Problem 3:

Determine the moment of inertia of the shaded area about the x-axis:

![Diagram of shaded area](image)

**Solution:**

First way: Double integration (choosing to integrate with respect to x first):

\[
\begin{align*}
\frac{dA}{dA} &= dxdy \\
I_x &= \int_A y^2 dA \\
I_x &= \int_0^a \int_0^{ky^2} y^2 dxdy = \int_0^a y^2 (b - ky^2) dy = \frac{2}{15} a^3 b
\end{align*}
\]

Since when \( y = 0, k = 0 \) and when \( y = a, k = b/a^2 \)

\[
I_x = \frac{2}{15} a^3 b
\]
Second way: Double integration (choosing to integrate with respect to y first):

\[ dA = dx\,dy \]
\[ I_x = \int_A y^2\,dA \]
\[ I_x = \int_0^b \int_0^{\sqrt[k]{3k^2}} y^2\,dy\,dx = \int_0^b \frac{1}{3k^{3/2}} = \frac{2}{15}a^3b \]
\[ I_x = \frac{2}{15}a^3b \]

Third way (Single integration – Finite length strip):
Case 1: Horizontal strip parallel to the x-axis:

All parts of the differential area element are the same distance from the x-axis

\[ I_s = \int y^2\,dA = \int_0^a y^2\,(b - ky^2)\,dy = \frac{2}{15}a^3b \]

Case 2: Vertical strip perpendicular to the x-axis:

Since all parts of the element area are not at the same distance from the x-axis, we find the moment of inertia by considering the differential area about the x-axis:

\[ d(A) = \frac{1}{12}(dx)\,y^3 + y(dx)\left(\frac{y}{2}\right)^2 = \frac{1}{3}(dx)\,y^3 \]
\[ I_x = \int d(I_x) = \int \frac{1}{3}\,y^3\,dx = \int_0^b \frac{y^{3/2}}{k^{3/2}}\,dx = \frac{2}{15}a^3b \]