

## Non-Newtonian flow behaviour in particulate suspensions under magnetic fields

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### Abstract

Magneto-rheological fluids are particulate suspensions which show a dramatic increase in viscosity when placed under an external magnetic field. This is due to the magneto-static interaction forces induced between the particles by the external field, which lead to the formation of elongated aggregates of particles aligned in the field direction. The highly non-Newtonian flow behaviour of these fluids can be represented by the Bingham fluid model, where there is solid-like behaviour with a field-dependent yield stress. We have experimentally examined the small strain behaviour of this solid-like, pre-yielded state, and observed a linear, elastic stress-strain response up to the yield point. The corresponding modulus  $G_0$  was found to depend on the magnetic flux density  $B$  as  $G_0 \sim B^{0.7}$ . A theoretical calculation of this modulus was also performed, based on the interacting dipole approach.

### Introduction

A magneto-rheological suspension (MRS) consists of polarisable particles dispersed in a non-polarisable carrier fluid, and displays a dramatic increase in flow resistance under a magnetic field. The basic mechanism is that the field induces a magnetic dipole within each particle, and the resulting interaction forces between particles lead to the formation of elongated aggregates which increase the viscosity (Fig. 1). There are several recent review papers available [1-3]. It should be noted there is also an analogous suspension system, electro-rheological fluid (ERF), whereby an external electric field induces electrostatic polarization of each particle, the interparticle forces again leading to aggregates and an increased viscosity.

These materials are currently the object of intense research attention, as well as considerable applications development work in industry (eg tunable vibration damping systems, based on the adjustable flow properties). A key outstanding issue in our fundamental understanding of these systems is a theoretical constitutive framework within which predictions can be made about the flow behaviour under arbitrary field and flow conditions. Careful measurements are required of the rheological response in closely controlled flow conditions, and this is the contribution of the present work.

We have carried out a series of rheological tests on MRS using laboratory prepared magneto-rheological samples and a modified controlled stress rheometer, with particular focus on the response at the start-up of shear flow. The following sections will describe the materials and apparatus used, and then present the results and discussion.

### Experimental

#### Material

The particles were carbonyl-iron powder (average particle size 4 $\mu$ m), manufactured by ISP Corp (grade S3700). These were dispersed in a 0.1Pas silicone oil (Dow Corning). The volume fraction was 30%. The particles were mixed thoroughly, and prior to any testing the bottle was vigorously shaken to re-

disperse the system. Inspection of the samples after the tests showed minimal clumping of particles.

### Equipment

The Paar Physica MCR300 rheometer was used in the parallel plate mode (diameter 20mm, typically 1mm gap), as illustrated in Fig. 2. The rheometer has been modified so that a uniform magnetic field can be applied to the sample (perpendicular to the plates) which is produced by an electromagnet under the bottom plate. This consists of a coil of 495 windings, through which is passed a DC current of up to 2A. The test cell is enclosed in a chamber comprising of iron elements, which redirect the magnetic field to the bottom stage, thus generating a magnetic circuit.

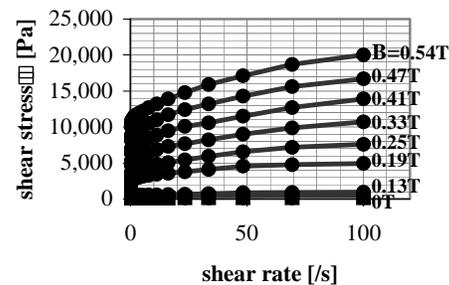


Figure 1. Flow curves for a typical magneto-rheological suspension, consisting of carbonyl-iron particles dispersed in silicone oil (volume fraction 30%). Each curve corresponds to a particular magnetic field strength  $B$  – magnitudes indicated on graph (units : Teslas).

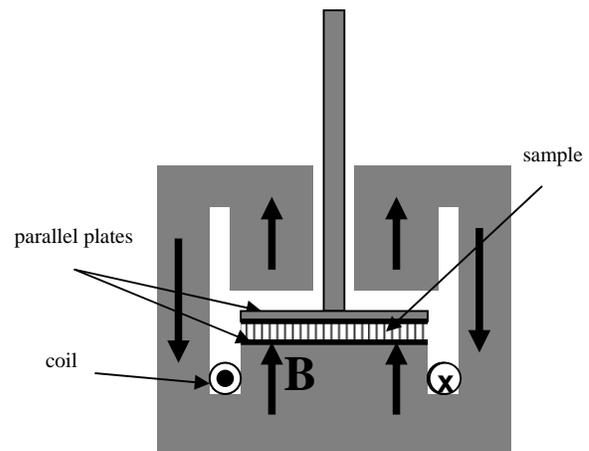


Figure 2. Schematic of the magneto-rheological test cell. The sample is sandwiched between the parallel circular plates, with a magnetic flux ( $B$ ) applied normally to the plates. The upper disk is rotated, inducing shear flow in the sample.

## Procedure

Prior to the commencement of the tests below, the constant magnetic field was applied to the sample for 5 minutes (with no deformation) to enable the particulate columnar microstructure to fully develop. The following tests were performed :

- (i) Steady shear flow – shear rates up to  $100\text{s}^{-1}$  (under constant magnetic field).
- (ii) Transient response after shear start-up under slow constant shearing (yielding behaviour). Typical shear rates :  $\sim 0.01$  to  $\sim 1\text{ s}^{-1}$ .

## Results and Discussion

The steady shear results are shown in Fig 1. These are similar to previous reports in the literature [5-7], and it is confirmed that the sample does indeed show the magnetorheological response. The presence of a yield stress, with a magnitude dependent on the magnetic field strength, is apparent.

A typical result from the shear start-up tests is illustrated in Fig. 3, for the case of applied shear rate of  $0.01\text{s}^{-1}$  and constant magnetic flux density of  $0.47\text{T}$ . It is seen that as the strain is increased, there is an elastic-type deformation at low strains (due to the tilting and deformation of the particle columns, spanning the two rheometer plates – Fig. 6). However, as the strain reaches a critical value (approximately 0.3 in the case of Fig. 3), the stress begins to plateau, eventually reaching a steady state value (corresponding to steady shear behaviour). At this stage the microstructural deformation process has likely reached a steady state, involving the rupture and re-formation of the chains or columns under the constant shearing.

Similar curves were obtained at different shear rates over the range  $0.01\text{ s}^{-1}$  to  $10\text{s}^{-1}$ . Under small strains, the shear stress  $\tau$  was

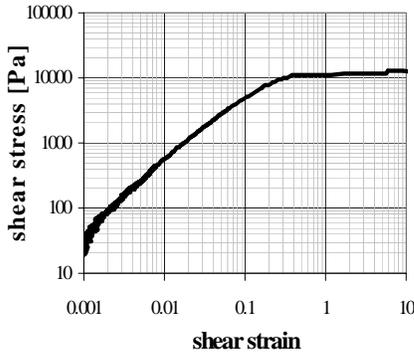


Figure 3. A typical shear start-up test, with stress plotted as a function of strain. The shear rate was  $0.01\text{s}^{-1}$  and the magnetic field  $0.47\text{T}$ .

found to increase linearly with strain  $\gamma$  : that is, the exponent  $\alpha$  in  $\tau \sim \gamma^\alpha$  was found to lie in the range 0.95 to 1.05 for all tests carried out. Therefore, we can use a modulus of elasticity  $G_0$  to describe the linear response at small strains, defined as follows :  $G_0 = \tau / \gamma$ . The variation of  $G_0$  with magnetic flux density  $B$ , evaluated at a strain of 0.01, is plotted in Fig. 4. Fig. 4 shows that  $G_0$  increases with magnetic flux density  $B$ . This result is expected since the interparticle interaction forces become stronger as  $B$  is raised. The exponent  $b$  in the relationship  $G_0 \sim B^b$  was found to be 0.70. Following is a theoretical treatment of this solid-like deformation of MRS under small strains, to examine the origin of this exponent of 0.70.

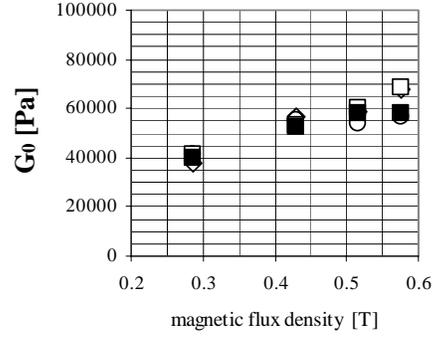


Figure 4. Variation with magnetic flux density  $B$  of modulus  $G_0$  which characterizes the initial solid-like response in the shear start-up curves. Measurements performed under different shear rates are shown: shear rate of  $0.01\text{ s}^{-1}$  (open circles),  $0.1\text{ s}^{-1}$  (open diamonds),  $1\text{ s}^{-1}$  (open squares) and  $10\text{ s}^{-1}$  (filled squares).

## Theory

The starting point for the theoretical modelling is the magnetostatic interaction force between two spherical particles of radius  $a$  under a magnetic flux density of  $B$ , as illustrated in Fig. 5.

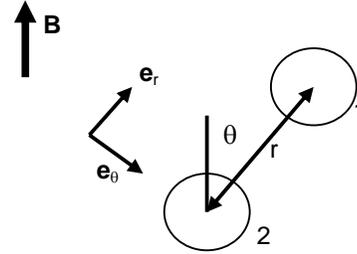


Figure 5. Two polarised spherical particles, under the magnetic flux density  $B$ .

The angle  $\theta$  is measured from the field direction, and  $r$  is the centre-to-centre separation. Using the unit vectors  $\mathbf{e}_r$  and  $\mathbf{e}_\theta$  as shown, the interaction force experienced by sphere 2 due to sphere 1 ( $\mathbf{F}_{12}$ ) is given by

$$\mathbf{F}_{12} = k P^2 r^{-4} [ f_1(\theta)\mathbf{e}_r + f_2(\theta)\mathbf{e}_\theta ] \quad (1)$$

$P$  is the strength of the electric dipole induced in each particle (ie the magnitude of the particle's polarisation, assumed constant), and is given by  $P = 4\pi a^3 \beta B$  where  $\beta = (\mu_p - \mu_c) / [\mu_0 \mu_c (\mu_p + 2\mu_c)]$  and  $\mu_p$  and  $\mu_c$  are the magnetic permeabilities of the particle material and the carrier fluid respectively. The functions  $f_1(\theta)$  and  $f_2(\theta)$  describe the orientational dependence of the interaction force and are given by  $f_1(\theta) = 3\cos^2\theta - 1$  and  $f_2(\theta) = \sin 2\theta$ . The constant  $k$  is given by  $k = (\mu_0 \mu_c) / 4\pi$  where  $\mu_0$  is the vacuum permeability.

The force given by eq(1) will act to attract particles which are aligned parallel with the field direction. The particles will be repelled if they are placed side-to-side, perpendicular to the field

direction. This model is often referred to as the “point dipole model”.

We now turn to the case where we have a single-width chain of particles, with the particles held to their neighbours by the interaction force ( $F_{12}$  - eq(1)). As illustrated in Fig. 6, this chain is subject to a shear strain through application of a horizontal shearing force  $F$ .

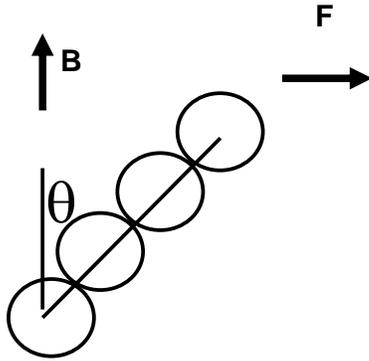


Figure 6. Chain of particles held together by the electrostatic interaction force (eq(1)). A shearing force  $F$  is applied to the chain, which is assumed to deform affinely as shown for small strains.

For small values of shearing strain (ie small values of  $\theta$ ) it is possible to analytically calculate the force-strain relationship. This enables us to obtain the elastic modulus, if we assume that the system consists of similarly strained chains connecting the plates of the rheometer, and which do not interact with each other. (isolated chains). Writing  $\phi$  for the volume fraction of the particles, we obtain the following theoretical result for the modulus  $G_0$ :

$$G_0 = 3/(16\pi) \phi k P^2 / a^6 \quad (2)$$

Substituting typical values for the experimental parameters we find that eq(2) predicts  $G_0$  to be of the order of  $10^4$  Pa under a 0.5 T magnetic flux density. This result underestimates the magnitude of the modulus (cf Fig. 4). In addition, the quadratic dependence on  $P$  (the dipole strength of each particle) implies a quadratic dependence on the flux density  $B$ . This is different to the  $\beta=0.7$  exponent observed experimentally (see Fig 4 and discussion thereof). Even a more sophisticated model, accounting for magnetic saturation effects in the region of the interparticle gap, predicts an exponent of  $\beta=1.5$  [8].

Clearly, the exponent in the experiments (0.70) is much lower than the exponents predicted by these theories. The most likely explanation for this is that the particles may have gone into the magnetic saturation regime under the fields used—in fact this would be more likely to happen with concentrated suspensions as used here. Indeed, magnetisation measurements by Martin and co-workers [9] on a similar particulate system (but at a lower volume fraction of 0.028) indicate that under 0.2 T fields the material is already heavily in saturation (Fig. 12 of that paper). Another factor leading to the discrepancy between the exponents may be that interactions other than field-induced forces, such as excluded volume interactions between the particles as they rearrange themselves within the aggregates, are also contributing to the solid-like mechanical resistance to shearing at the small strains. Indeed, it has been observed microscopically [1,2] that, rather than single-width chains (as we have modelled – Fig. 6),

the particles tend to arrange themselves into thicker columnar structures aligned in the field direction.

## Conclusions

Magneto-rheological fluids show a distinctive, Bingham plastic-type response under magnetic fields, with the yield stress depending on the field strength. We have examined the small strain behaviour of the pre-yielded, solid-like material. We observed a linear stress-strain response up to the yield point, with the corresponding ‘elastic’ modulus  $G_0$  depending on the magnetic flux density  $B$  as  $G_0 \sim B^{0.7}$ . This exponent was significantly less than those obtained from theoretical calculations based on the single-width chain model, indicating that interactions other than the induced magnetostatic dipole-dipole forces may be playing a role in the mechanical resistance of the particle aggregates to an applied shear strain.

## Acknowledgments

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