

Laminar Flow in a Periodic Serpentine Channel

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Abstract

This paper presents a study of laminar flow in serpentine channels with periodically repeating elements, with application to heat transfer passages within complex compact heat exchangers. A methodology has been developed to determine the fully-developed flow and heat transfer behaviour in such channels with a constant wall heat flux using Computational Fluid Dynamics (CFD). Utilising this approach, flow characteristics are investigated for a fixed geometry, with $L/d = 4.5$ and $R_c/d = 1$, for a range of Reynolds numbers up to 200. Pressure loss and heat transfer within a repeating module is compared with that expected from fully-developed flow in a straight pipe of equivalent path length. Dean vortices are generated in the flow and their rotation sense and intensity are dependent on the bend direction and Reynolds number. They are shown to suppress recirculation around bends and to be responsible for the high heat transfer rate with a relatively low pressure drop. The effect of the length to diameter ratio (L/d) on pressure loss and heat transfer performance is also reported for a Reynolds number of 110, with $R_c/d = 1$.

Introduction

This paper describes a method to study fully-developed flow in geometries that are periodic in the flow direction. The geometry under examination is a serpentine duct of circular cross-section with a constant heat flux applied at the walls. A serpentine channel geometry, as shown in Figure 1, has been studied as it is representative of the channel geometries used in complex compact heat exchangers. The work presented in this paper focuses on flow characteristics, with some comments also on heat transfer.

The definition of serpentine channels follows the work of Liu *et al.* [8]. The channel consists of a number of repeating modules that are periodic in nature. We analyse the fully-developed flow in this system, as this is analogous to a heat exchanger passage, consisting of many modules in series.

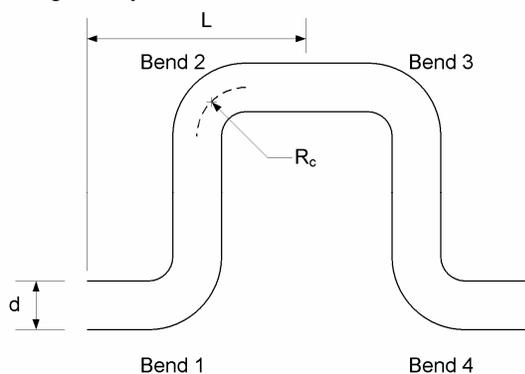


Figure 1: Repeating module of the serpentine geometry. Non-dimensional geometrical parameters of interest are L/d and R_c/d .

Relevant Literature

A method to study fully-developed flow and heat transfer in channels with periodically varying shape was first developed by Patankar *et al.* [11] for the analysis of an offset-plate fin heat

exchanger. Their method takes advantage of the repeating nature of the flow field to minimise the extent of the computational domain. The method of Patankar *et al.* [11] assumes that for a periodic geometry, the flow is periodic with a prescribed linear pressure gradient being applied to drive the flow. The outlet velocity field and its gradient are wrapped to the inlet to produce periodic boundary conditions. Flow velocities within the geometry are then calculated using momentum and mass conservation equations, assuming constant fluid properties.

Webb and Ramadhyani [16] and Park *et al.* [10] analysed fully-developed flow and heat transfer in periodic geometries following the method of Patankar *et al.* [11]. Webb and Ramadhyani [16] studied parallel plate channels with transverse ribs; they presented a comparison with the performance of a straight channel, and reported an increase in both the heat transfer rate and pressure drop as the Reynolds number is increased. Park *et al.* [10] incorporated optimisation of the heat transfer rate and pressure drop into their study of the flow and thermal fields of plate heat exchangers with staggered pin arrays.

A significant amount of research has focussed both on channels with internal obstructions and tortuous channels. The rationale behind this work has been to determine the configurations that lead to the most vigorous mixing and highest rates of heat transfer. Examples of research into such wavy channels are provided by Popiel and van der Merwe [12] and Popiel and Wojtkowiak [13] who studied experimental pressure drops for geometries with an undulating sinusoidal shape or U-bend configuration. In these papers, the effects of Reynolds number, curvature, wavelength and amplitude on the friction factor were investigated in laminar and low-Reynolds-number turbulent flow. An interesting observation made by these authors is that when the friction factor is plotted against the Reynolds number, there is either no definite transition from laminar to turbulent flow, or a delayed transition relative to that of a straight pipe. This is also reflected in the work of Johnston and Haynes [6], whose results demonstrate a smooth transition from laminar to turbulent flow in plots of both friction factor and Colburn j -factor versus Reynolds number. It is hypothesised by Popiel and van der Merwe [12] that a smooth transition to turbulence occurs due to the secondary flows produced within the complex geometry. Dean [3] originally observed that the mixing effects of these secondary flows are steadily replaced by the development of turbulent secondary flow. This effect of secondary flows is also discussed in the work of Chen *et al.* [2].

Shah and London [14] investigated laminar flow forced convection in ducts of various cross-sectional shapes. Many other researchers have investigated channels or ducts with undulating or corrugated surfaces, including the work of Fabbri [4], Greiner *et al.* [5] and Tauscher and Mayinger [15].

Modelling Methodology

The work of Patankar *et al.* [11] stands out as the formative reference for the calculation of fully-developed velocity and temperature fields in two-dimensional streamwise-periodic geometries. Here we describe a method to calculate fully-developed flow and heat transfer in a broader range of geometries

than that considered by [11], such as the serpentine channel, where the flow is not aligned with the streamwise coordinate. We use ANSYS CFX-5.7, a finite-volume code that solves the Navier-Stokes equations using a coupled solver. All calculations are performed using a second order bounded differencing scheme for the convective terms. The method has been developed for steady, laminar, incompressible, single phase flow of a Newtonian fluid with constant physical properties.

In order to model the periodic behaviour computationally, one of two methods may be adopted. The first method is similar to that of Patankar *et al.* [11], where the components of velocity and their gradients at the outlet of the geometry are re-applied to the inlet ('wrapping'). This requires the use of periodic boundary conditions for the flow variables and temperature, coupled with the ability to prescribe the mass flow rate and to wrap a normalised temperature profile. This option is not currently available in ANSYS CFX-5.7.

A second method to achieve a fully-developed periodic solution forms the basis of the methodology presented in this paper. The flow variables (u, v, w, T) at the outlet are reapplied to the inlet without any corresponding gradient information. For the case of constant wall heat flux, outlet temperatures are scaled to yield the desired bulk mean temperature at the inlet, and then applied. The solution then approaches the fully-developed state iteratively, but is never perfectly developed because the gradients are not wrapped. To overcome this, a sufficient number of repeating units of geometry need to be included in order to ensure that the flow is fully-developed in the unit of interest.

For computational efficiency and accuracy, a structured mesh has been utilised in the discretisation of the model domain. This discretisation is carried out to allow reduction of the system of differential equations that govern fluid flow and heat transfer to a set of coupled algebraic equations. Grid independence studies have been conducted to ensure that the mesh generated for each model is refined to the extent that the resulting solution is no longer influenced by the size of the grid. The resulting grid-independent mesh contains 1600 cells in any cross-section perpendicular to the axial direction. An indication of the mesh resolution is shown in Figure 2.

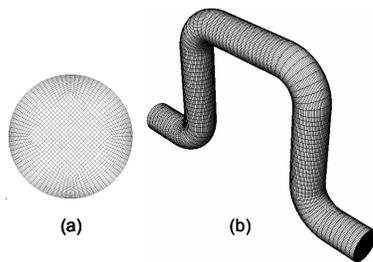


Figure 2: Mesh density (a) on the duct cross-section, (b) along the duct.

Studies were also performed to assess the required entrance length to ensure a fully-developed solution in the unit of interest. Flow and heat transfer was assessed in two, three and four repeating units. The method of Patankar *et al* [11] was also used as a reference to assess the velocity fields for the current geometry. Results from the entrance length study show that as the entrance length is extended (i.e. the number of repeating units is increased) the difference between the wrapped solution and the truly period one decreases. Improved levels of agreement were seen with an increase in the number of units up to three, with little difference beyond this. Typically, we find that three repeating units are sufficient to achieve fully developed flow, with all data being taken from the central unit.

Results and Discussion

We present results for $L/d = 4.5$ and $R_c/d = 1$, with Reynolds numbers up to 200. At higher Reynolds numbers the flow becomes unsteady. We measure performance of the serpentine channel by comparing pressure drop and rate of heat transfer in these channels to that achieved by fully-developed flow in a straight pipe of equal path length. We define a measure of the pressure loss with a normalised friction factor given by:

$$f^* = \frac{(p_{0,m} - p_{i,m})_{serpentine}}{(p_{0,m} - p_{i,m})_{straight}} \quad (1)$$

where $p_{o,m}$ and $p_{i,m}$ the mean outlet and inlet pressures, respectively.

The heat transfer performance is measured by:

$$Nu^* = \frac{Nu_{serpentine}}{Nu_{straight}} \quad (2)$$

where $Nu_{serpentine}$ and $Nu_{straight}$ are the average Nusselt numbers for the serpentine channel and an equivalent length of pipe, respectively.

Figure 3 shows the normalised friction factor and heat transfer performance for the geometry studied for a range of Reynolds numbers. It can be seen that the normalised friction factor increases only slightly with increasing Reynolds number. The heat transfer performance, however, is increased by a factor of three over the range of Reynolds numbers studied.

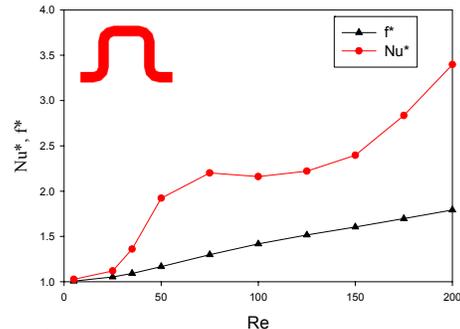


Figure 3: f^* and Nu^* as a function of Reynolds number for $L/d = 4.5$, $R_c/d = 1$ and $Pr = 6.13$.

A detailed examination of the flowfield shows that a number of interesting flow phenomena arise in these channels. A transfer of momentum towards the outer wall following each bend causes rotation of the fluid transverse to the bulk flow. These secondary flows, first observed by Dean [3] in helical channels, are known as Dean vortices. Every second bend in the serpentine geometry alternate in direction, i.e. bends 2 and 3 in Figure 1 are in the same sense. The direction of rotation of the Dean vortices also alternates after every second bend. Examples of the secondary flows, as well as directional change in vortex rotation, are shown in Figure 4.

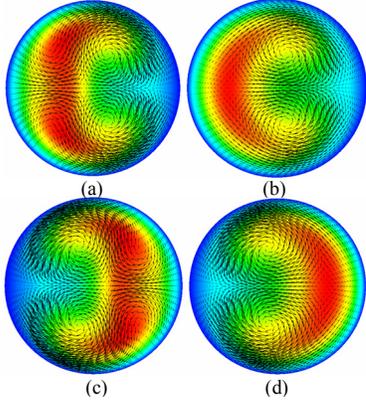


Figure 4: Speed contours, with tangential velocity vectors superimposed, at various downstream locations for a Reynolds number of 200 for $L/d = 4.5$ and $R_c/d = 1$ at: (a) Inlet, (b) 1d downstream of bend 1, (c) 1d downstream of bend 2, (d) 1d downstream of bend 3.

The effect of Reynolds number on the secondary flow patterns at the entrance of a repeating unit ($2.5d$ upstream of bend 1) is shown in Figure 5. The work of Dean [3] characterised the behaviour of secondary flow in helical channels by defining a non-dimensional number (the Dean number) given by:

$$Dn = Re \left(\frac{d}{R_c} \right)^{3/2} \quad (3)$$

For a fixed ratio of duct size (d) to radius of curvature (R_c), the Dean number is directly proportional to the Reynolds number. At low Reynolds numbers, shown in Figure 5(a), there is negligible secondary flow. The flow tends towards the straight pipe solution in this case, as the viscous forces are large in comparison with inertial forces, and act to suppress secondary flow formation. As the Reynolds number increases, inertial forces become more significant, and a pair of vortices develops. The strength of these vortices increases with Reynolds number, as is evident by the tangential velocities vectors seen in Figures 5 (b) to (d). As the Reynolds number increases further, a second pair of vortices develops on the inside of the bend (see Figure 6). A similar transition has been seen by others, for example, Mees *et al.* [9]. In this case the emergence of the second pair of vortices corresponds with the plateau in the heat transfer performance observed in Figure 3. Development of the second pair of vortices creates regions of slower flow at the walls, decreasing the rate of heat transfer.

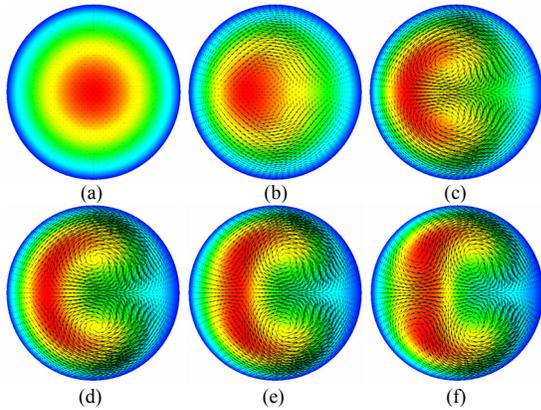


Figure 5: Speed contours, with tangential velocity vectors superimposed, at the inlet for $L/d = 4.5$, $R_c/d = 1$ and Reynolds numbers of: (a) 5, (b) 50, (c) 100, (d) 150, (e) 175, and (f) 200.

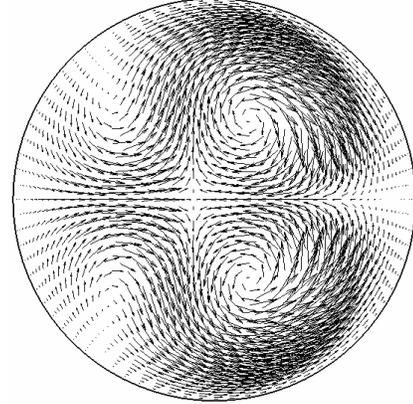


Figure 6: A tangential velocity vector plot at the inlet for a Reynolds number of 200 for $L/d = 4.5$, $R_c/d = 1$ showing two vortex pairs.

Levy *et al.* [7] proposed a method for detection and visualisation of vortex cores. They identified a correlation between velocity and vorticity in that, near vortex cores, the angle between these two vectors is small. They defined “normalised helicity” as:

$$H_n = \frac{\mathbf{v} \cdot \boldsymbol{\omega}}{\|\mathbf{v}\| \|\boldsymbol{\omega}\|} \quad (4)$$

This quantity has limiting values of ± 1 , where the angles between the velocity and vorticity vectors are zero, and the sign depends on the direction of rotation.

The volume-average of the absolute value of helicity, given by:

$$\bar{H}_v = \frac{1}{V} \int_V |H_n| dV = \frac{1}{V} \int_V \frac{|\mathbf{v} \cdot \boldsymbol{\omega}|}{\|\mathbf{v}\| \|\boldsymbol{\omega}\|} dV \quad (5)$$

is shown in Figure 7. The absolute value of helicity excludes rotation direction information from these results, and a high value corresponds to strong vortical structures. An increase in the volume-averaged helicity is seen up to a Reynolds number of approximately 125. The ensuing decline can be attributed to the development of the second pair of vortices. These vortices are not closed, and lead to a reduction in the volume average of the absolute value of helicity but do give rise to the improved heat transfer performance seen at higher Re in Figure 3.

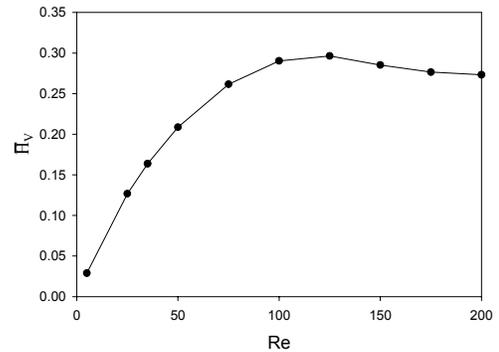


Figure 7: Effect of Reynolds number on the volume integral of the absolute value of normalised helicity for $L/d = 4.5$, $R_c/d = 1$.

Flow structures exhibiting a single pair of vortices display the highest vortex strength immediately after bends 1 and 3 (Figure 1). This occurs because vortices are allowed to progress through bends with the same direction of rotation, thus acting to increase the vortex strength. Flow development within straight sections of the geometry after each bend causes a decrease in vortex strength. For this reason, as the vortex strength increases, the distance that vortices progress into the straight sections of the pipe also increases, as shown in Figure 8.

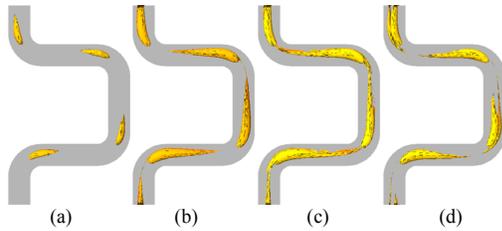


Figure 8: Isosurface plots of regions where the absolute value of helicity is 0.8 for Reynolds numbers of: (a) 50, (b) 100, (c) 150, and (d) 200.

The development of two pairs of vortices at Reynolds numbers above 150 occurs only after bends 2 and 4, which turn in an alternating direction from the previous bend. Bends 1 and 3 act to re-form the two-cell vortex structure, thus resulting in decreased vortex strength.

The influence of the ratio of unit size to diameter (L/d) on f^* and Nu^* is shown in Figure 9 for a Reynolds number of 110. It can be seen that up to $L/d = 12.5$, an increase in L/d leads to an increase in heat transfer performance and a reduction in the normalised friction factor.

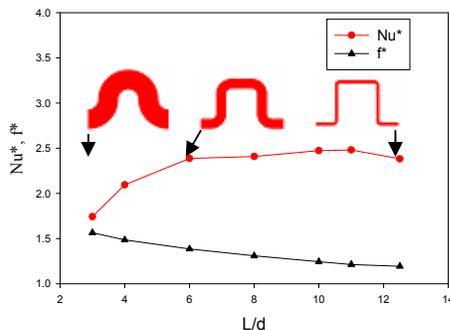


Figure 9: f^* and Nu^* as a function of L/d for a Reynolds number of 110, $R_c/d = 1$ and $Pr = 6.13$.

Geometries with high values of L/d have longer straight sections, in which flow development reduces the strength of the vortices, reducing their influence at the following bend. Ultimately, for large L/d , the flow in the straight sections becomes fully-developed, and the normalised friction factor and heat transfer performance are independent of the bend direction and upstream behaviour. The pressure drop approaches that of fully-developed flow in a straight pipe due to the increase in flow development as L/d increases. Dean vortices are evident in the flow, but the Reynolds number is too low for the development of a four-cell vortex structure.

A significant observation of this study is that flow separation does not occur. This is true for the studies in which the Reynolds number and L/d were varied. We explain this by the presence of Dean vortices. Recirculation was expected to occur on the inside wall after each bend, particularly at higher Reynolds numbers, due to the centrifugal shift of momentum to the outside wall. However, the Dean vortices direct flow back toward the inside wall, and act to suppress the onset of recirculation. The absence of recirculation zones accounts for the small increase in f^* . Increased heat transfer performance may therefore be obtained with a surprisingly low pressure drop penalty.

Conclusions

A methodology has been developed and validated to study fully-developed flow behaviour in serpentine channels. Flow characteristics within such channels are complex, leading to high rates of heat transfer, whilst low pressure loss is maintained. Dean vortices act to suppress the onset of recirculation around each bend, and are the main contributing factor to these high

levels of heat transfer performance, and low normalised friction factor. For $L/d = 4.5$, $R_c/d = 1$ and $Pr = 6.13$, two pair of vortices are observed at Reynolds numbers above 150. This flow structure occurs immediately after bends that turn in an opposite direction to the one previous.

The influence of L/d on heat transfer and pressure drop has been shown for a fixed Reynolds number. Increasing L/d increases the rate of heat transfer and decreases the pressure drop relative to that of fully-developed flow in a straight pipe.

Acknowledgments

The authors would like to acknowledge Meggitt UK, the ARC for an APAI scholarship for Nathan Rosaguti and Patrick Sharkey of CFX for assistance with the wrapping code.

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