This assignment is worth 30% of your final grade. You will submit both a paper report and the matlab code you have generated. The marks you receive will be based on both, with approximately equal weighting.

Download the file `Project.zip` from the subject web page. The objective is to navigate a lander vehicle through an obstacle field, and back into orbit. Doing so with minimal fuel use is desirable, so you have enough fuel for future adventures.

The vehicle is shown in Figure 1. You should model it as a rigid body, with inertia approximated by a solid sphere of radius $L$ and mass $m$ (see the table below). The configuration coordinates are horizontal and vertical position, and angle from vertical $\theta$. The vehicle can be controlled via thrust-vectoring. You may assume that a rocket applies a thrust $T$ at an angle $\alpha$ from the body axis, incident on a point $L$ from the centre of mass, imparting both a rotational moment and COM acceleration.

Thrust $T$ and the derivative of vector angle $\alpha$ are the control inputs. Ultimately, $T$, $\alpha$ and $\dot{\alpha}$ will all have limits on their range. You may ignore the fact that fuel use will change the mass of the vehicle.

There is a gravitational force, weaker than Earth’s, and an atmospheric drag. A simple model of drag is $F_d = \frac{1}{2}c_d A \rho v^2$, where the parameters are given in the table above and giving a force in Newtons when using the units listed in the table. Neglect any lift forces generated by the vehicle.

![Figure 1: Lander vehicle schematic](image-url)
Parameter & Value \\
--- & --- \\
Mass, $m$ & 5000kg \\
Total fuel & 500kg \\
Fuel consumption, $SFC$ & $3 \times 10^{-4}$kg/s/N \\
Drag Coefficient $c_d$ & 0.4 \\
Cross sectional area, $A$ & 25 m$^2$ \\
COM to thruster Length $L$ & 3m \\
Maximum fuel flow rate & 40kg/s \\
Atmospheric Density, $\rho$ & 1kg/m$^3$ \\
Gravitational acceleration constant, $g$ & 5N/kg \\

1. (10 Marks, Checkpoint: Wk 11) Motion planning:

(a) Write a collision checker that indicates whether the line segment between two points $x_1$ and $x_2$ intersects an ellipse, given in the form $(x - c)^T A (x - c) \leq 1$. Treat this as a simple problem of quadratic minimisation.

(b) Calculate the optimal cruise speed in terms of cost of transport (kg of fuel used per metre travelled) for straight-line paths of different angles from vertical. Implement this with a few (3 or 4) iterations of Newton’s method. Hint: find the required thrust as a function of $\theta$.

(c) Write a PRM combined with $A^*$ to find an approximately-optimal collision-free path, using the metric from Question 1b. Hint: you may use the matlab function knnsearch to select candidate connection points.

2. (10 Marks, Checkpoint: Wk 12) Design (separately) a state-feedback LQR controller to regulate the vehicle to your planned trajectory with full state feedback, and an extended Kalman filter to estimate the state of the vehicle (measurement model TBD). Combine these into an LQG controller. Motivate your choice of control and estimator weights in terms of performance and robustness. Evaluate response to a gust disturbance and robustness to uncertainty in the moment of inertia.

3. (10 Marks, Checkpoint: Wk 13) Implement a model-predictive control (MPC) system to regulate the system to the path with strict constraints on actuation (TBD). This involves solving a quadratic program as part of your control loop. You may use matlab qp, CVX or yalmip, but you must formulate the control problem as a standard-form quadratic program yourself. Investigate the effect of different input constraints, as well as the use of state constraints. Compare performance, robustness, and disturbance rejection to the LQG controller.