Shaft Design - conveyor drive pulley

A  Problem description
A shaft has to be designed to suit a drive pulley of a conveyor belt system. The overall dimensions are shown on the diagram below. From experience the customer requires that the inclination of the shaft at the keyways be no more than 1/1200 (tangent or radians). From past installations it has been noted that for this shaft deflection, fatigue failures of the pulley’s side cheeks, are minimal. The pulley is required to be capable of 20 years operation, at full load, 300 rpm, 2 shifts per day.

B  Executive Summary (to be finalised)
Fatigue requirements on the shaft alone would permit the use of steel shaft of a minimum 100 Dia to be used. The deflection requirements at the keys dictate a larger shaft diameter of 130 mm Dia. and a minor shaft diameter of 35. It is proposed to machine the ends of the shaft to allow the use of an adequate and more economic bearings of 35 mm ID. It is estimated that a double row spherical roller bearings can be used, but the actual size and type of bearing is to be determined next.

A factor of safety of 1.5 has been used in both the calculations for stress and deflection. There are arguments that that FS should not have been used to determine the shaft diameter that limits deflection.

C  Estimated no of cycles over life of shaft
The no of cycles over the expected life of this shaft is expected to be:

\[
\text{No of cycles} = 2.33 \times 10^9 = 20 \times 360 \times 18 \times 60 \times 300 \text{ years} \times \text{days per year} \times \text{hour per day} \times \text{revs per hour}
\]

Thus we will require fatigue strength at the endurance limit of the steel shaft.

D  Layout of shaft installation
Shown below are a schematic side and an end view of the drive pulley for which a new shaft is required. The belt tensions at full load have been determined with strain gauges. A photo of a similar but larger drive pulley assembly is shown at the end of this report.
E Vector sum of force on bearings and keys
We have two loads on the shaft due to the belt tensions. Note that the power that can be transmitted to the belt is a function of the tension on the low side of the belt, the coefficient of friction and the wrap angle.
It is typical of conveyor systems to have a tensioner on the low side of the belt, tight control of the wrap angle and at times the use of several drive pulleys one after the other.

Using the cosine rule we can arrive at the total combined force on the shaft. The vertical component will be ignored for the present for the present.

\[
\begin{array}{lcccc}
\text{High belt tension} & \text{Th} & 50000 & \text{N} \\
\text{Low belt tension} & \text{TL} & 20000 & \text{N} \\
\text{Force per bearing} & 2F & 34.18 & \text{SQRT}(50^2+20^2-2*50*20*COS(30*(\pi/180))) \\
\text{and per key} & R & 17088 & \text{N}
\end{array}
\]

Where the force F at each key and the the reaction R at each bearing are equal and opposite.

F Shear force and moment diagrams along shaft
given the distribution of shear forces, being equal and opposite, between the bearings and the keys the moment along the shaft is maximum and constant, from one keyway location to the other.

\[
\text{Distance R to F - RF mm} = 125
\]

\[
\text{Max bending moment - Mx Nmm} = 2135954.48 \text{ RF} F
\]

G Torque
The whole torque will most likely be transmitted at to the first key. The second key may transmit some of the torque, but no allowance can be made without some specific design feature that can ensure it.

\[
\text{torque = difference in belt tension multiplied by pulley radius}
\]

\[
\text{Torque being transmitted - Tx} = 6000000 \text{ (Th-TL)*200}
\]

H Estimating endurance limit assuming approximate values for diameters and fillet radius
Endurance limit of shaft at the key, where stress concentration, max moment and max torque coincide
where: $S_u = K_a \cdot K_b \cdot K_c \cdot K_d \cdot S_{u0}$

- $S_u$: N/mm$^2$
- $K_a$: load type, reverse bending = 1.0
- $K_b$: Surface area factor = 0.73
- $K_c$: Surface finish, fine turned finish = 0.78
- $K_d$: temperature effect <100 degree, =1.0
- $K_e$: stress concentration, torque with key = 0.33
- $K_{eb}$: """" bending = 0.45
- $K_f$: reliability 1 in 90 chance of failure = 0.814

Fatigue notes

Therefore: $Se = 115.1 \cdot S_u \cdot 0.5 \cdot K_b \cdot K_c \cdot K_f$ N/mm$^2$

excluding stress conc factors, which will be added where shaft dia is calculated

Eq 17a - l. notes

Eq 17b - l. notes

Eq 18 - l. notes

I Assumed stepped outline - leading to for deflection analysis

assumed stepped shaft layout:

To determine angular deflection at a place of interest, by the Castigliano’s method we need to differentiate the total energy in a component (due to all moments) with respect to the moment at that location of interest.

It is consistent with this method to place fictitious moments anywhere and include them when summing the total energy. Since any fictitious moments would have a value of 0, that is the value that has to be substituted for them when the total energy is evaluated. The reason for this strategy is that placing a 0 moment at a particular location gives us a method of calculating the angular deflection at that location, if a moment does not exits there.

$$\theta_m = \frac{Fb}{EIm} \left( l - 2b \right)$$

$\theta_m$: is the deflection required at location of moment $m$

$n$: is the ratio of diameters at the step in the shaft

$$\phi D_2 = \phi n D_1$$

Shaft dia to meet deflection req. 150 mm min

$$D_{defl} = \left( \frac{64 \cdot F \cdot FS \cdot 125}{\pi \cdot 206800 n^4} \cdot (500 - 250) \cdot 1200 \right)^{1/4}$$

$D_{defl}$: mm

$F$: \text{Force}\ N

$FS$: \text{Torque}\ \text{Nm}

$E$: \text{Young's modulus}\ \text{GPa}

$I$: \text{Moment of inertia}\ \text{mm}^4

$D_1$: \text{Shaft dia}\ \text{mm}

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J Check that a suitable bearing can carry the load and be of 50 mm ID.
(a) Simply supported beam with concentrated loading

\[ R_1 = F \left( 1 - \frac{a}{l} \right) \]
\[ R_2 = F \left( \frac{a}{l} \right) \]

\[ q = R_1 (x-a)^{-1} - F (x-a)^{-1} + R_2 (x-l)^{-1} \]

\[ V = R_1 - F (x-a)^0 + R_2 (x-l)^0 \]
\[ V_{\text{max}} = \text{MAX}(R_1, R_2) \]

\[ M_{\text{max}} = Fa \left( 1 - \frac{a}{l} \right) \]
\[ M = R_1 x - F (x-a)^1 + R_2 (x-l)^1 \]
\[ \theta = \frac{F}{2EI} \left\{ \frac{1}{3} \left( \frac{a}{l} \right)^3 - \frac{a^3}{l} - la^2 \right\} \]
\[ \gamma_{\text{max}} = \frac{F}{3EI} \left\{ \frac{1}{2} \left( \frac{a}{l} \right)^2 - \frac{a^2}{l} - la^2 \right\} \]

\[ y = \frac{F}{6EI} \left\{ \frac{1}{3} \left( \frac{a}{l} \right)^3 - \frac{a^3}{l} - la^2 \right\} \]