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Adaptive multi-point sequential sampling methodology for highly nonlinear automotive crashworthiness design problems

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1. Abstract
Automotive crashworthiness design is a highly expensive and non-linear problem. In metamodel-based crash design problem, the prediction error of the metamodel may induce a local or a wrong optimum. In the past few years, the multi-point objective-oriented sequential sampling methods have been demonstrated an efficient way to improve the fitting accuracy and find the true optimum. However existing infilling criteria are restricted to specify the number of the sequential samples obtained in each iteration. It is not practical for complex engineering design problems. In this paper, a new adaptive multi-point sequential sampling method is developed. The sequential sample size is determined by the prediction states of the fitting metamodels. To demonstrate the benefits, the new proposed method is applied to a highly nonlinear crashworthiness design problem. Results show that the proposed method can mitigate the effect of the prediction error, and more efficiently identify the crashworthiness design solution compared to the conventional approach.
2. Keywords: Metamodel-based optimization, objective-oriented sequential sampling method, adaptive multi-point strategy, crashworthiness design.

3. Introduction

Finite element (FE) simulations have been a useful tool for replacing the physical tests in crashworthiness design. However, high fidelity FE models are often computationally intensive, taking hours and even days to complete one computation cycle. A common approach to address this challenge is to employ metamodeling method predicting the simulation responses. The metamodel provides a cheap-to-run surrogate model to approximate the complex simulations [1]. The effectiveness of different metamodeling techniques vary based on the different modeling criteria, amount of available samples, and the behavior of the simulation responses [2].
However in complex engineering optimizations, the primal challenge is how to determine the number of samples required and how to allocate samples. Comparing to traditional one-stage DOE methods (Orthogonal experimental design, Uniform Design, Latin Hypercube Design et. al), sequential sampling methods have been identified as a more efficient strategy. In previous investigations, the sequential sampling criteria can be classified into two categories: model-oriented and objective-oriented. The model-oriented methods focus on the goal of creating a globally accurate metamodel, while the objective-oriented sequential sampling strategies have been demonstrated to have a higher efficiency of finding the global optimum [3]. The most widely used objective-oriented sequential sampling criteria, Efficient Global Optimization (EGO) algorithm, is first developed by Jones [4].
The EGO method only finds one point in one iteration, resulting in many sequential cycles before reaching convergence. To take advantage of the parallel computation capability and save the total amount of iterations, a multi-point sampling strategy is needed. Schonlau [5] defined the concept of multi-point sequential sampling method. Viana [6] extended the Probability of Improvement function to include multiple points at the same time. Zhu and Zhang [7] developed a new double-loop strategy to find q samples via Kriging Believer method.
However existing infilling criteria are restricted to specify the number of the sequential samples obtained in each iteration. It is not practical for complex engineering design problems. In this paper, a new adaptive multi-point sequential sampling method is developed. The following section reviews the concept of multi-point sequential sampling methods, and introduces the proposed adaptive strategy. A new infilling criterion is developed to determine whether there is a need to find one more sample. In Section 5, to demonstrate the effectiveness, the proposed adaptive multi-point sequential sampling method is applied to an automotive crashworthiness design problem. Finally, the discussions and conclusion are summarized in Section 6.

4: Adaptive multi-point sequential sampling methodology for complex engineering optimization

4.1. Multi-point Sequential sampling method for constrained optimization problem
For a constrained engineering optimization problem, the mathematical formulation can be defined as:

$$\begin{aligned} \min: \quad & y(\mathbf{x}) \\ \text{s.t}: \quad & g_i(\mathbf{x}) \le \beta_i, \qquad i = 1, 2, \ldots, k \end{aligned} \tag{1}$$

where $\mathbf{x}$ is the design variables; y and g_i represent the objective response and constraint responses; β_i is the ith constraint threshold. When the objective and constraint responses are replaced by metamodels ($\hat{y}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$), considering the metamodeling imperfection, the prediction error affects the optimization accuracy and constraint feasibility, especially in high-dimensional and highly-nonlinear engineering problems.

The objective-oriented strategy can spread new samples to balance the optimization exploration and accuracy improvement. Evaluating the effects of prediction error on the objective responses $\hat{y}(\mathbf{x})$ and the constraints $\hat{g}_i(\mathbf{x})$, the generalized expected improvement function (GEI) of a constrained optimization problem can be defined as [7].

$$\begin{aligned} \max: \quad & \mathrm{GEI}(\mathbf{x}) = \sqrt{\mathrm{REI}(\mathbf{x}) \cdot \mathrm{EV}(\mathbf{x})} \\ \text{where}: \mathrm{REI}(\mathbf{x}) &= \left(y_{\min} - \hat{y}(\mathbf{x})\right) \cdot \Phi\left(\frac{y_{\min} - \hat{y}(\mathbf{x})}{\sigma_y(\mathbf{x})}\right) + \sigma_y \cdot \phi\left(\frac{y_{\min} - \hat{y}(\mathbf{x})}{\sigma_y(\mathbf{x})}\right) \\ \mathrm{EV}(\mathbf{x}) &= \left[\prod_{i=1}^{k} \Phi\left(\frac{\beta_i - \hat{g}_i(\mathbf{x})}{\sigma_{gi}(\mathbf{x})}\right)\right]^{1/k} \end{aligned} \tag{2}$$

where k is the number of constraint responses; $y_{\min}$ is the minimal objective response of the sampled points; $\hat{y}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$ indicate the predicted value of the objective and constraint response respectively; $\phi(\mathbf{x})$ and $\Phi(\mathbf{x})$ represent the probability density function and cumulative density function of a standard normal distribution.

It is an efficient way to choose the global and quasi-local optimums of the GEI function as the sequential [8]. It should be noted that existing multi-points methods are developed to obtain a constant number q of sequential samples. But in real engineering problems, it is difficult to guess how many samples are needed in each cycle. A complex problem with a small q still needs many iterations, while with an over large q may induce intensive simulations. The following section will introduce a new adaptive multi-point sequential sampling method. The number q in each iteration is decided by the prediction states of the optimization problems adaptively.

4.2. Adaptive multi-point sequential sampling method for complex engineering optimization problem

The infilling criterion is the most important factor in sequential sampling process. In order to improve the sequential sampling efficiency, the weighted contribution of a new point is developed to replace the conventional generalized Expected Improvement function. The modified qGEI function is formulated as:

$$q\mathrm{GEI} = \begin{cases} \mathrm{GEI}, & q = 1 \\ \left(\dfrac{\mathrm{GEI}}{q\mathrm{GEI}_1}\right)^m, & q > 1 \end{cases} \tag{3}$$

where q is the number of the sequential samples obtained in each iteration; $q\mathrm{GEI}_1$ represents the qGEI value of the 1st sequential sample (q = 1); m is the power number. After the first point is found, the power function m of GEI downplays the relative contributions of the new points. As shown in Figure 1, when m = 1, the qGEI function represents the relative GEI value. As the m value increases, the regions with small GEI will be diminished. If m is set to 2, the point where the GEI value is less than 10% of the $q\mathrm{GEI}_1$ will be neglected. If m is set to 4, the point where the GEI value is less than 35% of the $q\mathrm{GEI}_1$ will be neglected. Using the qGEI function in the sequential sampling process, more efforts will be made in the regions with higher contribution.

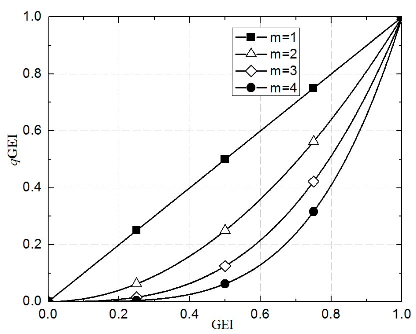


Figure 1: The influence of the m value in the qGEI function

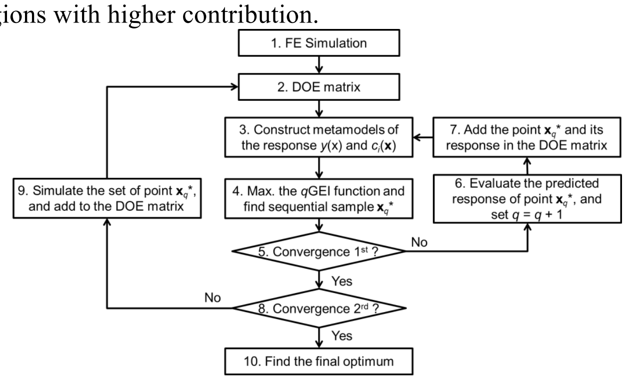


Figure 2: The flowchart of the proposed adaptive multi-point sequential sampling process

The true solution is likely to be near the inferior optimum of the infilling criterion, rather than the global optimum with the largest EI function**Error! Reference source not found.**. In this paper, the concept of the Kriging Believer strategy is adopted in the adaptive sequential sampling method. Different from any strategies in previous studies, the Kriging Believer strategy treats the predicted response as the true response during the sequential sampling process. The sequential samples obtained in each iteration are allocated to the global and quasi-local optimums.

The flowchart of the proposed method is shown in Figure 2.

Step 1: Generate a set of samples, and extract the simulated responses of these N training samples.

Step 2: Update the DOE matrix of the n samples (N initial DOE samples and all sequential samples).

Step 3: Based on the true observations at $\mathbf{x}_n$ and predicted responses at $\mathbf{x}_q$, the Kriging models of the objective responses $\hat{y}(\mathbf{x})$ and constraint responses $\hat{c}_i(\mathbf{x})$ are constructed. $\mathbf{x}_q$ is the sequential samples in the q^{th} iteration.

Step 4: Maximize the infilling criterion qGEI and find the next sequential sample $\mathbf{x}_q$.

Step 5: Check the convergence. If the 1st stopping criterion is satisfied, go to **Step 8**.

Step 6: Evaluate the predicted response $\hat{y}(\mathbf{x})$ and $\hat{c}_i(\mathbf{x})$ of the newly added point $\mathbf{x}_q$, and set $q = q+1$.

Step 7: Add the sample $\mathbf{x}_q$ into the training DOE samples.

Step 8: Check the convergence. If the 2nd stopping criterion is satisfied, the sequential sampling process is converged and goes to **Step 10**.

Step 9: Simulate the obtained samples $\mathbf{x}_q$ by FE models, and add these points into the training samples n.

Step 10: After the sequential sampling process is terminated, the final design solution will be found.

5. Engineering application in a crashworthiness design problem

In this section, the benefits of the proposed adaptive sequential sampling method are demonstrated in a complex crashworthiness design example. Two different strategies are considered in this section:

- Conventional multi-point sequential sampling method with a constant q (GEI_cq): the sequential sampling method found q samples in each iteration. The sequential infilling criterion is defined by Eq. (2).
- Proposed adaptive multi-point sequential sampling method (GEI_aq): the sequential samples found in each iteration are determined by the prediction states, and the infilling criterion is defined by Eq. (3).

5.1. Crashworthiness design application

In the automotive crashworthiness design, FE simulations are used to predict crash performances. Since full size automotive simulation models are computationally expensive, metamodeling techniques are widely utilized to build surrogate models. In this section, a frontal impact design problem is utilized to demonstrate the effectiveness of the proposed adaptive multi-point sequential sampling method in real engineering design. The FE model is shown in Figure 3. The average mesh size is 5 mm. For the frontal impact investigation, the regulations and test configurations in the China National Crash Legislation of frontal impact (GB11551-2003) are followed. Considering the strain rate sensitivity of the sheets in high speed impact, stress versus plastic strain curves under different strain rates are defined in a load table. These curves are obtained from physical tension experiments.

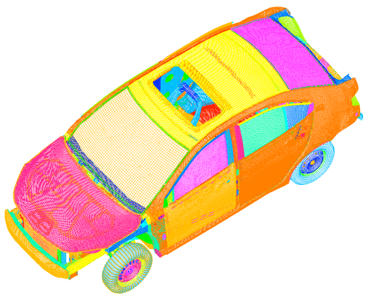
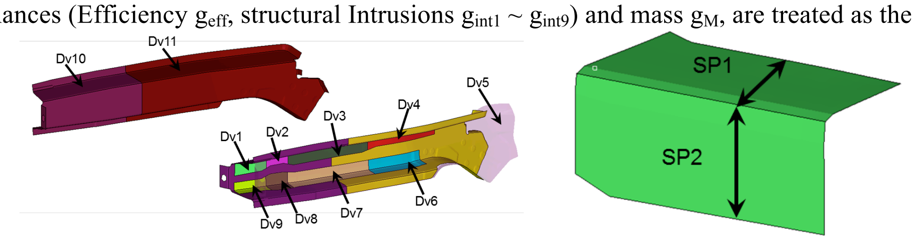


Figure 3: Full-size finite element model of frontal impact simulation

The frontal side rail is the critical part in absorbing frontal impact, as shown in Figure 4. The sheet gauge and the component shape are important for absorbing the impact energy. Considering the symmetry of the rail structure, 11 sheet gauges and 16 shape variables are chosen as the design variables, as shown in Table 1. In this crashworthiness design problem, the Effective Acceleration f_{acc} is defined as the objective response, while ten crash performances (Efficiency g_{eff}, structural Intrusions $g_{\text{int1}} \sim g_{\text{int9}}$) and mass g_M, are treated as the constraints.



(a) Gauge variables (b) Shape variables

Figure 4: Design variables of the crashworthiness design

Table 1: Design variables of the automotive crashworthiness design

	Component	Variables	DV	Original/mm	LB/mm	UB/mm
Gauge DVs	Upper Reinf. 1	Dv1	x1	1.20	0.60	1.60
	Upper Reinf. 2	Dv2	x2	1.20	0.60	1.60
	Upper Reinf. 3	Dv3	x3	1.20	0.60	1.60
	Upper Reinf. 4	Dv4	x4	1.20	0.60	1.60
	Frontal side rail outer	Dv5	x5	1.40	1.00	1.80
	Lower Reinf. 1	Dv6	x6	1.20	0.60	1.60
	Lower Reinf. 2	Dv7	x7	1.20	0.60	1.60
	Lower Reinf. 3	Dv8	x8	1.20	0.60	1.60
	Lower Reinf. 4	Dv9	x9	1.20	0.60	1.60
	Frontal side rail inner 1	Dv10	x10	1.40	1.00	1.80
	Frontal side rail inner 2	Dv11	x11	1.50	1.00	1.80
Shape DVs	Upper Reinf. 1 SP1	Dv12	x12	0.00	0.00	15.00
	Upper Reinf. 1 SP2	Dv13	x13	0.00	0.00	15.00
	Upper Reinf. 2 SP1	Dv14	x14	0.00	0.00	15.00
	Upper Reinf. 2 SP2	Dv15	x15	0.00	0.00	15.00
	Upper Reinf. 3 SP1	Dv16	x16	0.00	0.00	15.00
	Upper Reinf. 3 SP2	Dv17	x17	0.00	0.00	15.00
	Upper Reinf. 4 SP1	Dv18	x18	0.00	0.00	15.00
	Upper Reinf. 4 SP2	Dv19	x19	0.00	0.00	15.00
	Lower Reinf. 1 SP1	Dv20	x20	0.00	0.00	15.00
	Lower Reinf. 1 SP2	Dv21	x21	0.00	0.00	15.00
	Lower Reinf. 2 SP1	Dv22	x22	0.00	0.00	15.00
	Lower Reinf. 2 SP2	Dv23	x23	0.00	0.00	15.00
	Lower Reinf. 3 SP1	Dv24	x24	0.00	0.00	15.00
	Lower Reinf. 3 SP2	Dv25	x25	0.00	0.00	15.00
	Lower Reinf. 4 SP1	Dv26	x26	0.00	0.00	15.00
	Lower Reinf. 4 SP2	Dv27	x27	0.00	0.00	15.00

5.2. Sequential improvement and optimization results

All structural performances are interpolated by Kriging method. The optimization formulation is defined as:

$$\begin{aligned} \min: \quad & f_{acc} \\ s.t.: \quad & c_1 = g_{\text{int}1} / \beta_1 \le 1 \\ & \vdots \\ & c_9 = g_{\text{int}9} / \beta_9 \le 1 \\ & c_{10} = 2 - g_{eff} / \beta_{10} \le 1 \\ & c_{11} = g_M / \beta_{11} \le 1 \end{aligned} \tag{4}$$

where β_i represents the ith constraint target. Based on 180 samples generated by the Latin Hypercube method, the metamodel-based optimization results are shown in Table 2. But when the optimization solution is confirmed by the FE simulation model, there has a large discrepancy between predicted and simulated objective response f_{acc}. And two constraint responses (c_{10}, c_{11}) violate the design limits. The prediction error misleads to find an infeasible solution. In order to mitigate the prediction error, the multi-point sequential sampling method is used.

Table 2: The optimization results based on initial DOE samples

Opt. Result		Target	Kriging-based	Simulation confirmation
Objective	f_{acc}	min.	0.91	1.04
Constraints	c_1	≤ 1	0.28	0.33
	c_2		0.35	0.48
	c_3		0.58	0.77
	c_4		0.18	0.25
	c_5		0.54	0.66
	c_6		0.75	0.81
	c_7		0.45	0.61
	c_8		0.70	0.62

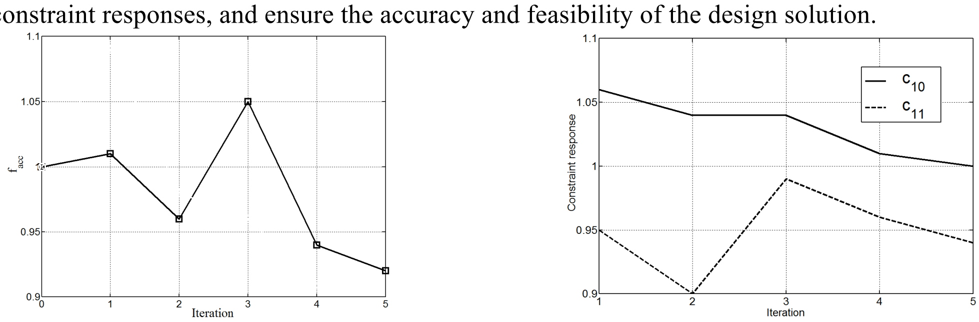
c_9	0.82	0.88
c_{10}	0.99	1.15
c_{11}	0.95	1.01

To demonstrate the benefits of the proposed method, the conventional multi-point sequential sampling strategy GEI_cq with a constant q is also adopted in this example, formulated as:

$$\begin{aligned} \max: \quad & \mathrm{GEI\_cq}(\mathbf{x}) = \sqrt{\mathrm{REI}(\mathbf{x}) \cdot \mathrm{EV}(\mathbf{x})} \\ \text{where}: \quad & \mathrm{REI}(\mathbf{x}) = \left(f_{acc} - \hat{f}_{acc}(\mathbf{x})\right) \cdot \Phi\left(\frac{f_{acc}^{\min} - \hat{f}_{acc}(\mathbf{x})}{\sigma_f(\mathbf{x})}\right) + \sigma_f \cdot \phi\left(\frac{f_{acc}^{\min} - \hat{f}_{acc}(\mathbf{x})}{\sigma_f(\mathbf{x})}\right) \\ & \mathrm{EV}(\mathbf{x}) = \left[\prod_{i=1}^{11} \Phi\left(\frac{1 - \hat{c}_i(\mathbf{x})}{\sigma_{ci}(\mathbf{x})}\right)\right]^{1/11} \end{aligned} \tag{5}$$

In this infilling criterion, $f_{\mathrm{acc}}^{\min}$ represents the minimal objective response value of the sampled points, and $q = 5$ samples are newly added in each iteration. The limit criterion is utilized in this crashworthiness design problem: when $\mathrm{GEI\_cq}_1$ is less than 1%, the sequential sampling process will be terminated.

The GEI_cq method is converged after 5 iterations. The optimization solution is obtained based on initial training samples and the newly added samples. The solution is confirmed by FE simulation. Figure 5 illustrates the convergence history of the GEI_cq method. The objective response f_{acc} and two critical constraint responses c_{10}/c_{11} are monitored. The objective response f_{acc} reduced from 1.00 to 0.92, achieving 8% improvement, while two critical constraint response c_{10} and c_{11} are successively approaching to the design target 1. It demonstrates that the multi-point sequential sampling method GEI_cq can mitigate the prediction error in both objective response f_{acc} and all constraint responses, and ensure the accuracy and feasibility of the design solution.



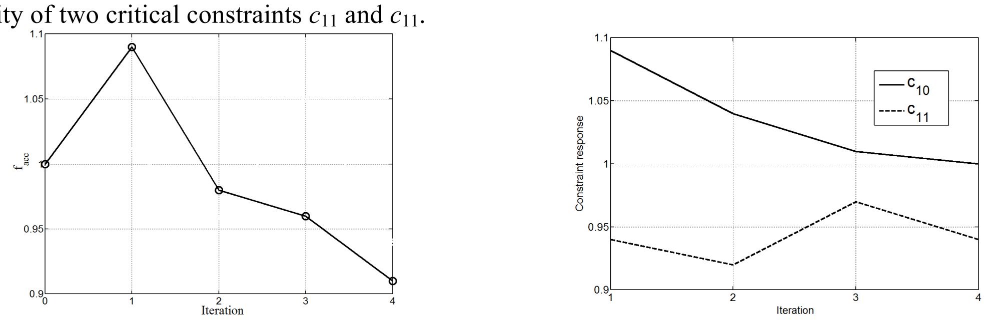
(a) Objective response f_{acc} (b) Constraint responses c_{10} and c_{11}

Figure 5: Convergence history of the GEI_cq method

The newly proposed adaptive multi-point sequential sampling method GEI_aq do not need to define a number q, and can find a proper amount of sequential samples based on the prediction states of the fitting models. The infilling criterion of the proposed method is defined as:

$$\max: \mathrm{GEI\_}aq = \begin{cases} \mathrm{GEI\_}cq, & i = 1 \\ \left(\mathrm{GEI\_}cq / \mathrm{GEI\_}aq_1\right)^2, & i > 1 \end{cases} \tag{6}$$

Similar to the GEI_cq method, when the $\mathrm{GEI\_aq}_1$ value is less than 1%, the sequential improvement process terminates. The convergence histories of the proposed GEI_aq method are shown in Figure 6. The confirmed objective response reduced from 1.00 to 0.91, while two critical constraints satisfy the design requirements. The FE simulated results shows that the proposed GEI_aq can improve the objective response f_{acc}, and ensure the feasibility of two critical constraints c_{11} and c_{11}.



(a) Objective response f_{acc} (b) Constraint responses c_{10} and c_{11}

Figure 6: Convergence history of the proposed GEI_aq method

The sequential samples obtained by these two methods are compared in Figure 7. In the 1st sequential iteration, the

GEI_cq with a constant q explored and found 5 new samples to improve the fitting states. But when the proposed adaptive method GEI_aq is used, 8 samples are allocated in the design space. It demonstrates that based on initial 180 training samples, the fitting responses of the crashworthiness design problem has large prediction error, and the number q used in the GEI_cq method is not enough. After the 1^{st} iteration, the interpolation accuracy of the crash responses has been improved. Fewer points are needed in the next iterations. The conventional GEI_cq method with a constant q allocated more and more samples on the points with lower contribution. In summary, the proposed method is converged in the 4^{th} iteration and 16 sequential samples are newly added. Comparing to the conventional GEI_cq method, the proposed strategy converge to the true crashworthiness solution faster. It demonstrates a higher efficiency in the complex engineering design problem.

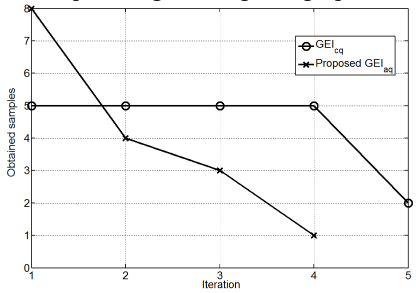


Figure 7: The samples obtained by two different sequential sampling methods

6. Discussions and conclusions

A few observations are made:

- The proposed adaptive multi-point sequential sampling method can decide the sample size by the prediction states of the design responses. It is beneficial for the problems where simulation models are computationally expensive and the parallel computing ability can be utilized to calculate many simulations at the same time.
- The crashworthiness design is a highly nonlinear problem. It is found that comparing to conventional sequential sampling method, the proposed adaptive strategy not only can improve the objective response (Effective Acceleration f_{acc}) and ensure the feasibility of ten crash constraint responses, but also can converge to the true solution in fewer iterations. It demonstrates the effectiveness and the efficiency of the newly proposed method.

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