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Gradient based structural optimization with fatigue constraints of jacket structures for offshore wind turbines

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1. Abstract

Investigating the fatigue life of support structures of offshore wind turbines is imperative to avoid unexpected failure. Therefore, in the context of structural optimization, including fatigue constraints is crucial, as the optimized design will meet the design criteria early in the design process without the need for extensive manual post-processing. Ultimately, the optimized design may be lighter and thus reduce both production and installation cost. The aim of this work is to present such a gradient based optimization method with fatigue constraints of jacket structures for the preliminary design phase. The key challenge is to efficiently deal with the very large number of non-linear fatigue constraints and the very large time-history loads that are used in the design of offshore support structures. In this paper main emphasis will be on the analytical design sensitivity analysis used in the optimization. Sensitivities determined by the direct differentiation method and by an aggregated adjoint method will be presented and evaluated.

2. Keywords: Structural optimization, fatigue constraints, sensitivity analysis

3. Introduction

In recent years a clear tendency in wind energy industry is to install larger wind turbines further away from the coast [4]. Being further away from the coast will, in most cases, mean favorable wind conditions but also deeper waters. This of course calls for larger and more complex support structures. The dominating type of support structure is the monopile. However, beyond shallow waters the jacket structure is often applied instead. Currently the support structures can account for as much as 20% of the total cost of the wind turbine [1, 9]. However, as the need for more complex jacket structures is inevitable, new and robust methods for designing lightweight and cost efficient support structures are required. Here, numerical optimization methods that can incorporate a wide range of design criteria can aid engineers during the design phase. In this work, we focus on developing fatigue constraints, which can be incorporated into the design optimization process. By including fatigue constraints in the early design phases, engineers may require less time for manual post-processing while also designing lighter structures.

Fatigue is already an integral part of the design of jacket structures from the conceptual phase to the final design. The offshore industry has a readily good statistically understanding of the environmental conditions and thus the fatigue loads during the expected lifetime. To further incorporate the operational conditions in the design of support structures for offshore wind turbines, we need rational, trustworthy, and efficient methods to evaluate and optimize for fatigue loading.

Although optimization with fatigue constraints can be a very powerful tool, it is a relatively unexplored domain. Some of the first who contributed to this area of research were Grunwald and Schnack [5], who formulated a shape optimization method to maximize the crack initiation phase of a simple test specimen. Their method was restricted to 2D problems under constant amplitude loading, using fatigue constraints based on equivalent stresses. Their findings were somewhat discouraging; they succeeded in their fatigue optimization but the results were similar if they applied the much simpler minimum equivalent stress optimization. In addition, their optimization for fatigue was computational inefficient. Computational inefficiency is a key problem in optimization for fatigue that also applies today. Shortly after, Zeiler and Barkey [11] strongly suggested that optimization for fatigue was so well-developed that industry could start taking advantage of the method. They used a gradient based optimization method to optimize stiffness and damping of a greatly simplified six degree-of-freedom model of an automobile subjected to Formann crack-growth constraints. Their methods are also limited to proportional loading. More recently Martini and Tobias [7] applied non gradient based fatigue optimization on industrial components, gaining a better result when optimizing for fatigue than when optimizing for stress. The authors also made clear that fatigue optimization is now so well-established that it should be used in industry.

In this paper we present a method of gradient based 3D structural optimization with high-cycle fatigue constraints. The aim is to reduce the overall mass of a structure, having diameter and thickness of each member as design variables. The methods are intended for the preliminary design phase, that is, after the general topology of the structure has been determined. The constraints are based on Palmgren-Miners linear damage hypothesis. Computational efficiency is preserved through the use of gradient based optimization, where the design sensitivity analysis is performed using analytical expressions. The optimization is carried out using Sequential Linear Programming (SLP) with a global convergence filter [2].

The paper presents a brief insight into the theory behind the analytical design sensitivities. The implementation of these is later verified using central difference approximations of the presented constraint formulations. The optimization algorithm is developed for support structures of offshore wind turbines, but can easily be applied to many mechanical components under high-cycle fatigue. Lastly, a brief discussion of the method in its current state is given and ideas on how to elevate the current model are presented.

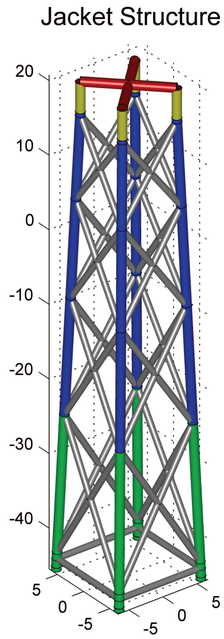

Figure 1: The OC4 reference jacket [10]. The five different colors represent five different sets of tube dimensions. Two shear forces and two bending moments are applied in the intersection of the red beams located at the jacket top. The dimensions shown on the figure are in meters.

4. Fatigue Analysis

It is important to use an adequate cumulative damage theory when determining the fatigue damage in variable amplitude loading. The damage is defined as a fraction of the life of the structure. To predict the fatigue life, the fractions are summed using an accumulation rule. Even though many advanced and non-linear accumulation rules exist, none can fully represent the complicating aspects of variable amplitude loading [8]. Therefore, Palmgren-Miner's linear damage hypothesis is applied in this study. This rule does not take sequential effects and interaction of events into account, even though it can potentially have a large influence on the fatigue life of the structure. However, these shortcomings are deemed acceptable for the preliminary design-phase of jacket structures. Also, this is how the current recommended offshore practice [3] addresses fatigue. The material data for fatigue is given by Wöhler diagrams. A Wöhler diagram (S-N curves) represents the number of cycles to fatigue failure in high-cycle regime as a function of the stress amplitude.

4.1 Load Spectrum

Large time-history loads are used in the prescribed standards for design of fatigue life of wind turbine support structures [6]. This makes the fatigue investigation of support structures of offshore wind turbines very time consuming, even more so in design optimization, where all iterations may require a new simulation. Including large time-history loads and reducing the stress and displacement spectra through multiaxial Rainflow counting can be a good approach, because as a rule of thumb ten percent of the loads cause more than ninety percent of the damage [8]. However, multiaxial rainflow counting has not yet been implemented.

The current study only includes a load spectrum consisting of one minute of operational time. This is partly so because the aim of this work is to investigate design sensitivity analysis methods on fatigue constraints and not to present validated designs for jacket structures. As the time-history load is not reduced through methods such as Rainflow counting the time-history load is still sufficiently challenging for the problem at hand. One minute of operational time corresponds to 6,000 load combinations, resulting in 5,999 stress and displacement cycles as no reduction is done. Henceforth the total number of cycles are referred to as N_i.

The load spectrum in the authors possession does not include the torsional loads M_z and normal loads F_z, meaning that two shear loads, F_x and F_y, and two bending moments M_x and M_y represents the wind loads, see Figure 2. However, the developed design sensitivity analysis is capable of capturing the normal load and torsional moments if a more detailed time-history load is applied. It is believed that the normal and torsional loads will have a significant impact on the fatigue analysis, especially as the jacket is designed for large wind turbines in deep waters. Furthermore, hydrostatic wave loads may have a large impact on the fatigue on deep waters, but they are not included in this preliminary work.

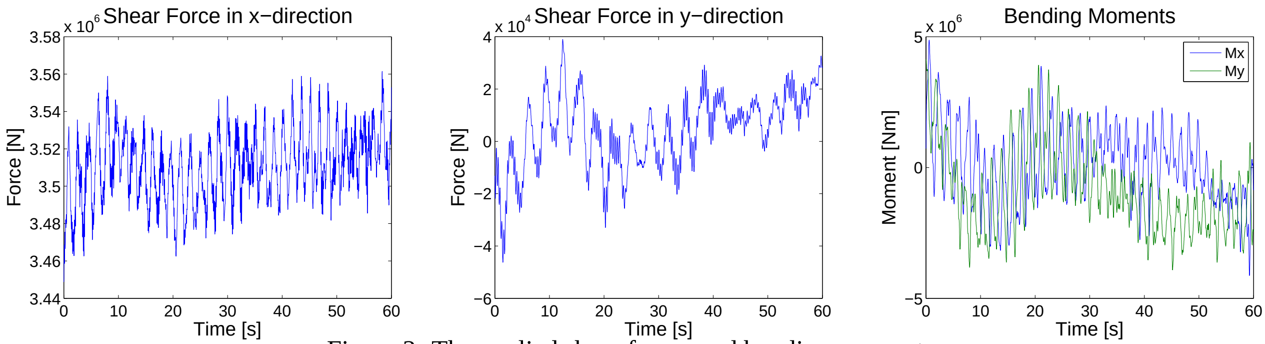

Figure 2: The applied shear forces and bending moments.

For every load time-history the displacements and stresses must be determined. In this work, the static stress analysis is conducted by use of the finite element method with linear assumptions. To apply the varying stresses and displacements in the fatigue analysis and the design sensitivity analysis, they must be reduced to a set of reversals.

4.2 Accumulated Damage

A log-log Wöhler diagram and the Basquin equation are utilized in order to determine the local damage caused by the loads:

$$\sigma_a(\mathbf{u}(\mathbf{x}),\mathbf{x}) = \sigma'_f(2N_{f_\sigma})^{b_\sigma} \tag{1}$$

$$\tau_a(\mathbf{u}(\mathbf{x}),\mathbf{x}) = \tau'_f(2N_{f_\tau})^{b_\tau} \tag{2}$$

σ_a and τ_a represent normal and shear stress amplitudes, respectively. $\mathbf{u}$ is the global displacement vector and $\mathbf{x}$ is the vector of all design variables v, that is $\mathbf{x} = [d_{grey}, d_{green}, d_{blue}, d_{yellow}, d_{red}, t_{grey}, t_{green}, t_{blue}, t_{yellow}, t_{red}]$. N_f is the number of cycles to failure, σ'_f is the fatigue strength for one reversal and b_σ is the regression slope, called the fatigue strength exponent, for normal stress. Since the loading conditions are multiaxial, it is very difficult to predict where the highest accumulated damage will occur. For this reason, the damage must be evaluated at many local points k for each stress cycle i. Accordingly, the local accumulated damage g_k can be calculated using Palmgren-Miners linear damage rule in combination with Eq. 1-2:

$$g_k(\mathbf{u}(\mathbf{x}),\mathbf{x}) = \sum_{i=1}^{N_i} \frac{n_i}{N_{f_i}} = \sum_{i=1}^{N_i} \left(\frac{n_i}{\frac{1}{2}\exp\left(\frac{\ln(\sigma_{a_i}(\mathbf{u}(\mathbf{x}),\mathbf{x})/\sigma'_f)}{b_\sigma}\right)} + \frac{n_i}{\frac{1}{2}\exp\left(\frac{\ln(\tau_{a_i}(\mathbf{u}(\mathbf{x}),\mathbf{x})/\tau'_f)}{b_\tau}\right)} \right) \leq \bar{g} \tag{3}$$

Here n_i is the number of reversals the structure is subjected to with the corresponding stresses. Fatigue failure is expected to occur at $\bar{g} = 1$. The subscript k refers to the specific constraint number, as Eq. 3 constitutes the fatigue constraints.

5. Problem Formulation

The optimization problem under consideration is to reduce the overall mass m of a given preliminary design taking fatigue constraints into account. All other structural criteria are not included in this preliminary study. The requirements for the preliminary design are that the topology and choice of material are fixed during the entire optimization procedure. The design variables are tube diameter d and thickness t. As five symmetry conditions are enforced to produce a double symmetric jacket design for easy manufacturing, the number of design variables are kept low. The cost function is defined as:

$$f(\mathbf{x}) = \sum_{i=1}^{n_e} \rho A_i(\mathbf{x}) L_i = m \tag{4}$$

Here ρ is the material density and n_e is the number of elements. A_i and L_i are the cross sectional area and length of element i, respectively. The finite element mesh is set up such that each element represents a Bernoulli-Euler beam between two joints. Evaluating the fatigue in the cross section in each end of each element will thus approximate the fatigue in the welds where failure is expected to occur. The optimization problem is defined as:

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}) \tag{5}$$

$$\text{subject to} \quad g_k(\mathbf{u}(\mathbf{x}),\mathbf{x}) \leq \overline{g} \quad \forall k \tag{6}$$

$$\underline{d} \leq d_s \leq \overline{d} \quad \forall s \tag{7}$$

$$\underline{t} \leq t_s \leq \overline{t} \quad \forall s \tag{8}$$

The overline and underline represent the upper and lower limits and the subscript s represents the symmetry group, or element patch, in which the design variable belongs. As there is a very large number of highly nonlinear constraint functions, g_k, the optimization can be quite difficult to control. Moreover, as there are 6,000 different load combinations in the applied time-history load, it is imperative that the number of design iterations is kept to a minimum in order to reduce the computational demand.

6. Design Sensitivity Analysis

In order to use gradient based methods, design sensitivity analysis (DSA) needs to be performed, that is, the gradients of the cost function and the constraints with respect to the design variables must be determined. The DSA is performed analytically to ensure accurate and fast gradient evaluations.

6.1 Derivative of the cost function

As the cost function defined in Eq. 4 is an explicit function of a given design variable x_v, it is easily determined as:

$$\frac{df(\mathbf{x})}{dx_v} = \sum_{i=1}^{n_e}\left(\rho\frac{dA_i(\mathbf{x})}{dx_v}L_i\right) \tag{9}$$

6.2 Derivative of the constraint function

The constraint function defined in Eq. 3 is a function of the design variables, and also the displacements which are in itself a function of the design variables. This relationship will no longer be shown in the equations. Two different DSA methods will be presented; the direct differentiation method and an aggregated adjoint method. Using the direct differentiation method, the full derivative of the constraint with respect to a design variable x_v is given as:

$$\frac{dg_k}{dx_v} = \sum_{i=1}^{N_i}\left(\frac{\partial g_k}{\partial x_v} + \frac{\partial g_k}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dx_v}\right) \tag{10}$$

The derivative of the displacement with respect to the design variables is the computational demanding part of this equation. This part is omitted when using the adjoint method. The partial derivatives are determined using the chain rule of differentiation:

$$\frac{\partial g_k}{\partial x_v} = \sum_{i=1}^{N_i}\left(\frac{\partial g_k}{\partial \sigma_a}\frac{\partial \sigma_a}{\partial x_v} + \frac{\partial g_k}{\partial \tau_a}\frac{\partial \tau_a}{\partial x_v}\right) \tag{11}$$

$$\frac{\partial g_k}{\partial \mathbf{u}} = \sum_{i=1}^{N_i}\left(\frac{\partial g_k}{\partial \sigma_a}\frac{\sigma_a}{\partial \mathbf{u}} + \frac{\partial g_k}{\partial \tau_a}\frac{\tau_a}{\partial \mathbf{u}}\right) \tag{12}$$

The stress sensitivities are found analytically. In the adjoint formulation, a Lagrange multiplier vector, λ, is introduced to omit the implicit and computational demanding $d\mathbf{u}/dx_v$. The full derivative using the adjoint method is given as:

$$\frac{dg_k}{dx_v} = \sum_{i=1}^{N_i}\left(\frac{\partial g_k}{\partial x_v} - \lambda\frac{d\mathbf{K}}{dx_v}\mathbf{u}\right) \tag{13}$$

$\mathbf{K}$ is the global stiffness matrix. The Lagrange multiplier vector is solved as:

$$\mathbf{K}\lambda = \frac{\partial g_k}{\partial \mathbf{u}} \tag{14}$$

The calculation costs of the Lagrange multipliers are severely affected by the very large number of constraints. The amount of constraints can be reduced to one by aggregation functions, making the adjoint formulation very effective. The aggregation function sums all n_k constraints into a global constraint. The applied aggregation functions are the Kreisselmeier-Steinhauser, the mean p-norm and the p-norm method. Depending on which aggregation method used, the global constraint is either an over or underestimate of the highest real constraint value. In the following, the formulation using the p-norm aggregation function is outlined. The single global constraint is then given as:

$$g^{p-norm} = \left(\sum_{k=1}^{n_k} \left(w_k (g_k - f^0) \right)^p \right)^{1/p} \tag{15}$$

w_k is a weight factor, f^0 is an ideal value and p is a curve fitting factor. The constraint sensitivity using the adjoint method and p-norm aggregation is thus given as:

$$\frac{dg^{p-norm}}{dx_v} = \sum_{i=1}^{N_i} \left(\frac{\partial g^{p-norm}}{\partial x_v} - \lambda^{p-norm} \frac{dK}{dx_v} \mathbf{u} \right) \tag{16}$$

Where λ^{p-norm} is attained in a similar way as before.

7. Framework

The authors have established a framework for optimization of a 5 MW reference wind turbine jacket from UpWind [10] to demonstrate the proposed method. The highly idealized jacket is modeled as a Bernoulli-Euler 3D beam finite element model in MATLAB, see Figure 1. The initial design variables are seen on Table 1. The jacket is simplified as a fixed-free model and only include wind loads. The wind loads are based on very simplified dynamic multibody simulations of the wind-induced response of the turbine. These simulations present two shear forces and two bending moments at the jacket top. A total of 6,000 force and moment combinations are applied in the analysis. These loads represent a mean wind speed of 10 m/s applied in a constant direction, that is, orthogonal to the turbine blades.

Table 1: Initial beam dimensions of the jacket.

Symmetry Group	**Red**	**Yellow**	**Blue**	**Green**	**Gray**
Diameter	1.20m	1.20m	1.20m	1.20m	0.80m
Thickness	0.040m	0.040m	0.035m	0.050m	0.020m

7.1 Modeling Limitations

In its current form, the constraints do not take sequential effects, multiaxial effects, environmental effects, and non-proportionality effects into account. Moreover, the finite element formulation does not take material or geometric non-linearities into account. As the jacket is in high-cycle regime, the assumption of linear material behaviour is sound. The applied time-history loads determined by time-marching multibody simulations are very simplified. The largest errors are that the wind is applied in a constant angle and that the normal loads and torsional moments are not included. No hydrostatic loads are enforced on the submerged part of the jacket and the soil-structure interaction is simplified as fixed-free. Furthermore, the applied loads do not change when the design variables change. However, for proof-of-concept of the initial method, these assumptions are deemed acceptable.

8. Results

The design sensitivities are verified using central finite difference with a fixed perturbation of 1/100,000 of the original design variables. Results for two diameter and two thickness sensitivities are shown on Table 2 in root mean square percentage error. The remaining sensitivities have similar marginal deviations. The author's find the results very promising, especially since a fixed perturbation was applied. No optimization results are shown, as they will not reflect anything realistic until at least more representative time-history loads and the prescribed Det Norske Veritas design guidelines are applied.

Table 2: Root mean square percentage error compared to central difference approach.

DSA	dg/dx_1	dg/dx_2	dg/dx_6	dg/dx_7
Direct Differentiation Method	0.0004%	0.0018%	0.0007%	0.0022%
p-norm Adjoint Method	$6.5e^{-7}$%	0.0004%	$6.8e^{-6}$%	0.0004%

9. Discussion

Two different methods of performing the design sensitivity analysis have been presented. The suggested method depends entirely on the problem at hand. In the direct differentiation method accuracy is preserved. This method can, however, be time consuming when the optimization contains many design variables. The aggregated adjoint method is much faster at the cost of some accuracy. Both methods can, however, be applied for fatigue optimization of jacket structures for offshore wind turbines.

In its current state of development the algorithm will, to some extent, always favor a high moment of inertia. This means that the diameter will increase and the thickness will be lowered in each tube member in order to reduce mass. However, as no buckling constraints are included, poor choices of bounds on the design variables will result in buckling and ultimately total collapse of the structure. This fatigue optimization should not stand on its own; all analyses prescribed by Det Norske Veritas should still be carried out to ensure a reliable structure. Optimizations run by the authors indicate that the damage is currently underestimated. This can be explained by several observations: The simplified load time-history currently used does not include normal loads or the torsional moments induced by the wind. Furthermore, only one minute of a load time-history with a mean wind from a constant angle is used and then scaled to represent the desired lifetime. Including several load time-histories from different angles with different mean wind speeds will produce far more fatigue damage, and this will result in a better representation of the actual accumulated damage. Moreover, including hydrostatic loads and a complex soil-structure interaction model will also present a higher accumulated damage. Including offshore design guidelines will obviously also enforce a safety factor on the fatigue damage. Lastly, including additional constraints such as maximum displacement and eigenfrequency constraints will be very beneficial for the overall method. When the method is elevated to include some or all of the aforementioned, the authors believe that it can serve as a very powerful and efficient tool for optimizing a jacket structure under operational conditions. Furthermore, the method can easily be applied in other fatigue driven structural design problems such as aerospace and automobile industries.

10. Acknowledgements

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