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# Fatigue Life Optimisation of Damage Tolerant Structures using Design Space Exploration

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**1. Abstract**
In parametric design studies, the strength of a structure is often considered as the primary design criterion, and consequently the optimal (best) structural design is often chosen as the one that minimises the maximum stress generated. However, for structures whereby failure is governed by fracture or fatigue, residual strength and fatigue life, as distinct from stress, need to be considered as the explicit design objectives.
In this study, the design space for fatigue life for different structural configurations is evaluated to demonstrate the utilities of design space exploration for damage tolerance design optimisation. This was illustrated using the problem of the optimum design of a cutout shape with boundary cracks under biaxial load. The minimum fatigue life associated with the cracks was evaluated for each cutout geometry.
The design surface for fatigue life establishes that a design based on damage tolerance parameters poses a well-behaved optimisation problem with a well-defined minimum/maximum region. The design space was found to be flat for fatigue life, enabling the specification of design tolerances. The optimum values of the fatigue life obtained from the design space agreed well with those determined using various optimisation methods. It is shown that a design space exploration can provide a systematic way to reduce the weight of a structure by adopting a 'feasible non-optimal' solution that meets the design criteria, rather than aiming for the 'optimal' (best) solution.
**2. Keywords:** Design space; Shape optimisation; Damage tolerance; Fatigue life; Finite element analysis.

## 3. Introduction

The late twentieth century saw the development of sophisticated structural analysis methods, which led to the use of light weight structures with low design safety factors. This resulted in high operational stress levels. High service stresses increase the likelihood of crack initiation and propagation. Furthermore, structures are now being fabricated using high strength materials that have a relatively low resistance against crack propagation. This problem is of prime concern in the aerospace industry where weight reduction is an important consideration. This led to the inception of damage tolerance design philosophy in which the presence of cracks and defects in a structure is taken into account. To address this, we previously developed a range of damage tolerance optimisation techniques based on a heuristic algorithms [1-5], and applied it for maximising the residual strength and fatigue life of structures.

Design optimisation including damage tolerance parameters is an inherently iterative process. One challenge often faced by a designer is to automate the evaluation of several potential designs. Design space exploration can be of immense aid in obtaining a collection of 'preliminary' improved designs that (partially) meet the design specifications and can assist in further optimisation of structures. The aim of the present study is to demonstrate the advantages of design space exploration for durability based designs. The effectiveness and utilities of design space exploration in the context of damage tolerance optimisation are demonstrated. In particular, one purpose of this paper is to evaluate the characteristics of the design space fatigue life. The optimum solutions obtained via the design space study are compared with those predicted by different structural optimisation methods. The utility of design space exploration in designing light-weight structures is also emphasised.

A design space for structural designs is a collection of structural responses (i.e. the values of objective and constraint functions) for various structural geometries and/or configurations (expressed by combinations of design variables). One way to perform a detailed (iterative) design study is to visualise the partial or entire design space. The initial step for this is similar to that of optimisation. The 'design problem' is to be cast as an equivalent 'standard optimisation problem', in terms of a set of design variables and objective and constraint functions. The design space can then be determined by analysing the structure for each possible combination of the design variables (design point). Design space studies for multiple design criteria can aid in assessing the relative roles of multiple design objectives. For example, the variation of the minimum fatigue life and the maximum stress for different shapes provides the designer with an insight into the relative performance of these shapes. A designer has the option of choosing a design in the vicinity of an optimum solution rather than selecting the 'fatigue life optimum point' itself as the final design. This compromise may be necessary to meet a maximum strength requirement, or to satisfy specific operational constraints. In this paper, we will evaluate the characteristics of the design space of one of the primary damage tolerance criteria, i.e. fatigue life of a structure via a simple example.

## 4. Example Problem

4.1 Problem Description

Design space exploration is illustrated through the simple problem of 'the optimum design of a cylindrical (through-the-thickness) cutout located in a rectangular block under biaxial loading'. This specific problem was selected as it has been used in the previous optimisation studies by the present and other authors in the literature [3, 4, 6, 7]. Hence, this will enable us to correlate and compare the 'optimum point(s)' observed in the design space with those obtained using the different optimisation methods.

The problem geometry, loading and boundary conditions are shown in Figure 1. It is a three-dimensional rectangular block, 320 mm wide, 320 mm high, with a thickness of 20 mm, and has a circular through-the-thickness cutout at its centre. The diameter of the initial cutout was 20 mm. The material of the block was assumed to be an aluminium alloy (2219-T851) with a Young's modulus of 71 GPa and a Poisson's ratio of 0.3. A one-eighth model of the block along with the loads and constraints was considered, because the geometry, loading, and constraints are symmetric about the three planes (*xy*, *yz*, and *xz*), as shown in Figure 1. Symmetry boundary conditions were imposed on the planes (*xz, yz* and mid-*xy* planes) by constraining the appropriate displacements ($u_x$, $u_y$ and $u_z$) and rotations ($\theta_x$, $\theta_y$ and $\theta_z$). All the planes (*xy*, *yz* and *xz*) mentioned in the rest of the paper refer to Figure 1.

A simple constant amplitude fatigue loading was assumed. The block was subjected to fluctuating (cyclic) stresses in the horizontal (*x*) and vertical (*y*) directions. The mean stresses for the present problem were 75 and 150 MPa, respectively, in the *x* and *y* direction, and the corresponding stress amplitudes were 25 and 50 MPa, respectively. The minimum fatigue life associated with the cracks was taken as the design criterion or objective function.

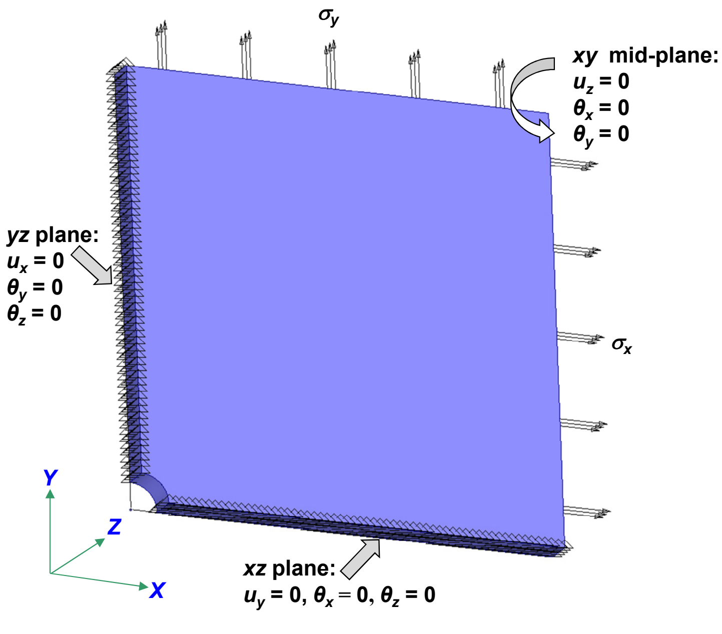


Figure 1: Schematic of the one-eighth model of a cylindrical cutout in a rectangular block under biaxial load ($u_x$, $u_y$, and $u_z$ denote the displacements along the *x*, *y* and *z* axes, respectively, and $\theta_x$, $\theta_y$ and $\theta_z$ denote the rotations about the respective axes).

4.2 Crack Modelling

A number of surface cracks were modelled on the hole boundary. All the cracks were assumed to be semi-elliptical flaws emanating from the hole surface with their major axes (*c*) parallel to the axis of the hole (*z* axis) and minor axes (*a*) normal to the hole surface, see Figure 2. An initial crack spacing approximately equal to the smallest crack size was used to achieve an effective modelling of the stress intensity factor and fatigue life variation along the structural boundary. Here we modelled 21 three-dimensional semi-elliptical cracks along the surface of the cylindrical hole (for one quarter) resulting in an initial crack spacing of ~0.75 mm. Each crack on the structural boundary was assumed to grow in the direction of the major and minor axes from an initial size of ($c_i$, $a_i$) to its final size of ($c_f$, $a_f$). The initial surface flaws were assumed to be of size, $c_i$ = 5 mm, $a_i$ = 2 mm, and the final acceptable flaw size was taken as, $c_f$ = 8 mm, $a_f$ = 4 mm. These specific flaw cases were chosen, because these were the representative cases previously studied [3, 4]. All the cracks were grown simultaneously for fatigue life design space evaluation.

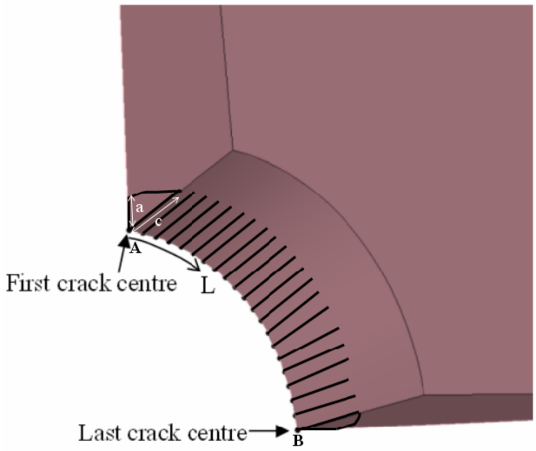


Figure 2: Locations of the three-dimensional semi-elliptical cracks along the hole surface (one-eighth model)

4.3 Geometry Representation
The geometric representation of the hole shape in the *xy* plane is given by:

$$\frac{x^p}{a^p}+\frac{y^p}{b^p}-1=0 \tag{1}$$

where *a, b,* and *p* are the shape parameters. The shape of the hole is altered by varying these parameters. Any combination of them can be chosen as the design variables for optimisation. As such, this geometric description is ideally suited to the present problem of the optimum design of a cutout in a rectangular block under biaxial loading. Hence, in the present study, Equation 1 was used to generate the design points on the hole surface for a given combination of design variables (*a*, *b*, and *p*).

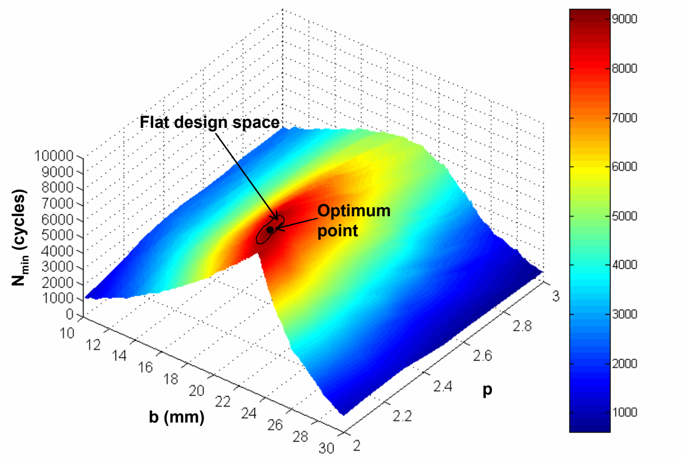


Figure 3: Fatigue life study: Design space plot of the objective function (the minimum fatigue life $N_{min}$) with hole geometric parameters, hole size (*b*) and curvature index (*p*)

## 5. Design Space Exploration

The design parameters, hole dimension (*b*) and curvature index/exponent (*p*), were varied to generate a set of design points to investigate the nature of variation of the minimum fatigue life associated with all the cracks on the boundary for different cutout shapes. The major axis *b* was varied from 10 to 30 mm, and the index *p* was varied from 2 to 3, generating a total of 1287 design points. This enabled a reasonably accurate representation of the design space, which was also used for validating the previous optimisation results. The design space for the minimum fatigue life ($N_{min}$) is presented in Figure 3. As expected, the surface takes the shape of an 'inverted ship hull', which is intuitive. The maximum point (optimal hole) corresponds to a fatigue life of 9214 cycles. The size ($b$) and curvature ($p$) of the optimal hole shape are 22.8 mm and 2.1, respectively. The fatigue life optimal cutout is shown in Figure 4. It is noteworthy that for fatigue life, the optimal points exist on the plane $p$ = 2.1, and the optimal shape represents a 'super-ellipse'.

The design space of fatigue life of the cutout in Figure 3 is found to be flat for fatigue life. This 'flat' design space can be thought of as a set of local 'optimums' clustered in a small region. Since all of these optimum points have approximately the same value of the objective function ($N_{min}$), it is thus appropriate to conclude that this class of problems has a 'global optimum region' instead of a global optimum point. Furthermore, the flatness of the design space establishes that the design is robust in a Taguchi sense. The width of the flat region is within 10% of the average value of the design variables. So this is within the commonly adopted manufacturing tolerances. This insensitivity of the optimum point or flatness of the optimum region implies that it is feasible to extract the maximum fatigue life taking into account the variability in typical industrial manufacturing processes.

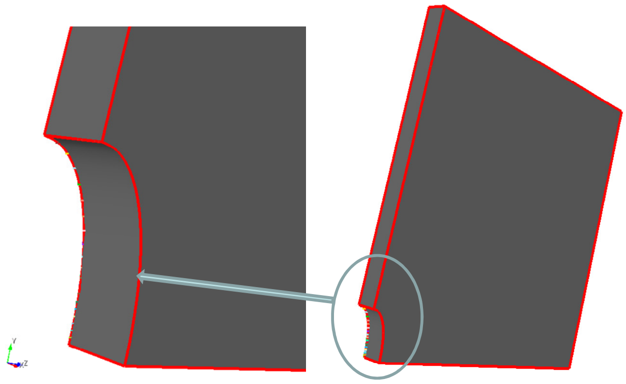

Figure 4: Fatigue life optimal shape with $b$ = 22.8 mm and $p$ = 2.1, with life optimised shape being larger than the initial cutout, leading to weight reduction

**6. Relationship of Design Space Study with Structural Optimisation**

Design space exploration and optimisation are closely related in that in structural optimisation we move through the design space using an algorithm to improve a current design, whereas in a design space study we attempt to obtain the overall variation in the design objective function with structural geometry/shape. Indeed, one of the earlier optimisation algorithms, known as the 'random search method', utilises a similar concept [8]. For many optimisation problems the optimum solution may not be unique and often depends on the initial (starting) shape, especially if multiple (local) optimum points exist. In such cases an initial examination of the nature of the design space can help set a starting solution that would (eventually) converge to an improved (local) optimum point. This can lead to a significant improvement in the structural performance in cases where there is a considerable variability among the objective functions associated with different local optimum shapes.

In contrast, for the present problem of the cutout shape design with fatigue life as the design objective, an initial design space evaluation can save computational time. The realisation that the design space around the optimum point is 'flat' means that once a design point is in the 'near' optimal zone, any solution in the neighbourhood could be taken as an acceptable design, because the fatigue life of the structure will not improve appreciably by further refining the solution to locate the 'precise' optimum point. In the context of practical structural designs, the extent of flatness in the design space can be used to specify the manufacturing tolerances during the design stage without compromising the fatigue life of the resultant structure.

Design space analysis can also be used for verifying the reliability and assessing the performance of optimisation algorithms before applying them to design optimisation of a relatively complex structure. In the previous studies, we performed the same damage tolerance based cutout optimisation problem using two 'fundamentally' different optimisation methods, a Biological method [3, 4] and nonlinear programming methods [6, 7]. The optimum results from the design space evaluations for fatigue life are compared with those obtained using the different optimisation methods [3, 4, 6, 7].

The hole dimensions and curvatures obtained using a heuristic and a gradient-based method are compared with those observed from the design spaces in Table 1. To compare the optimum point identified using the design space study for fatigue life, the hole shape optimisation was performed using the 3D Biological algorithm [4] and the nonlinear programming method [7]. The initial and final flaw sizes were the same as used in the design space study, i.e. $c_i$ = 5 mm, $a_i$ = 2 mm, and $c_f$ = 8 mm, $a_f$ = 4 mm. The fatigue life at the optimum point and the cutout geometry parameters are presented in Table 1. All the three approaches essentially predicted the same 'near' optimum solution.

Table 1: Comparison of the fatigue life optimisation results with the design space study

| **Methods** | **Hole major axis (*b*)** (mm) | **Hole curvature (*p*)** | **Objective function ($N_{min}$)** (cycles) |
|---|---|---|---|
| Biological method [4] | 22.909 | - | 9043 |
| Nonlinear programming method [7] | 22.876 | 2.132 | 9154 |
| Design space study | 22.8 | 2.1 | 9214 |

**7. On Weight Reduction and Optimum Design**

A design space plot can further help in lightening a shape by exploring alternative designs. There are cases when the design is deemed to be acceptable, but the structure is thought to be too heavy. This can be illustrated using the present example. Since in this example, a rectangular block supposedly from a generic structural component is used, the total weight of the structure is unknown. So the volume of the cutout shape is used instead to identify the weight savings. An increase in the cutout volume will lead to weight reduction of the resultant structure. Figure 5 presents the volume of the cutout (normalised relative to the volume of the initial circular cutout) for different shapes. By combining Figures 3 and 5, a non-optimal design point could be chosen that would have a lower weight, yet maintaining an adequate fatigue life.

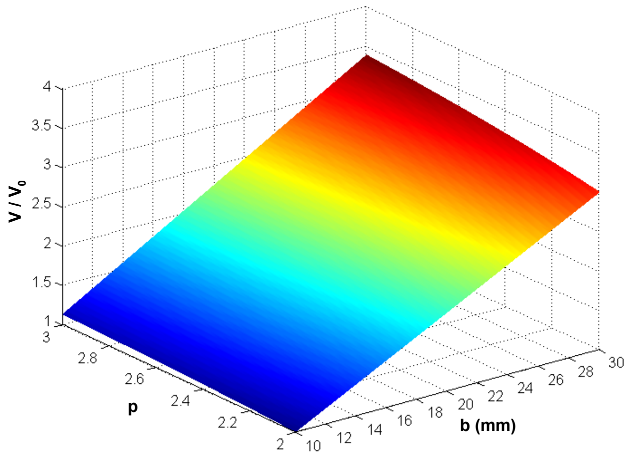


Figure 5: Volume (normalised) of the cutout at various design points

To illustrate this concept, let us consider a series of cutout shapes, all having the same (optimum) curvature index of $p$ = 2.1. Figure 6 shows the variation of the (minimum) fatigue life with the cutout volume ratio ($V/V_0$) for the optimum curvature index ($p_{opt}$ = 2.1). The fatigue life increases as the hole enlarges and it reaches a maximum value of 9,214 cycles at $V/V_0$ = 2.322. Beyond this optimum point the fatigue life reduces with an increase in the cutout volume. In this case an optimum shape leads to a weight saving of ~2.32 times the volume of a circular hole, in addition to a significant gain in the fatigue life (~7.9 times that of a circular hole).

However, the shape can be further lightened if the desired fatigue life ($N_{design}$) is lower than the maximum fatigue life that can be achieved by adopting an optimal shape. This is illustrated in Figure 6. For example, if the design life is $N_{design}$ = 7000 cycles, then a line AB corresponding to $N_{min}$ = 7000 cycles can be drawn in Figure 6. Any point above AB will constitute an acceptable design with a life $N_{min} > N_{design}$. In such a case the shape corresponding to point B ($V/V_0$ = 2.474) will provide the largest (acceptable) hole shape, or the lightest shape satisfying the fatigue life design limit. In this case, adopting a 'non-optimal feasible' design can lead to a further weight saving of ~6.5% over the fatigue life optimal shape. This weight reduction can be enhanced if the design fatigue life is further lowered, see Figure 6.

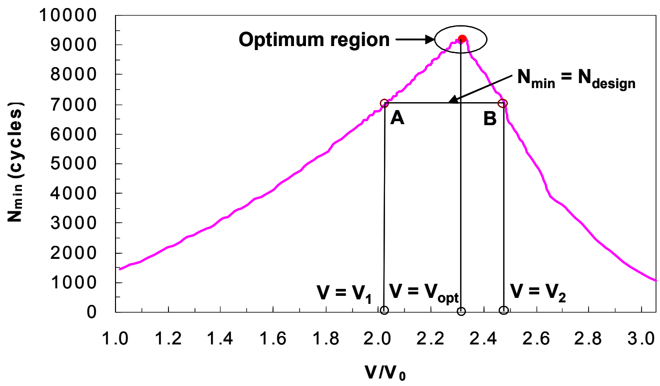


Figure 6: Variation of the minimum fatigue life with (normalised) volume for a super elliptical ('near' elliptical) hole with optimum curvature index ($p_{opt}$ = 2.1)

**8. Conclusions**

In this paper, a design space exploration study has been undertaken to understand the nature and variation of the damage tolerance based objective functions with structural geometry and to illustrate the utility of design space exploration in the context of durability based design optimisation. The design space study was first demonstrated using the problem of the optimisation of a cutout shape under biaxial load with fatigue life as the design criteria. The shape of the cutout was parametrically represented using super ellipses. The minimum fatigue life associated with the flaws along the structural boundary was evaluated for various hole geometries to construct the design space.

The design surface for fatigue life it resembles an 'inverted ship hull'. These shapes confirm that the design based on fatigue life, indeed poses a well-behaved optimisation problem, i.e. a well-defined maximum region exists.

One benefit of a design space study is that it can provide an 'overall view' of the objective function distribution. From earlier studies it was concluded that for this category of problems multiple 'local' optimums can exist. The present study has shown that a set of 'local' optimum solutions can exist in a 'close' neighbourhood, rather than lying apart as found in many other classes of structural optimisation problems. It is therefore contended that this class of damage tolerance optimisation problems has a 'global' optimum region, rather than a single global optimum point. This feature of damage tolerance optimisation has not been previously reported.

It was found that the design space is flat, which supported the earlier findings using the various optimisation methods. This signifies that from an engineering design point of view, the structural responses of various geometries in the 'near' optimal region will not be considerably different. Thus, it may be sufficient to choose one of the shapes in the 'near' optimal region as the final design. This can immensely reduce optimisation effort and computational time, and also enable us to extract the optimum performance accounting for manufacturing tolerances, as there will not be any need to precisely locate the (local/global) optimum solution. We can also lighten a structure by removing material appropriately from a 'near' optimal geometry without significantly degrading its durability related structural performance.

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