Assessing sensitivities of maneuver load alleviation parameters on buckling reserve factors using surrogate model based extended Fourier amplitude sensitivity test

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1. Abstract
To assess the impact of manuever load alleviation parameter changes on the buckling reserve factor a multidisciplinary high fidelity analysis is necessary. To this end flight maneuver calculation, linear static analysis using a global Finite Element Model and a structural sizing need to be performed. The use of surrogate modeling techniques helps to avoid the time consuming high fidelity analysis but still provides accurate results. In the frame of global sensitivity analysis the surrogate model is used to apply variance based extended Fourier amplitude sensitivity test. The contribution of input parameter variation of the manuever load alleviation system to the variance of the surrogate model output in terms of buckling reserve factors is measured. The sensitivity study is performed on the upper cover of a backward swept composite wing and the results are compared to those of the high fidelity analysis. Note that the variation of maneuver load alleviation parameters is nowadays assessed by external loads resulting from the flight maneuver calculation only. The presented approach includes reserve factors and hence provides an insight into the structural response. In this way those maneuver load alleviation parameters are found that affect the structure in terms of buckling reserve factors the most and can be used for future design changes and weight reductions.

2. Keywords: Surrogate model, sensitivity analysis, maneuver load alleviation, buckling, reserve factor

3. Introduction
Recent developments in the field of multidisciplinary analysis have led to an increased investigation of Maneuver Load Alleviation (MLA) effects on the aircraft wing [7]. Especially during early design phase of the aircraft the use of an active load alleviation system has a significant impact on the overall aircraft design and flight performance evaluation [8]. In this scope the present work demonstrates a surrogate model based sensitivity analysis on MLA parameters for the wing upper cover. The surrogate model is constructed using an already existing data basis which contains MLA parameter values and local skin buckling Reserve Factor (RF) values provided by the Airbus Operations GmbH. Note that the preparation of such a data basis takes in general several months and includes the work of different departments and disciplines. Figure 1 demonstrates how the high fidelity process looks like and in which way this long process is replaced by a surrogate model.

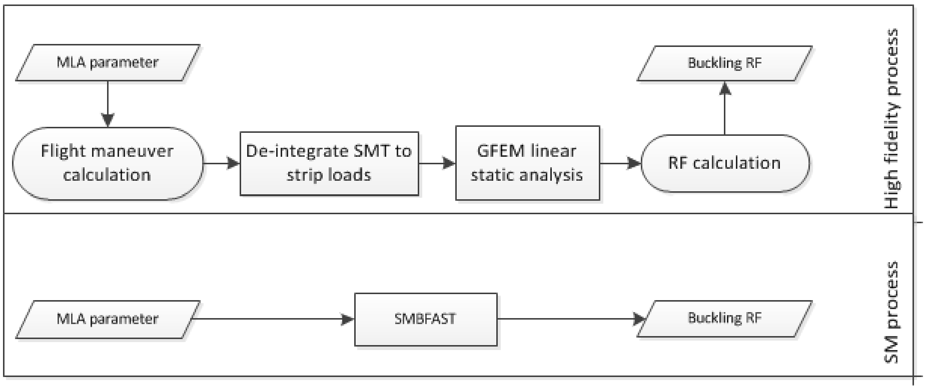

Figure 1: High fidelity versus surrogate model process overview

The surrogate model replaces the flight maneuver calculation, the de-integration of the Shear Moment Torque (SMT) values to strip loads along the wing and the linear static analysis using a Global Finite Element Model (GFEM), which consists of 1- and 2-dimensional elements, and finally the RF calculation.

High Dimensional Approximation (HDA) is used for the construction of the surrogate model and is based on the works of [5] and [6]. HDA uses a linear expansion of non-linear functions and works in a similar way as artificial neural networks. Internal validation of the surrogate model is performed using the concept of cross validation and the Root Mean Square (RMS) error as a failure index. The constructed surrogate model can be used to assess the effect of MLA parameter changes on the structure in terms of RF quickly. Previous works from [3] and [4] have shown a proper use of extended Fourier Amplitude Sensitivity Test (eFAST) applied on a surrogate model in order to perform sensitivity analysis. The combined use of both approximation and sensitivity analysis technique is called Surrogate Model Based Fourier Amplitude Sensitivity Test (SMBFAST).

4. Maneuver load alleviation vs local skin buckling reserve factor

The MLA is used for the reduction of the loads acting on the wing during flight maneuvers. For this purpose the vertical load factor n_z at center of gravity is measured by an accelerometer and is compared against a threshold value. If the threshold value is reached inner and outer ailerons are deflected upwards for positive n_z values. For negative n_z values the ailerons are deflected downwards. The aileron deflections lead to a redistribution of the lift along the wing with an additional reduction of the wing bending moment. This effect is shown on a simple example in Figure 2 for the left wing of a schematic passenger aircraft. Here the aileron upward deflections shift the center of pressure inboards which yield a reduction of the wing root bending moment. Note that the resulting pitching moments are compensated with additional deflections of the elevators included in the MLA law.

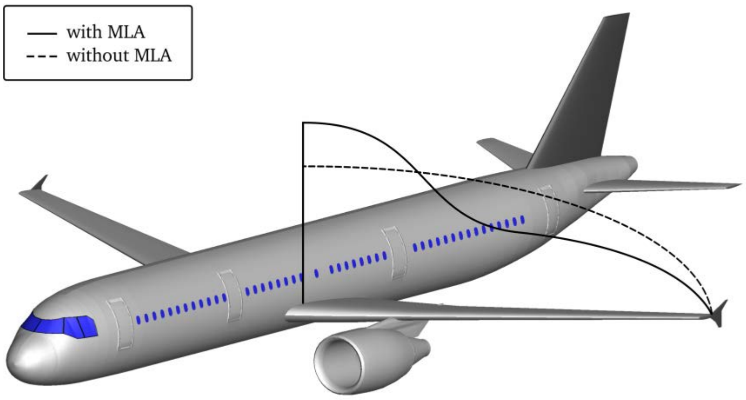

Figure 2: Schematic change of lift distribution with and without MLA

The MLA is integrated into the flight maneuver calculation software at Airbus and consists of different control laws depending on the flight condition. The load factor is the main driving parameter for the activation of the MLA and in addition positive/negative load factor values steer the upward/downward deflection of the ailerons. But still the available control laws prescribe aileron deflections based on the Mach number Ma and calibrated airspeed v_{cas} as well. Hence in the scope of this study the chosen MLA parameters for analysis are 3 of its steering quantities and the resulting deflection angles of the inner aileron ϑ_{in} and the outer aileron ϑ_{out}. Eq. (1) shows the relationship between the SMT values by which the change of lift distribution can be seen and the MLA as a function of previous 5 parameters. The same order of MLA parameters is used for the construction of the surrogate model.

$$SMT_{MLA} = MLA(\vartheta_{in}, \vartheta_{out}, n_z, Ma, v_{cas}) \tag{1}$$

The change of the lift distribution along the wing with an active MLA leads to different wing bending moments which again influence the wing torque due to the sweep angle. This combined bending and torque leads to compression and shear loads in the skin panels of the wing upper covers while the supporting stringers are axially loaded only. The RF value is an estimator in order to evaluate if the structure can sustain these loads. The RF value is the comparison of the allowable stress value against the stress values of the applied loads, cf. Eq. (2). The structure is statically sized if the value is higher than 1 and fails for values lower than 1.

$$RF = \frac{\sigma_{allowable}}{\sigma_{applied}} \lesseqgtr 1 \tag{2}$$

After the internal loads are computed on GFEM level many ways exist to compute the RF values during the structural sizing. For local skin buckling an analytical approach based on energy method is used for the presented study. The different stringer bays are computed individually for local skin buckling what reduces the region of analysis to 1 stringer and 2 skin panels. The idealized stiffened panel is shown in Figure 3. The skin panels are simply supported at all edges with the stringer itself providing simply support. The approach of Rayleigh-Ritz is used then to solve the energy equation. More about the solution of stability problems is found in [10].

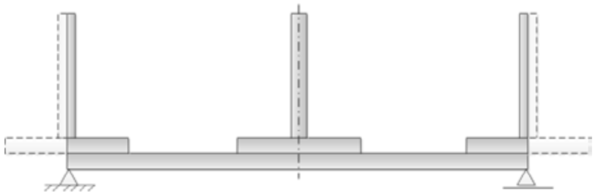

Figure 3: Schematic view of a stiffened panel

5. Surrogate model based extended Fourier amplitude sensitivity Test

In order to perform SMBFAST the software toolkit MACROS is used which is developed by the company DATADVANCE. The chosen method for the construction of the surrogate model is called HDA. Previous observations of [5] have shown superiority in time performance and accuracy compared to techniques like artificial neural networks, Kriging and radial basis functions. Unlike artificial neural networks which is based on a single type of basis functions HDA is based on linear expansion of different types of functions which are namely sigmoid, radial basis and linear functions. The linear expansion of the basis functions is done according to the approach shown in Eq. (3). Here the exact function $f(X)$ is approximated by a superimposition of mentioned basis functions φ_j and their weighting values α_j. The number of superimpositions p is estimated by a boosting algorithm until the accuracy is converging, cf. [6].

$$f(X) \approx \tilde{f}(X) = \sum_{j=1}^{p} \alpha_j \varphi_j(X) \tag{3}$$

For the purpose of linear expansion techniques like elastic net and regularization are used for parameter initialization and an additional hybrid learning algorithm is used to improve these initial parameters. The latter is based on regression analysis and gradient based optimization.

In order to check the accuracy of the surrogate model the data basis is partitioned into a training set n_{train} and a test set n_{test} following the principles of cross validation. The training set is used to build the model while the purpose of the latter is its validation. The RMS error is recommended by [9] for the cross validation of the surrogate model and takes into account the exact sample values y_i and the predicted values $\tilde{f}(x_i)$ as shown in Eq. (4).

$$RMS = \varepsilon\left(\tilde{f}(X)\middle|X_{test}\right) = \sqrt{\frac{1}{n_{test}}\sum_{i=1}^{n_{test}}\left(y_i - \tilde{f}(x_i)\right)^2} \tag{4}$$

eFAST is applied on the constructed surrogate model in order to assess the sensitivity of input parameter variations on the output values. eFAST is a variance decomposition method for global sensitivity analysis, cf. [3]. The presented core ideas are mainly based on the work of [1] with a proposal for computational implementation described in [2].

The variance value s^2 takes into account the model output y_i as well as its mean value $\bar{y}$ and is an estimator for the change of the model output resulting by input changes. Eq. (5) shows the variance value in a general notation.

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \tag{5}$$

The idea using eFAST is to split the total variance value into partial variances according to the contribution of each input dimension. This variance splitting is performed in the frequency domain using a Fourier transformation of the surrogate model. The input parameters are then varied at frequencies according to [1] for which the Fourier coefficients are determined. If an input parameter has high influence on the model output the amplitude of the oscillation in the frequency domain is high. The surrogate model is expanded into a Fourier series in the frequency domain as follows:

$$\tilde{f}(s) = \sum_{k=-\infty}^{+\infty}\{A_k\, cos(ks) + B_k\, sin(ks)\} \tag{6}$$

Here the Fourier coefficients A_k and B_k are defined as:

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \tilde{f}(s)cos(ks)dx \;\; ; \;\; B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \tilde{f}(s)sin(ks)dx \tag{7}$$

Using these Fourier coefficients the partial variance value s_i^2 for each input dimension is calculated in the following way:

$$s_i^2 = 2\sum_{k=1}^{\infty}(A_k^2 + B_k^2) \tag{8}$$

Finally the main effect S_i is determined using the partial and total variance values:

$$S_i = \frac{s_i^2}{s^2} \tag{9}$$

The main effect can take values between 0 and 1 while higher values indicate more influence of the input parameter on the model output. For example a main effect value of 0.3 for an input parameter means that this parameter contributes by 30% to the total variance.

The main effect takes only the individual contribution of each input parameter variation on the model output into account. On the other hand the total effect S_{Ti} which is only available using the eFAST considers the interaction effects S_{ij} between the input parameters as well. Interaction effects quantify the impact on the output variance when multiple parameters are varied simultaneously. For instance the main effect of one input parameter can tend to zero whereas its total effect can reach high values due to its high effect on the model output in combination with another input parameter. Eq. (10) shows how the interaction effects are added to the main effect of each input dimension.

$$S_{Ti} = S_i + \sum_{j=1}^{n} S_{ij}\, , with\; j \neq i \tag{10}$$

6. Available data basis for the construction of the surrogate model

The data basis for the construction of the surrogate model is provided by the Airbus Operations GmbH and consists of MLA parameter values and local skin buckling RF values. For the MLA parameters input as well as output quantities of the control law are chosen. The main input quantities that affect the results of the MLA are the load factor n_z, the Mach number Ma and the calibrated airspeed v_{cas}. The resulting inner and outer aileron deflections are taken into account as well. The RF values are computed for each skin panel of the wing upper cover but the minimum value for each load case overall skin panels is considered only.

1522 different steady longitudinal maneuvers are provided using different flight parameters and which correspond to EASA CS 25.331 requirements. The load cases are purely mechanical ones and don't consider any thermal or pressure contribution.

Table 1 summarizes the 5 different input dimensions and the output dimension with the range of their values. In this way the input matrix for the HDA has the size 1522x5 and the output vector 1522x1.

Table 1: Available data basis for surrogate model construction

RF	ϑ_{in} / deg	ϑ_{out} / deg	n_z	Ma	v_{cas} / m/s
$[RF_{min}; RF_{max}]$	[0; -25]	[15; -30]	[-1; 2.5]	[0.4; 0.96]	[123; 193]

While the input parameters are mentioned in their original scale the RF values are stated in form of RF_{min} and RF_{max}. Here RF_{min} and RF_{max} refer to the minimum and maximum values overall load cases which are already in the output vector.

7. Results using surrogate model based Fourier amplitude sensitivity test

For the interpretation of the results the surrogate model is discussed first and later on a conclusion on the main and total effects resulting from the eFAST follows. In order to construct the surrogate model the RMS error value is taken as an indicator for its reliability.

In the original data basis with 1522 load cases very high RF values exist which are like noise for the training set. Hence the data basis is reduced by the number of load cases using the RF values in order to minimize the RMS error. The HDA is run 4 times with a different threshold for the RF each. From the first to the second run all the load cases which yield a RF value that is higher than 1.77% * RF_{max} are removed which reduces the number of load cases in the data basis from 1522 to 783. At the same time the RMS error is reduced from 3.04 to 0.11. Further due to this load case selection all the negative load factors are removed and with this all the positive aileron deflections. One can conclude that the flight cases with negative load factors are not relevant for the structure in the scope of this study since these cases are noisy data in terms of very high RFs. The same principle of load case selection is applied to the following HDA runs, which is summarized in Table 2.

Table 2: Internal validation of surrogate model using data basis in original and reduced form

	HDA 1	HDA 2	HDA 3	HDA 4
Nr. Load Cases	1522	783	567	366
RF	RF_{max}	1.77% * RF_{max}	1.50% * RF_{max}	1.33% * RF_{max}
RMS	3.04	0.11	0.08	0.05

The optimal solution is found in the third HDA run because here we have a RMS error below 0.10 and still have an adequate number of load cases for later SMBFAST (according to DATADVANCE sample size needs to be at least around 100 times dimensionality of input matrix).
Figure 4 shows the absolute error between real and predicted RF values using the different HDA runs as described in Eq. (11). Here the predicted RF values resulting from the constructed surrogate models are compared to the real RF values overall available load cases. One can see that the absolute error value decreases from the first until the last run with a reducing number of load cases.

$$err_{RF} = RF_{real} - RF_{predicted} \tag{11}$$

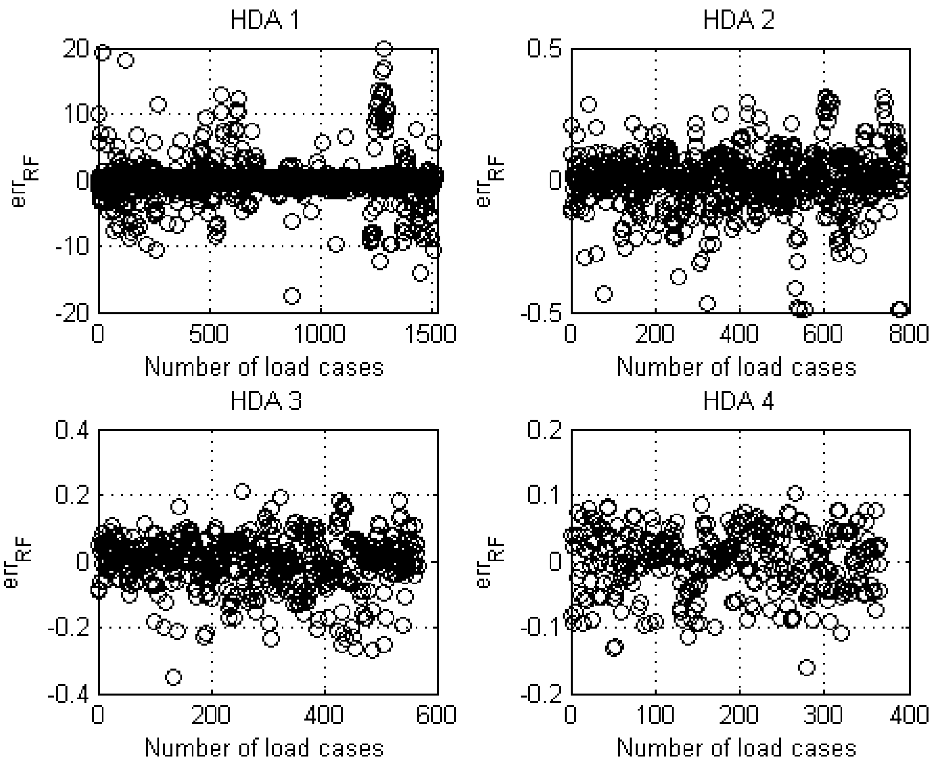

Figure 4: Comparison of HDA runs using err_{RF}

In addition Table 3 shows the change of the design space for each HDA run. The biggest change in the design space occurs from the first HDA run to the second one. In addition to the previously mentioned removal of positive aileron deflections and negative load factors here the smallest value for the calibrated airspeed has changed from 123 m/s to 141 m/s. The design space for the following HDA runs is still filled properly although the number of load cases is reduced by around 200 cases each.

Table 3: Change of the design space due to load case reduction by RF values

HDA loop	RF	ϑ_{in} / deg	ϑ_{out} / deg	n_z	Ma	v_{cas} / m/s
1	$[RF_{min}; RF_{max}]$	[0; -25]	[15; -30]	[-1; 2.5]	[0.4; 0.96]	[123; 193]
2	$[RF_{min}; 1.77\% * RF_{max}]$	[0; -25]	[0; -30]	[1.5; 2.5]	[0.4; 0.96]	[141; 193]
3	$[RF_{min}; 1.50\% * RF_{max}]$	[0; -25]	[0; -30]	[1.7; 2.5]	[0.4; 0.96]	[141; 193]
4	$[RF_{min}; 1.33\% * RF_{max}]$	[0; -25]	[0; -30]	[1.7; 2.5]	[0.4; 0.87]	[141; 193]

The third HDA run is used for the application of the eFAST. Figure 5 shows a comparison of the total and main effect values for the chosen MLA parameters. The diagram demonstrates that in terms of ranking both main and total effect are identical. In both cases changes of the load factor n_z result in the highest changes of RF values. The main effect value for the load factor is 0.037 which means that the load factor causes 3.7% of the total variance only. On the other hand the total effect value for the load factor is 0.73 which means that 73% of the total variance relates to this input parameter.
The Mach number and the calibrated airspeed are listed next in the ranking. One can conclude that the flight parameters itself which are more an input for the MLA law have much higher influence on the RF than the aileron deflections. Furthermore the total effects have much higher values than the main effects which yield high influence on the RF by varying the input parameters simultaneously. The combinational effect of the MLA parameters drives the RF value more than just changing one parameter and keeping the others constant. Hence the main effect contributions are negligible whereas the interaction effects are the main drivers.
Note that the shown behaviour of main and total effects is confirmed with results coming from high fidelity analysis inside Airbus Operations GmbH.

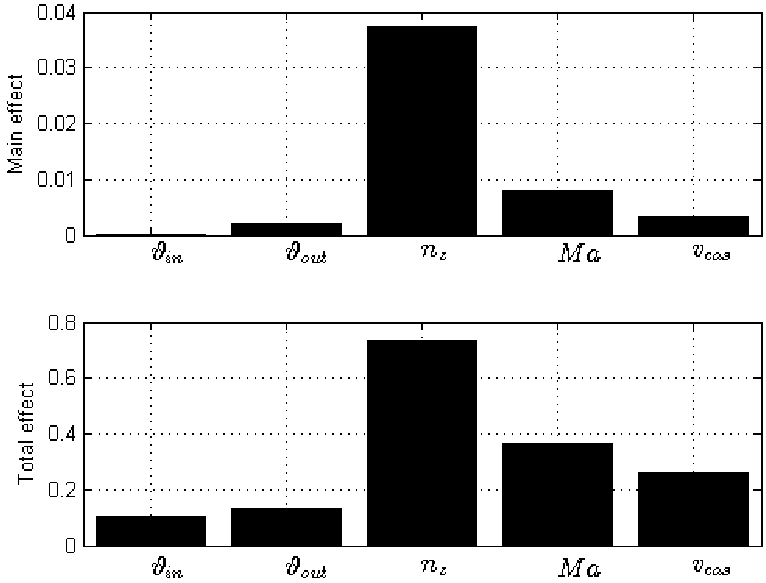

Figure 5: Main and total effects of MLA parameters on RF value

The application of eFAST on the constructed surrogate model from the first HDA run has shown different ranking and values for the total and main effects compared to the results shown in Figure 5. One can conclude that the results of the SMBFAST are strongly dependent on the quality of the sampled data basis and can lead in the worst case to conclusions which are not consistent with engineering judgment.

8. Summary and outlook

HDA is used in order to construct a cheap and fast to evaluate surrogate model compared to the overall industrial process. The application of eFAST yield in main and total effect values which are confirmed with high fidelity analysis inside Airbus Operations GmbH. The study has shown much higher influence of the flight parameters on the RF values compared to the aileron deflections for different flight points. But still for a given flight point the

load factor, the Mach number and the calibrated airspeed are constant while the aileron deflections can be modified. Hence the aileron deflections are the only parameters that can be varied in order to affect the RF values and thus to reach a reduction of the structural weight.
In future applications the constructed surrogate model can be used for instance for optimization of the aileron deflections as shown in Figure 6. Here the surrogate model of the third HDA run is used in order to plot the evolution of the RF value with the outer aileron deflection as the variable parameter and the other parameters hold on constant value. The RF values in the plot are normalized to 1. In Figure 6 the change of RF from its minimum to its maximum value is around 13% which can result in weight savings even if they are small. Hence in this way one can choose a certain flight condition and find the optimum value for the aileron deflections using surrogate model based optimization algorithms.

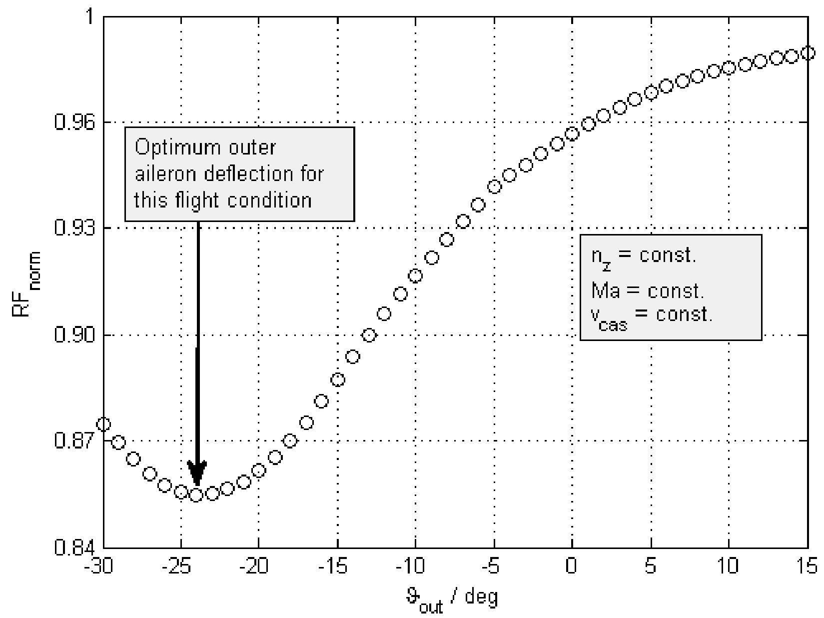

Figure 6: Evolution of normalized RF by outer aileron deflection changes for constant flight parameters

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