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Integrated multi-scale vibro-acoustic topology optimization of structure and material

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Abstract
The paper deals with the problem of the integrated layout design of macro and micro structure taking into consideration vibro-acoustic criteria. A multi-scale topology optimization model is presented for minimization of the sound power radiation from the vibrating composite structure with different designable periodic microstructures in different domains of the macrostructure. An extended multi-material interpolation model based on SIMP and PAMP is developed to implement the concurrent multi-scale topological design of the structure and material, and to achieve optimum distribution of the prescribed number of materials at both the micro and macro scale. The equivalent material properties of the macrostructure are calculated using homogenization method. The method of MMA is utilized to solve the multi-scale optimization model with respect to vibro-acoustic criteria. Numerical examples are given to validate the model and the method developed.
Keywords: structure and material; vibro-acoustic criteria; multi-scale; integrated method; topology optimization

1. Introduction

Vibration and noise attenuation is one of the most concerned problems in vibro-acoustic field, where structural topology optimization can act as a strong tool. Most of the present work concerning vibro-acoustic design generally focuses on topological optimization of the macrostructure including the distribution of the materials and damping [1-3]. A few works concerned topology optimization of the macrostructure or the microstructure of materials to achieve the optimum vibro-acoustic properties [4-5], which normally only involves single scale design. In recent years, some methods of concurrent topology optimization at structure and material scale have been proposed including the Hieratical topology optimization [6-7] and the PAMP model [8-9]. However, the former method may give rise to a large challenge in manufacturability for the microstructural configuration varies from point to point in the macro design domain. As for the latter, only single base material is taken into consideration to produce composite composed of porous microstructure. Moreover, design objectives of the above-mentioned methods generally involves extreme structural properties such static compliance or thermal elasticity [6-9] without respect to vibration and noise attenuation. In order to exert the potential of the structure and material to the largest extent, it is necessary to establish a multi-material and multi-scale model and further develop an effective and efficient method to combine the vibro-acoustic topology design of the macro and micro structure simultaneously. In this paper, section 2 will discuss the two scale multi-material interpolation model and optimization model with respect to vibro-acoustic criteria will be analysed in section 3. Then some numerical examples will be given to validate the viability of the presented method in section 4 and a brief conclusion is made in section 5.

2. Two-scale Multi-material Interpolation Model

The extended multi-material interpolation model at the micro scale based on the present SIMP [10] and PAMP model is illustrated as follows:

$$\mathbf{D}_e^{\mathrm{MI}} = {\kappa_n}^p \left\{ {\kappa_{n-1}}^p [{\kappa_{n-2}}^p (\quad) + (1-{\kappa_{n-2}}^p)\mathbf{D}_{n-2}^*] + (1-{\kappa_{n-1}}^p)\mathbf{D}_{n-1}^* \right\} + (1-{\kappa_n}^p)\mathbf{D}_n^* \tag{1}$$

$$\eta_e^{\mathrm{MI}} = {\kappa_n}^q \left\{ {\kappa_{n-1}}^q [{\kappa_{n-2}}^q (\quad) + (1-{\kappa_{n-2}}^q)\eta_{n-2}^*] + (1-{\kappa_{n-1}}^q)\eta_{n-1}^* \right\} + (1-{\kappa_n}^q)\eta_n^* \tag{2}$$

where $\mathbf{D}_0^*$, $\mathbf{D}_1^*$ …, $\mathbf{D}_{n-1}^*$, $\mathbf{D}_n^*$ and η_0^*, η_1^* …, η_{n-1}^*, η_n^* represent respectively the elasticity matrices and mass density of the multiple designate solid isotropic base materials numbered as 0, 1,…, n-1 and n. The symbol p and q are the penalty factors while p normally takes the value 3 or 4 and q usually takes the value 1 or 2 in order to achieve the clear zero-one design.

Theoretically speaking, with the above-mentioned multi-material interpolation model, multiple microstructures can be formulated. For convenience of illustration, in this article two microstructures in macro scale with multiple designate base materials are involved to perform topology design and variables like μ_1, μ_2,…, μ_{n-1}, μ_n to formulate the other microstructure in a similar way, based on which the material interpolation model at the macro scale can be expressed with the following equations if the macrostructure is meshed into N_e finite elements:

$$\mathbf{D}^{\mathrm{MA}} = \boldsymbol{\rho}^{p}\mathbf{D}^{\mathrm{H1}} + \left(1-\boldsymbol{\rho}^{p}\right)\mathbf{D}^{\mathrm{H2}} \tag{3}$$

$$\eta^{\mathrm{MA}} = \boldsymbol{\rho}^{q}\eta^{\mathrm{H1}} + \left(1-\boldsymbol{\rho}^{q}\right)\eta^{\mathrm{H2}}, \quad \boldsymbol{\rho} = \left[\rho_1, \rho_2, \quad, \rho_{N_e}\right]^{\mathrm{T}} \tag{4}$$

where $\mathbf{D}^{\mathrm{H1}}$, $\mathbf{D}^{\mathrm{H2}}$ and $\eta^{\mathrm{H1}}, \eta^{\mathrm{H2}}$ represent the equivalent stiffness matrices and mass density calculated via the homogenization method [11]. The symbol $\boldsymbol{\rho}$ denotes the relative volume density vector of the first microstructure in each element of the macrostructure, which may differ from point to point at macro scale, ranging from 0 to 1. Especially for the case where two designate solid base materials are taken into consideration, assuming the microstructure unit cell is discretized into n_e finite elements, the interpolation model at the micro scale above will be expressed as follows:

$${}_1\mathbf{D}^{\mathrm{MI}} = \boldsymbol{\kappa}^{p}\mathbf{D}_1^* + \left(1-\boldsymbol{\kappa}^{p}\right)\mathbf{D}_0^*, \ {}_1\eta^{\mathrm{MI}} = \boldsymbol{\kappa}^{q}\eta_1^* + (1-\boldsymbol{\kappa}^{q})\eta_0^*, \boldsymbol{\kappa} = \left[\kappa_1, \kappa_2, \quad, \kappa_{n_e}\right]^{\mathrm{T}} \tag{5}$$

$${}_2\mathbf{D}^{\mathrm{MI}} = \boldsymbol{\mu}^{p}\mathbf{D}_1^* + \left(1-\boldsymbol{\mu}^{p}\right)\mathbf{D}_0^*, \ {}_1\eta^{\mathrm{MI}} = \boldsymbol{\mu}^{q}\eta_1^* + (1-\boldsymbol{\mu}^{q})\eta_0^*, \boldsymbol{\mu} = \left[\mu_1, \mu_2, \quad, \mu_{n_e}\right]^{\mathrm{T}} \tag{6}$$

where $\mathbf{D}_1^*$ and $\mathbf{D}_0^*$ represent the constitutive matrices while η_1^* and η_0^* denote the mass density of the two designate solid isotropic base materials numbered as 1 and 0. In Eq. (5) and (6), the symbol $\boldsymbol{\kappa}$ and $\boldsymbol{\mu}$, varying from zero to unit vector, denote the relative volume density vector of the stiffer isotropic material (material 1) in one discretized micro unit cell of the first and second microstructure, which may differ from element to element,. Clearly the elasticity matrix of the element becomes $\mathbf{D}_1^*$ and $\mathbf{D}_0^*$ when the material volume density κ_i and μ_j $(i, j = 1, 2, \quad, n_e)$ take the values 1 and 0, respectively. Especially, if $\mathbf{D}_0^*$ equals zero matrix, the above interpolation model indicates topology optimization model based on single base material and porous composites may be acquired for holes will appear when κ_i or μ_j takes the value 0 in the result, which may correspond to the topology optimization using PAMP model. When the dynamic properties such as sound radiation power, fundamental frequency and band gap between eigenfrequencies of the macrostructure are taken into consideration, similar interpolation formulation may be employed to deal with the inertia part of the dynamic equations.

3. Optimization model with respect to vibro-acoustic criteria

Given the example of bi-material model used at both macro and micro scale, the macrostructure is assumed to be composed by two different designable composite materials, each of which is constructed by periodically arranged identical micro unit cells. And the micro unit cell is filled up with two prescribed solid isotropic base materials. The topology optimization model aimed at the best mechanical performance of the macrostructure such as the static compliance and sound radiation power may be formulated as:

$$\begin{aligned}
&\min_{\boldsymbol{\kappa},\boldsymbol{\mu},\boldsymbol{\rho}} \left\{\Pi\left(\mathbf{u}, \boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\rho}\right)\right\} \\
&s.t. \\
&\quad \mathbf{G}(\mathbf{u}, \mathbf{D}^{\mathrm{H}}(\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\rho})) = \mathbf{0}, \\
&\quad \frac{1}{N_e}\frac{1}{n_e}\left\{\left(\sum_{s=1}^{N_e}\rho_s\right)\left(\sum_{i=1}^{n_e}\kappa_i\right) + \left[\sum_{s=1}^{N_e}\left(1-\rho_s\right)\right]\left(\sum_{j=1}^{n_e}\mu_j\right)\right\} - \gamma_1 \le 0 \\
&\quad \sum_{i=1}^{n_e}\kappa_i V_i - V^2 \le 0, \ \left(V^2 = \gamma_2 V^{\mathrm{MI}}\right), \\
&\quad \sum_{j=1}^{n_e}\mu_j V_j - V^3 \le 0, \ \left(V^3 = \gamma_3 V^{\mathrm{MI}}\right), \\
&\quad \sum_{s=1}^{N_e}\rho_s V_s - V^4 \le 0, \ \left(V^4 = \gamma_4 V^{\mathrm{MA}}\right), \\
&\quad 0 \le \kappa_i, \mu_j, \rho_s \le 1, \ \left(i, j = 1, \quad, n_e; s = 1, \quad, N_e\right).
\end{aligned} \tag{7}$$

The symbol Π represents the design objective which may be expressed as the function of the structural response (e.g. displacement) vector $\mathbf{u}$. The first constraint in Eq. (7) is a general form of the governing equations of the macrostructure, from which the structural response such displacement vector may be solved. The symbol $\mathbf{D}^{\mathrm{H}}$ indicates the equivalent macro elasticity matrices of the metamaterial, which will be calculated using homogenization analysis at both micro and macro scales, relying on the topology design variable $\boldsymbol{\kappa}$, $\boldsymbol{\mu}$ and $\boldsymbol{\rho}$. The symbols $\gamma_1, \quad ,\gamma_4$ denote the material volume fractions under different scales, and γ_1 is the upper limit of the total volume fraction of the strong material (i.e. material no. 1) in the whole structure. As further illustration for the volume constraint equations, V^{MI} and V^{MA} denote respectively volume of the micro unit cell and the admissible design domain of the macrostructure, while V_i (or V_j) and V_s represent respectively the volume of one element in the micro unit cell and the macrostructure. It is necessary to note that not all the constraints above are essential so that we can reduce some of the inequations according to practical design requirement. Meanwhile even extra constraints such as the lower limits of the volume fractions can be added to the optimization model. Given the model established above, sensitivity analysis is performed and the MMA method [12] is employed in the optimum search in this paper. As a complementary step, the technology of density filtering [13] is used to help avoid the Checkboard problem [14].

4. Numerical Examples

4.1. Benchmark Example 1 - Micro-scale design of microstructure

To verify the validity of the presented integrated method applied to topology design of the vibro-acoustic metamaterial, microstructural topology optimization of the four-edge-clamped $1\mathrm{m}\times1\mathrm{m}\times0.01\mathrm{m}$ wall structure with a harmonic unit concentrated force working at its centre as shown in Figure 1. And the design objective is minimization of the sound radiation power caused by the vibration of the plate. Specially speaking, structural damping in this example is neglected. The macrostructure is assumed to consist of single composite material uniformly. Discretize the macrostructure and micro unit cell into 16×16 and 40×40 four-node Kirchhoff plate elements respectively. The micro unit cells will be filled with two designate isotropic base materials, of which the strong material(in dark color) has Young's modulus $E_1 = 2.1\times10^{11}\,\mathrm{Pa}$, mass density η_1=7800kg/m^3 and Poisson ratio $\nu_1 = 0.3$, while values of Young's modulus and mass density of the weak material (in light color) are tenth of those of the strong one except the identical Poisson ratio. Given volume constraint, volume fraction of the strong material is limited to not exceeding 50% of volume of the microstructure unit cell. The macro material volume density vector $\boldsymbol{\rho}$ is assigned to one mandatorily in each iteration step, and the micro material volume density vector $\boldsymbol{\mu}$ is initialized to zero to ensure the macrostructure is evenly composed by only identical microstructure. Comparison of the optimum topology (Table 1) of the bi-material microstructure (6×6 array of unit cells) under harmonic excitation with round frequency $\omega_p = 300\mathrm{rad/s}$ between utilizing the proposed integrated method and using only micro-scale design elucidates validity of the present work in this paper.

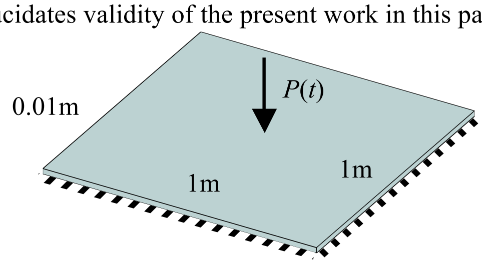
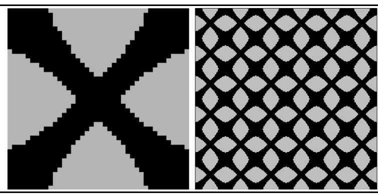
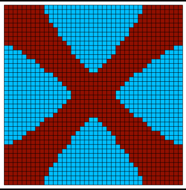
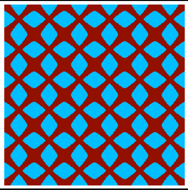


Figure 1: Four-edge-clamped plate loaded by harmonic concentrated force

Table 1: Comparison of microstructural design of vibro-acoustic metamaterial between using micro-scale optimization and using the presented method

$\gamma_1 = \gamma_2 = 0.5$	Sound power of the Optimum design/W	Microstructural topology
Results in the reference [6]	3.238×10^{-6}	

Results using new integrated method	4.099×10^{-6}	 

4.2. Benchmark Example 2 - Simultaneous multi-scale design for minimization of structural static compliance
The presented concurrent method of topology optimization is here employed to minimize the static compliance of the MBB beam (shown in Figure 2) as a comparison to the result demonstrated in a previous paper where the PAMP interpolation model was proposed, to validate the correctness of our innovative concurrent approach. According to the publication, all the variables involved in this example are non-dimensional. The concentrated vertical force working at the mid-point of the upper edge of the beam is $P = 1000$ and the length and the height of the beam are 4 and 1 respectively. Corresponding to original model, constitutive constants of the two base materials are as follows: Young's modulus $E_1 = 2.1\times10^5$ (in dark color) and $E_2 = 2.1\times10^{-5}$ (in light color), Poisson ratio $\nu_1 = \nu_2 = 0.3$. Given the axial symmetry condition, only the right half of the beam is taken as the macro design domain. Discretize the macrostructure and the micro unit cell respectively into 50×25 and 25×25 elements with two-dimensional four-node isoparametric element. Meanwhile the upper limit of total volume fraction of the base material is 0.25 for the macrostructure and 0.4 for the micro unit cell. The volume density vector $\boldsymbol{\mu}$ is initialized to zero, which helps to assure only the first microstructure contributes to the optimum configuration of the beam, to keep consistent with optimization using PAMP model. Optimum topologies of the macro and micro structure are shown in Table 2, where with comparison between our results and those using PAMP model, obviously good consistency can be confirmed. Hence the effectiveness and correctness of the proposed integrated method in this paper can be strongly validated.

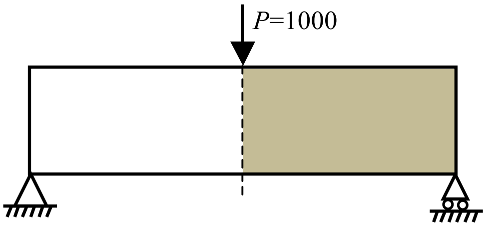
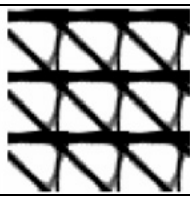
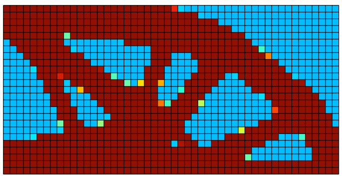
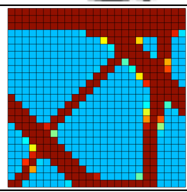
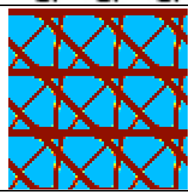


Figure 2: MBB beam

Table 2: Comparison of macrostructural and microstructural design of MBB beam for minimum static compliance between using micro-scale optimization and using the presented method

$\gamma_1 = 0.25$, $\gamma_2 = 0.4$	Optimum Compliance	Macrostructural topology	Microstructural topology
Results in the reference [10]	2234		 
Results using new integrated method	703		 

4.3. Example 3 - Simultaneous multi-scale design for minimization of sound radiation power
Simultaneous topology optimization with respect to vibro-acoustic criteria of the macro and micro structure, that is, structure and material, of the four-edge-clamped $1.2\text{m}\times1.2\text{m}\times0.01\text{m}$ plate loaded by harmonic uniform pressure on its top surface as shown in Figure 3, which amounts to 100N at each node, is considered in this example. And the design objective is to minimize of the sound radiation power produced by the vibration. Similar to Example 4.1, structural damping is neglected. The macrostructure and the micro unit cell are discretized into 30×30 and

25×25 four-node Kirchhoff plate elements, respectively. The micro unit cell will be filled with two designate base materials: Aluminium Alloy (in dark color) and Epoxy Resin (in light color) with Young's modulus $E_1 = 7.76\times10^{10}\,\text{Pa}$, $E_2 = 4.35\times10^{9}\,\text{Pa}$, mass density η_1=$2.73\times10^3\text{kg/m}^3$, η_2=$1.18\times10^3\text{kg/m}^3$, Poisson ratio $\nu_1 = \nu_2 = 0.3$, respectively. Regarding the constraints, upper limit of total volume fraction of the strong material in the whole macrostructure is 0.5, while upper limits of volume fraction of the strong material in the first and second microstructure are 0.5 and 0.25 respectively. No constraint of the volume fraction of the first microstructure in the macro design domain is imposed. Optimum topologies and sound power of the bi-material macrostructure and microstructure (4×4 array of unit cells) under harmonic excitation of different round frequencies including ω_p=100, 500, 800 and 2500 rad/s are shown respectively in Table 3 and Table 4 . Note that for the optimum macrostructural topology, elements in dark color represent the first optimum microstructure. Figure 3 shows the iteration history curve of the objective function under excitation frequency ω_p = 100rad/s. Our tests indicate ideal numerical stability of the presented method in certain range of low frequencies including 500 and 800 rad/s. Numerical oscillation was observed in the iteration history curve of the objective function at high frequencies, e.g. 2500 rad/s, as a result of which it failed to acquire clear 0 or 1 macro and micro topology. Possible reason is that the presented material interpolation model is a stiffness-dominated one while stiffness may not be the most influential factor when high frequency excitation is considered. Hence some improvement to the material interpolation model should be further researched in the future.

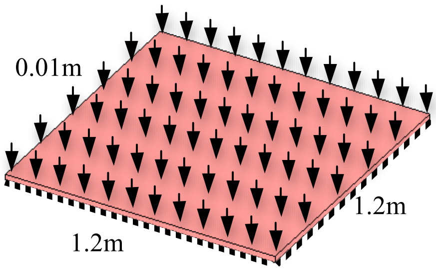
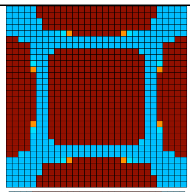
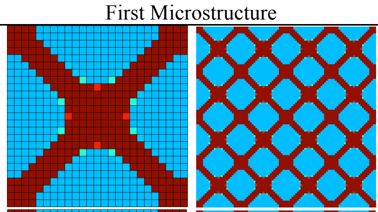
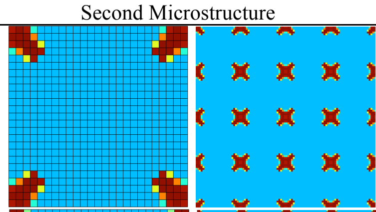
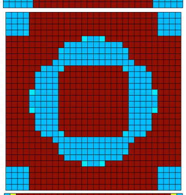
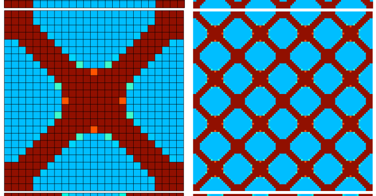
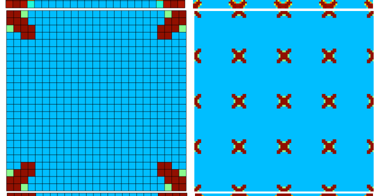
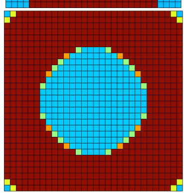
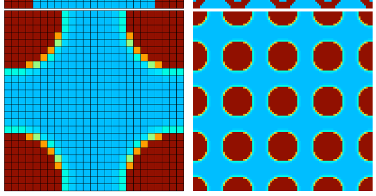
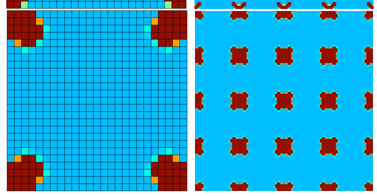


Figure 3: Four-edge-clamped plate with harmonic uniform pressure on top surface

Table 3: Optimum macrostructural and microstructural of the four-edge-clamped Kirchhoff plate under harmonic uniform on the surface

ω_p(rad/s)	Macrostructural topology	Microstructural topology: First Microstructure	Microstructural topology: Second Microstructure
100			
500			
800			

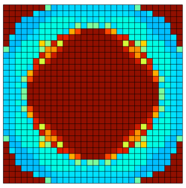
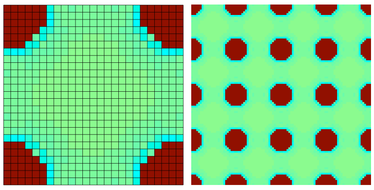
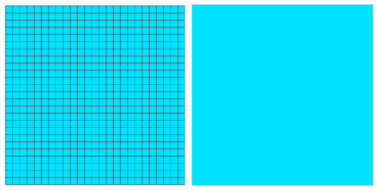
2500			

Table 4: Comparison of the sound power between the initial and the optimum topology design of the plate

ω_p(rad/s)	Sound Power/W	
	Initial design	Optimum design
100	0.100	0.008
500	3.316	0.226
800	16.332	2.426
2500	8.341	6.538

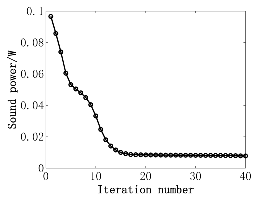


Figure 3: Iteration history curve of the objective function (ω_p = 100rad/s)

5. Conclusions

This paper presents an integrated multi-scale topology optimization method of finding the optimum macro and micro topology configuration of the structure and material simultaneously based on a multi-material interpolation model, with respect to the vibro-acoustic criteria. Numerical examples validate the reliability of the new method through comparison with research results of some benchmark examples. Our study shows effectiveness of the presented method in designing optimum vibro-acoustic structure and metamaterial aiming at minimization of sound radiation power of the vibrating structure. The presented work may offer new ideas and relevant theoretical basis for conceptual design of the structural and material for vibration and noise attenuation.

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