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Topology Optimization of Members of Dynamically Loaded Flexible Multibody Systems using Integral Type Objective Functions and Exact Gradients

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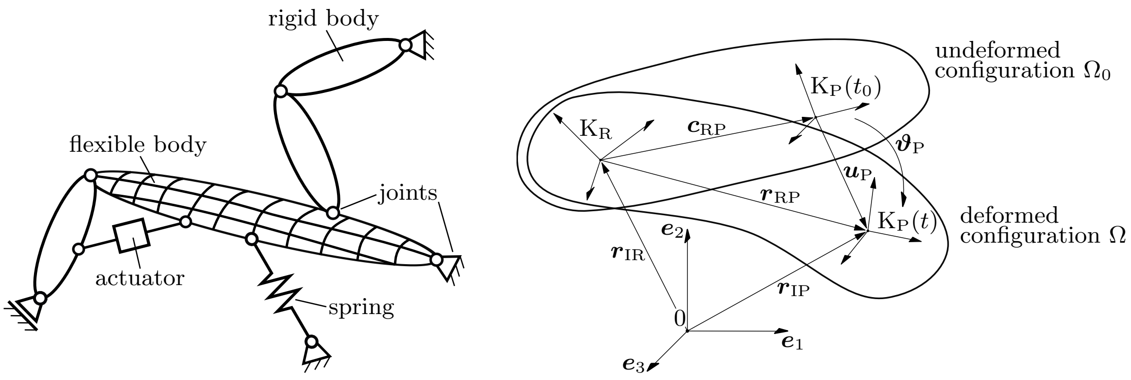
1. Abstract
In this work a procedure is presented to perform topology optimization of components of flexible multibody systems, which are modeled with the floating frame of reference formulation. For the topology optimization, the solid isotropic material with penalization (SIMP) method is used. In order to capture the actual loads on the flexible components in the optimization, integral type objective functions are employed and exact gradients are provided. The latter are computed by the adjoint variable method to handle the large number of design variables.

2. Keywords: Topology Optimization, Flexible Multibody Systems, Integral Type Objective Function, Sensitivity Analysis, Adjoint Variable Method

3. Introduction
For topology optimization of components of dynamically loaded flexible multibody systems the equivalent static load method is often employed, see [5, 7]. Thereby, the actual loads on the flexible components are first imitated by a finite set of equivalent static loads. These loads are derived from the results of a dynamic multibody system simulation and are assumed to be static and design independent. Then a static response optimization is carried out with regard to the equivalent static loads. Finally, the next optimization loop is started performing another multibody system simulation using the optimized design. By this procedure the computation of the objective function and its gradient is significantly eased. However, the gradient is only an approximation of the actual dynamic problem.
Depending on the origin of the loads, e.g. whether they are caused by external forces or due to the inertia of their self-weight, the simplifications made in the equivalent static load method can lead to severe differences in the gradients. These might cause the optimization algorithm to search in the wrong direction and converge to undesired solutions. Therefore, a topology optimization procedure is presented, which relies on integral type objective functions and exact gradients, which are computed using the adjoint variable method, see [3]. In this way the actual dynamical loads can be captured in the objective function and the gradient.
The paper is organized in the following way. Section 4 addresses the topology optimization of flexible multibody system. Some basics of the floating frame of reference formulation and the SIMP method are given and the optimization procedure is described. In Section 5 the gradient computation of functional type objective functions using the adjoint variable method is explained. Then, in Section 6 the procedure is tested performing a topology optimization of a piston rod of a flexible slider-crank mechanism, which is only loaded by its own inertia forces. Finally, Section 7 concludes with a brief summary and discussion.

4. Topology Optimization of Flexible Multibody Systems
The method of flexible multibody systems is a well established approach to model and analyze compliant mechanisms, which perform large rigid body motions. These systems are assembled from rigid and flexible bodies, spring and damper elements, and actuators, which are connected via joints and bearings, see Fig. 1(a). If the deformations are comparatively small the floating frame of reference formulation can be used to efficiently incorporate flexible bodies into the multibody system. In the following the basic ideas of the floating frame of reference formulation are briefly reviewed, for a detailed description see [10, 11]. In addition, the SIMP approach and the optimization procedure are described, which are used to perform topology optimization of components of flexible multibody systems.



(a) Flexible multibody system

(b) Kinematics of a single flexible body using the floating frame of reference formulation

Figure 1: Flexible multibody systems modeled with the floating frame of reference formulation

4.1 The Floating Frame of Reference Formulation

In the floating frame of reference formulation the flexible body motion is defined using two sets of coordinates, see Fig. 1(b). One set gives the position and orientation of a body-related reference frame K_R and describes the large nonlinear motion. The second set describes the deformation of the body with respect to K_R using a Ritz approach. That is, the displacement $\boldsymbol{u}_P$ at the point P and the rotation $\boldsymbol{\vartheta}_P$ of the coordinate system K_P fixed in P are approximated by

$$\boldsymbol{u}_P(\boldsymbol{c}_{RP}, t) = \boldsymbol{\Phi}(\boldsymbol{c}_{RP})\boldsymbol{q}(t) \quad \text{and} \quad \boldsymbol{\vartheta}_P = \boldsymbol{\Psi}(\boldsymbol{c}_{RP})\boldsymbol{q}(t) \tag{1}$$

as products of global shape functions $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ and time-dependent elastic coordinates $\boldsymbol{q}$. The global shape functions are often obtained from finite element (FE) models of the flexible bodies. More precisely, the equations of motion of a linear FE model of the flexible body

$$\boldsymbol{M}_e\ddot{\boldsymbol{q}}_e + \boldsymbol{K}_e\boldsymbol{q}_e = \boldsymbol{f}_e, \tag{2}$$

with the mass matrix $\boldsymbol{M}_e$, the stiffness matrix $\boldsymbol{K}_e$, the vector of applied loads $\boldsymbol{f}_e$ and the nodal degrees of freedom $\boldsymbol{q}_e \in \mathbb{R}^n$ have to be set up. Then, the global shape functions $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are obtained from a model reduction of (2) using, for instance, modal truncation or component mode synthesis, see [4].
After the kinematics of the bodies is derived the equations of motion of the flexible multibody system can be obtained in minimal coordinates by applying a principle of mechanics, such as d'Alembert's principle, and it yields

$$\boldsymbol{M}(t, \boldsymbol{y})\ddot{\boldsymbol{y}} = \boldsymbol{f}(t, \boldsymbol{y}, \dot{\boldsymbol{y}}). \tag{3}$$

Thereby, $\boldsymbol{M}$ is the global mass matrix and $\boldsymbol{y} \in \mathbb{R}^f$ are the generalized coordinates, which comprise both the rigid body degrees of freedom $\boldsymbol{y}_r$ and the elastic degrees of freedom $\boldsymbol{q}$. The right-hand-side vector $\boldsymbol{f}$ contains the applied forces, inner forces from elastic deformations as well as centrifugal, Coriolis, and gyroscopic forces.

4.2 The SIMP Approach

One way to perform topology optimization is to distribute a limited amount of mass in a discretized design domain such that an objective function, for instance the compliance, becomes minimal under certain loading conditions. A common method to relax and treat this kind of hard to solve mass/no mass integer optimization problem is the SIMP approach, see [1, 2]. Thereby, continuous density-like design variables $x_i \in [0, 1]$, $i = 1 \dots m$, are introduced and used to parameterize the material properties of the m subdomains. Following the penalization strategy in [9], the density and stiffness of an element i is computed as

$$\begin{aligned} \rho_i &= \begin{cases} c x_i^q \rho_0, & \text{for } x_{\min} = 0.01 \le x_i < 0.1, \\ x_i \rho_0, & \text{for } 0.1 \le x_i \le 1, \end{cases} \\ E_i &= x_i^p E_0. \end{aligned} \tag{4}$$

Thereby, ρ_0 and E_0 are the density and the stiffness of the solid material, while c, p and q are scalar parameters. In the topology optimization, the FE model (2), which is used to determine the global shape

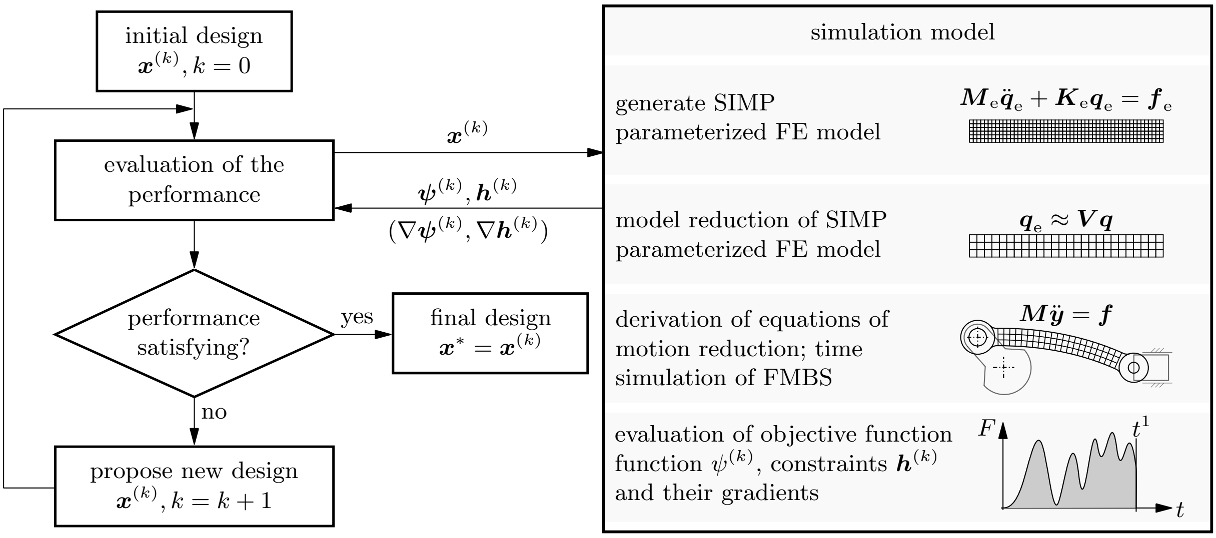


Figure 2: Schematic representation of the optimization procedure

functions $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, is parameterized by this SIMP approach, whereby each element is a subdomain.

4.3 Optimization Procedure

The established topology optimization procedure including a detailed description of the simulation model is shown in Fig. 2. Starting from the initial design $\boldsymbol{x}^{(0)}$, at first the performance of $\boldsymbol{x}^{(0)}$ is evaluated by a numerical simulation of the flexible multibody system. The simulation model contains the following steps.

At first, a SIMP parameterized FE model is generated in a reference domain. Then, a model reduction of the SIMP parameterized FE model is performed to reduce the number of elastic degrees of freedom and to determine the global shape functions. In the next step, the flexible multibody system is assembled from rigid and flexible bodies, i.e. the equations of motion (3) are derived in minimal coordinates, and thereafter the time simulation can be performed. In the last step of the simulation model the objective function and, if present, the constraint equations and their gradients are evaluated.

Continuing the iterative optimization procedure, it is checked whether the performance of the design $\boldsymbol{x}^{(k)}$ satisfies given criteria. If this is the case $\boldsymbol{x}^* = \boldsymbol{x}^{(k)}$ is accepted as the solution of the optimization problem and the optimization procedure is terminated. If the performance is not satisfying yet, an improved design $\boldsymbol{x}^{(k+1)}$ is proposed by the optimization algorithm and the next optimization loop is started. As optimization algorithm an elementary version of the Method of Moving Asymptotes [12] is used.

The whole optimization procedure from the generation of the FE model, the model reduction, the modeling of the flexible multibody system, the time simulation to the optimizer is established in Matlab.

5. Sensitivity Analysis of Functional Type Objective Functions

For a successful application of the optimization procedure presented in the previous section, the efficient gradient computation is of major importance. Due to the large number of design variables the usage of the finite difference method would result in excessively high computation times. Therefore, the gradients are computed using the adjoint variable method instead. It is derived in [3] for the sensitivity analysis of rigid multibody systems and can be transferred to flexible multibody systems as described in [6].

Given a parameterized flexible multibody system in minimal coordinates as initial value problem

$$\begin{aligned}
\text{equations of motion (ODE):}\quad & \boldsymbol{M}(t,\boldsymbol{y},\boldsymbol{x})\ddot{\boldsymbol{y}} - \boldsymbol{f}(t,\boldsymbol{y},\dot{\boldsymbol{y}},\boldsymbol{x}) = \boldsymbol{0},\\
\text{initial positions and velocities:}\quad & \boldsymbol{\Phi}^0(t^0,\boldsymbol{y}^0,\boldsymbol{x}) = \boldsymbol{0},\quad \dot{\boldsymbol{\Phi}}^0(t^0,\boldsymbol{y}^0,\dot{\boldsymbol{y}}^0,\boldsymbol{x}) = \boldsymbol{0},\\
\text{implicit final condition:}\quad & H^1(t^1,\boldsymbol{y}^1,\dot{\boldsymbol{y}}^1,\boldsymbol{x}) = 0,
\end{aligned}\tag{5}$$

then the gradient of functional type objective functions in form of

$$\psi(\boldsymbol{x}) = \int_{t^0}^{t^1} F(t, \boldsymbol{y}, \dot{\boldsymbol{y}}, \ddot{\boldsymbol{y}}, \boldsymbol{x})\mathrm{d}t \tag{6}$$

can be computed with the adjoint variable method as follows. At first, the adjoint variables, which are also referred to as Lagrange multipliers, $\tau \in \mathbb{R}$, $\boldsymbol{\mu} \in \mathbb{R}^f$ and $\boldsymbol{\nu} \in \mathbb{R}^f$ have to be determined at the final time t^1 from the equations

$$\tau^1 = \frac{F^1}{\dot{H}^1}, \quad \boldsymbol{\mu}^1 = -\tau^1 \frac{\partial H^1}{\partial \boldsymbol{y}^1} \quad \text{and} \quad \boldsymbol{M}^1 \boldsymbol{\nu}^1 = -\tau^1 \frac{\partial H^1}{\partial \dot{\boldsymbol{y}}^1}. \tag{7}$$

Thereafter, the adjoint differential equations for $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$, which read

$$\begin{aligned} \dot{\boldsymbol{\mu}} &= \left(\frac{\partial \mathrm{ODE}}{\partial \boldsymbol{y}}\right)^{\mathrm{T}} (\boldsymbol{\nu} + \boldsymbol{\xi}) - \frac{\partial F}{\partial \boldsymbol{y}}, \\ \boldsymbol{M}\dot{\boldsymbol{\nu}} &= -\boldsymbol{\mu} - \dot{\boldsymbol{M}}\boldsymbol{\nu} + \left(\frac{\partial \mathrm{ODE}}{\partial \dot{\boldsymbol{y}}}\right)^{\mathrm{T}} (\boldsymbol{\nu} + \boldsymbol{\xi}) - \frac{\partial F}{\partial \dot{\boldsymbol{y}}}, \end{aligned} \tag{8}$$

are derived and solved by a backward time integration from the final time t^1 to the initial time t^0. The auxiliary variables $\boldsymbol{\xi} \in \mathbb{R}^f$ are computed from the algebraic equation $\boldsymbol{M}\boldsymbol{\xi} = \partial F / \partial \ddot{\boldsymbol{y}}$. It should be mentioned that the adjoint differential equations and, hence, the effort to solve them do not depend on the design variables $\boldsymbol{x}$. However, the partial derivatives of the equations of motion with respect to the generalized positions $\boldsymbol{y}$ and velocities $\dot{\boldsymbol{y}}$ have to be provided.
After the backward time integration of Eq. (8) the adjoint variables $\boldsymbol{\eta}^0 \in \mathbb{R}^f$ and $\boldsymbol{\zeta}^0 \in \mathbb{R}^f$ are determined from the equations

$$\left(\frac{\partial \dot{\boldsymbol{\Phi}}^0}{\partial \dot{\boldsymbol{y}}^0}\right)^{\mathrm{T}} \boldsymbol{\eta}^0 = \boldsymbol{M}^0 \boldsymbol{\nu}^0 \quad \text{and} \quad \left(\frac{\partial \boldsymbol{\Phi}^0}{\partial \boldsymbol{y}^0}\right)^{\mathrm{T}} \boldsymbol{\zeta}^0 = \boldsymbol{\mu}^0 - \left(\frac{\partial \dot{\boldsymbol{\Phi}}^0}{\partial \boldsymbol{y}^0}\right)^{\mathrm{T}} \boldsymbol{\eta}^0. \tag{9}$$

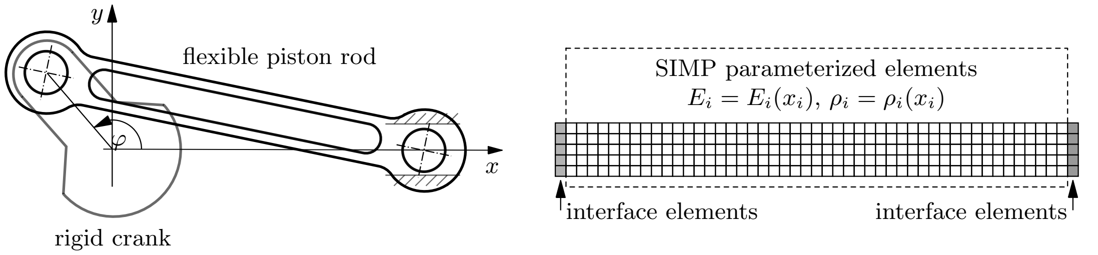
Finally, the gradient can be computed as

$$\nabla\psi = -\tau^1 \frac{\partial H^1}{\partial \boldsymbol{x}} - \left(\frac{\partial \boldsymbol{\Phi}^0}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} \boldsymbol{\zeta}^0 - \left(\frac{\partial \dot{\boldsymbol{\Phi}}^0}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} \boldsymbol{\eta}^0 + \int_{t^0}^{t^1} \left[\frac{\partial F}{\partial \boldsymbol{x}} - \left(\frac{\partial \mathrm{ODE}}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} (\boldsymbol{\nu} + \boldsymbol{\xi})\right] \mathrm{d}t, \tag{10}$$

whereby the adjoint variables and the partial derivatives of the equations of motion with respect to the design variables $\boldsymbol{x}$ are needed. For the computation of $\partial \mathrm{ODE} / \partial \boldsymbol{x}$ the global shape functions, used to approximate the deformations of the flexible bodies, have to be differentiated with regard to the design variables. To compute these derivatives efficiently Nelson's method is used, see [8].

6. Application Example

In order to test the proposed optimization procedure, the topology of a flexible piston rod of a slider-crank mechanism is optimized, see Fig. 3(a). The results are compared with those of a second optimization, in which the equivalent static load method is used. In the modeling of the slider-crank mechanism the slider block is omitted. Therefore, the piston rod is only loaded by its own inertia forces, which occur during the motion of the system.
The motion is composed of two stages. In the first one, the crank is accelerated within two seconds from a resting position until a constant angular velocity is reached. In the second stage, the angular velocity is kept constant for another second.
The design domain, in which the optimal topology of the piston rod shall emerge, possesses the dimensions $(1.0 \times 0.06 \times 0.01)\,\mathrm{m}$ and is discretized using 200×12 finite elements, see Fig. 3(b). Thereby, the interface elements, which include the elements of the first and the last column in the mesh, are solid. The inner 2376 elements are parameterized by the SIMP law (4) using the parameters $c = 10^5$, $p = 3$, $q = 6$, $E_0 = 0.5 \cdot 10^{11}\,\mathrm{N/m^2}$ and $\rho_0 = 8750\,\mathrm{kg/m^3}$.



(a) Flexible slider-crank mechanism (b) SIMP parameterized FE model of the design domain

Figure 3: Flexible slider-crank mechanism

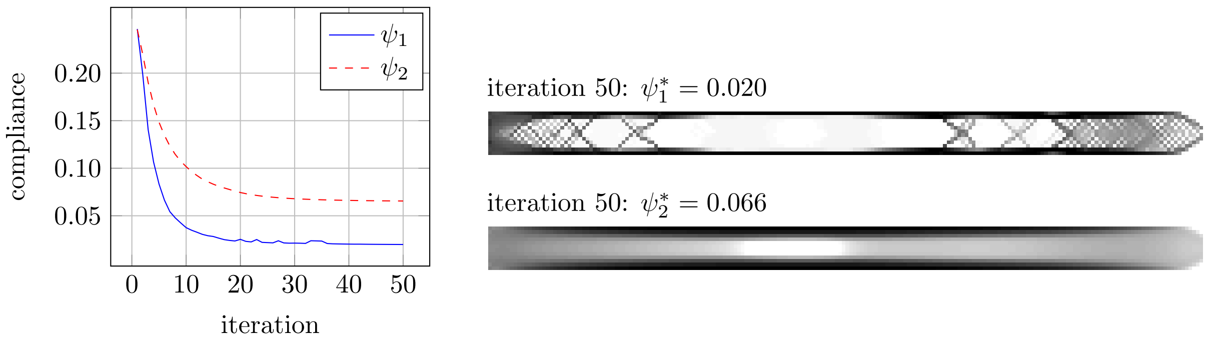


Figure 4: Optimization results comparing integral type objective functions and equivalent static loads

The optimization problem is formulated as minimal compliance problem with a volume restriction $V(\boldsymbol{x})/V_0 \leq 0.5$ and reads

$$\begin{aligned} \min_{\boldsymbol{x} \in \boldsymbol{R}^h} \quad & \psi_i(\boldsymbol{x}) \\ \text{subject to} \quad & \boldsymbol{h}(\boldsymbol{x}) = V(\boldsymbol{x}) - \frac{1}{2} V_0 \leq 0, \\ & 0.01 \leq x_i \leq 1, \quad i = 1 \dots h, \end{aligned} \tag{11}$$

whereby two different objective functions $\psi_i(\boldsymbol{x}), i = 1, 2$ are defined. On the one hand, being an example of functional type objective functions, the integral compliance

$$\psi_1(\boldsymbol{x}) = \int_{t^0}^{t^1} \boldsymbol{q}_{\mathrm{e}}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{e}} \boldsymbol{q}_{\mathrm{e}} \mathrm{d}t \tag{12}$$

shall be minimized. On the other hand, employing the equivalent static load method [5, 7] the compliance is evaluated and summed up at $n_{\mathrm{esl}} = 100$ time points t_j, which are uniformly distributed in $[t^0, t^1]$. Here, the objective function $\psi_2(\boldsymbol{x})$ is computed as follows

$$\psi_2(\boldsymbol{x}) = \frac{t^1}{n_{\mathrm{esl}}} \sum_{j=1}^{n_{\mathrm{esl}}} \boldsymbol{q}_{\mathrm{e},j}^{\mathrm{T}} \boldsymbol{K}_{\mathrm{e}} \boldsymbol{q}_{\mathrm{e},j}, \tag{13}$$

whereby the scaling t^1/n_{esl} is not necessarily needed but introduced to ease the comparison of ψ_1 and ψ_2. The elastic coordinates $\boldsymbol{q}_{\mathrm{e},j} = \boldsymbol{q}_{\mathrm{e}}(t_j)$ are recovered after the time simulation and are assumed to be constant in the gradient computation of the static response optimization.

Two optimizations are carried out, in which ψ_1 and ψ_2 are minimized. The resulting compliance histories and material distributions after 50 iterations are given in Fig. 4. It can be seen that in both optimizations, the evolving designs are stiffened with regard to bending loads. However, while the optimization of ψ_2 gets stuck in an intermediate layout with a majority of gray elements, a design emerges in the optimization of ψ_1, where most of the elements are either white or black and even checkerboard patterns are formed. This clear difference can also be observed in the compliance which is reduced from $\psi_1(\boldsymbol{x}^{(0)}) = \psi_2(\boldsymbol{x}^{(0)}) = 0.246$ to $\phi_1^* = 0.02$ and $\phi_2^* = 0.066$, respectively.

It can therefore be concluded that for the application example the simplifications made in the equivalent static load method do not apply and as a consequence the optimization yields an unsatisfying design. In contrast, the presented optimization procedure, which relies on an integral type objective function and exact gradients, is able to solve the problem. However, this ability comes with a price. While the optimization of ψ_2 takes only about 1.5 h, the optimization of ψ_1 needs roughly two days due to the increased effort in the gradient computation.

7. Summary and Conclusion

An optimization procedure is presented to perform topology optimization of flexible multibody systems, modeled with the floating frame of reference formulation. The flexible bodies are parameterized using the SIMP approach. In contrast to previous approaches, functional type objective functions are employed and exact gradients are provided. In the gradient computation the adjoint variable method is used to handle the large number of design variables. As testing and comparing example, a topology optimization is performed for a flexible piston rod of a slider-crank mechanism. The results show that it is possible with the presented method to optimize multibody systems, which do not meet the necessary requirements to use the equivalent static load method. However, even though an appropriate method is used to evaluate the gradients efficiently the total optimization time is comparatively high.

Acknowledgements

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