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Surrogate Models for Data-inspired Reliability Design

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1. Abstract
A design concept is proposed called 'Data-inspired Reliability Design', where measured data are aggressively used to improve the accuracy of structural reliability design, and surrogate models suitable for this design concept are investigated. Since the amount of measured data is limited due to the cost of sensors, the structural responses that are not measured should be predicted using the measured data. To best use the measured data, the hybrid surrogate models generated using both the measured data and simulation results are applied to predict structural responses in this study. For the hybrid models, the discrepancy between the measured data and simulation results is approximated using response surface methodology. The Gauss process model and an artificial neural network were used as the response surface, and the suitability of the response surfaces were checked for use as virtual sensors. To validate the hybrid surrogate models, the structural responses of a welded structure were predicted using both the measured responses and those analyzed using a simulation. As a result, the predicted responses agreed well with the measured ones. It can therefore be concluded that using the hybrid surrogate models is one way to predict structural responses instead of using sensors in the proposed design concept.
2. Keywords: Virtual sensor, Surrogate model, Gauss process model, Neural networks, Data-inspired design

3. Introduction
Today, pieces of infrastructure include many sensors for collecting operational and environmental data. The collected data are then used, for example, to control infrastructure machinery more efficiently, to determine the maintenance interval of parts appropriately, and to design more reliable machinery. As the number of sensors increases, we can gain more benefits from the collected data, because the amount of information obtained from the data increases. On the other hand, the increase in the number of sensors raises the initial cost of machinery. It is therefore better to obtain much information about operational condition and environment with the fewest sensors. To increase the amount of information without increasing the number of sensors, the surrogate model that predicts responses to be measured can be used as a virtual sensor.

Although the surrogate model should be accurate enough to be used as a virtual sensor, an accurate surrogate model is difficult to generate with few sensor outputs. To overcome this difficulty, we can employ the method for generating a surrogate model by using both measured outputs and simulation results. According to this hybrid modelling method, the discrepancy between real output and simulated output is explicitly included in the model of target output [1][2][3], so accordingly the target out is predicted by adding the discrepancy to the simulation result. The discrepancy is usually approximated with the Gauss process (GP) model [4], which is a response surface model, hence the parameters in the GP model, called hyper-parameters, are determined with known discrepancies. Since the GP model is based on a stochastic framework, the hyper-parameters can be determined by using the Markov chain Monte Carlo method (MCMC). The estimation of an unknown response with the GP model is, however, computationally demanding; hence, this high computational cost limits the usage of the GP model. In addition to the GP model, an artificial neural network is therefore applied to the approximation of the discrepancy in this study. Once an artificial neural network has been learned, a target response can be estimated quickly by using the learned neural network.

In the following sections, first, the design concept is introduced where the collected data can be effectively employed, and then a procedure is described for generating the GP model and estimating unmeasured responses with a hybrid modelling method. Furthermore, we propose a procedure for using an artificial neural network to interpolate the discrepancy instead of the GP model. These procedures are applied to predict the strains occurring on a typical welded structure, and the approximation accuracy is investigated for three validation scenarios.

4. Data-inspired Reliability Design
When developing a product, the structural reliability of the product is usually designed in accordance with a design standard for the product. The design standard provides the safety margin for structural reliability, such as the safety margin for fatigue resistance; hence, the structure of the product is designed so that the product is within the safety margin. If operational and environmental conditions, for example, applied loads, are uncertain for the product to be designed, the safety margin for structural reliability is set to a large value. As a result, the developed product will

be inappropriate for a sustainable society because it is too heavy, and heavy products need a large amount of material and energy for manufacturing and operating them. A decrease in the uncertainty of the conditions therefore leads to the development of products that are appropriate for a sustainable society. A better way to decrease the uncertainty is to clarify the uncertain conditions with the data collected during the bench test, test operation, and field operation. By repeating the data collection and improving the design standard for structural reliability, the reliability of products can be improved continuously as shown in Figure 1. We call this design concept "data-inspired reliability design".
The data-inspired reliability design can be applied to the reliability design of the products (such as wind turbines, construction machinery and trains) for which operational and environmental data are monitored. Figure 2 shows an example of the data analytics performed to confirm the safety margin for fatigue resistance of a wind turbine. The fatigue damage of the welded joints in the tower of the wind turbine was estimated with measured strain data, and then the estimated damage was compared with the damage that had been evaluated in accordance with a design standard. Since strain sensors are mounted on the tower at two different heights, the structural reliability of the tower was evaluated there. If the number of sensors is increased, the structural reliability can be evaluated at other positions. The number of sensors, however, increases the cost of products; hence, the number of sensors used for a product has to be limited. To overcome this limitation, virtual sensor technology, namely, surrogate models, can be used for collecting data to supplement real sensors.

5. Surrogate models

5.1. Bias-corrected model

The hybrid surrogate model using both measured data and simulation results is applied to predict unknown responses. There is generally a discrepancy between the output of computer simulation and measured output because simulation models approximate the target physical phenomenon and the measured output includes measurement noise. When formulating the prediction model of a target phenomenon, the measurement noise is usually modelled as a Gaussian noise with zero mean. On the other hand, for the discrepancy caused by model

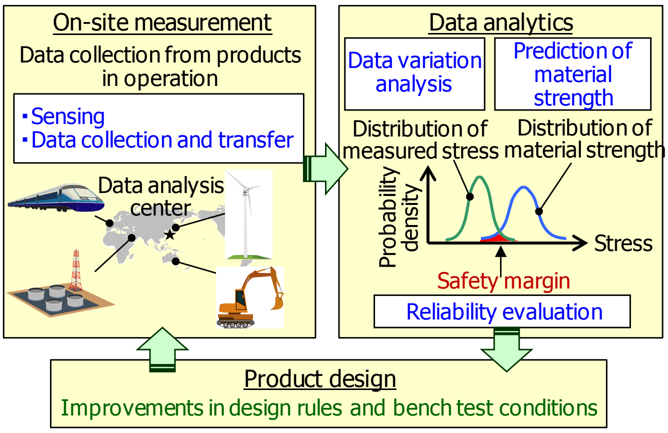


Figure 1: Data-inspired reliability design

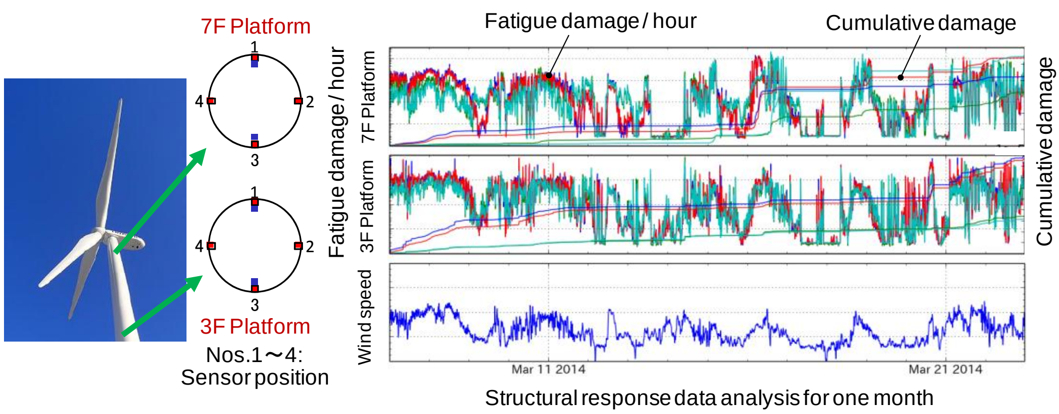


Figure 2: Fatigue damage estimated with the collected data for wind turbine

approximation, methods for modelling the discrepancy as a bias have been proposed in the last decade. In accordance with these methods, the real output of the target phenomenon that does not include measurement noise is formulated as

$$y^R = y^M + \delta \tag{1}$$

where y^R and y^M are the real and simulation outputs, respectively, and δ is the bias between the two outputs. When the outputs are strains that have to be evaluated at some positions and for some applied loads, Eq. (1) could be rewritten as follows.

$$y^R(\mathbf{x},\mathbf{F}) = y^M(\mathbf{x},\mathbf{F}) + \delta(\mathbf{x},\mathbf{F}) \tag{2}$$

where **x** and **F** are respectively the vectors of positions and applied loads.

5.2. Output prediction using Gauss process model

To estimate the real output $y^R(\mathbf{x}, \mathbf{F})$, two response surfaces for simulated outputs $y^M(\mathbf{x}, \mathbf{F})$ and bias $\delta(\mathbf{x}, \mathbf{F})$ were constructed by using the GP model. The mean and covariance of the applied GP model are

$$m(\mathbf{x}) = \beta \tag{3}$$

$$k(\mathbf{x},\mathbf{x}') = \sigma^2 \exp\left\{-\sum_{j=1}^{q} \omega_j \left(x_j - x'_j\right)^2\right\} \tag{4}$$

where β, σ, and ω_j are the hyper-parameters. In this study, these parameters were adjusted by using the MCMC method. Bayesian analysis was also applied to predict the expected mean of real output because the uncertainty of the hyper-parameters should be considered for the prediction. The whole prediction process proposed in this study is as follows.

1) Generate the GP response surface for interpolating simulated outputs $y^M(\mathbf{x}, \mathbf{F})$
2) Generate the GP response surface for interpolating bias $\delta(\mathbf{x}, \mathbf{F})$
3) Obtain posterior distributions for the hyper-parameters of the bias response surface by using Bayesian analysis
4) Obtain the expected mean of real output from the posterior distributions of the hyper-parameters and the bias

If the computational cost of the simulation is not expensive, we can skip prediction process step 1. The expected mean of real output was calculated with

$$\hat{y}^R(\mathbf{x},\mathbf{F}) = \frac{1}{N}\sum_{i=1}^{N}\left[y^M(\mathbf{x},\mathbf{F}) + \delta^{(i)}(\mathbf{x},\mathbf{F})\right] = \frac{1}{N}\sum_{i=1}^{N} y^{R(i)}(\mathbf{x},\mathbf{F}) \tag{5}$$

where $\delta^{(i)}(\mathbf{x})$ is a sample of the bias drawn by using Bayesian analysis in step 4, and N is the number of samples. The same as the expected mean of the real output, the expected mean of the bias can be estimated with

$$\hat{\delta}(\mathbf{x}) = \frac{1}{N}\sum_{i=1}^{N} \delta^{(i)}(\mathbf{x}) \tag{7}$$

5.3. Output prediction using artificial neural network

Artificial neural networks can be also used to predict the real response in accordance with the model bias correction; that is, the response surfaces of the simulated output and the bias can be generated using artificial neural networks as well as the GP model. However, conventional neural networks sometimes fit only teacher data, hence the generated response surface does not become smooth enough to use it, especially for the interpolation of the bias. The Bayesian framework for learning neural networks [5][6] is therefore applied when generating the response surfaces by the neural networks in this study. On the basis of the Bayesian framework, the objective function to be minimized for the learning of neural networks is formulated as follows.

$$M = \alpha E_{\mathrm{W}} + \beta E_{\mathrm{D}} \tag{8}$$

where E_{D} is error sum of squares and E_{w} is a generalization term that expresses the prior information for the weighing coefficients of the neural network to be learned. The prior information introduced here means that the response surface should be smooth; therefore, the weighing coefficients should preferably be small values. α and β in Eq. (8) are hyper-parameters that are determined by maximizing the likelihood of the hyper-parameters.

After the development of a formula, the equations for maximizing the likelihood are

$$\alpha = \frac{\gamma}{2E_W}, \quad \beta = \frac{N-\gamma}{2E_D} \tag{9}$$

where γ is the effective number of the weighing coefficients and means how many coefficients are accurately determined with teacher data. Given that the eigenvalues of matrix $\beta\nabla\nabla E_D$ are λ_a (a = 1～k), γ can be calculated by

$$\gamma = \sum_{a=1}^{k} \frac{\lambda_a}{\lambda_a + \alpha} . \tag{10}$$

By repeating the learning of the neural network and the update of the hyper-parameters sequentially, the smooth response surface of the bias was generated in this study.

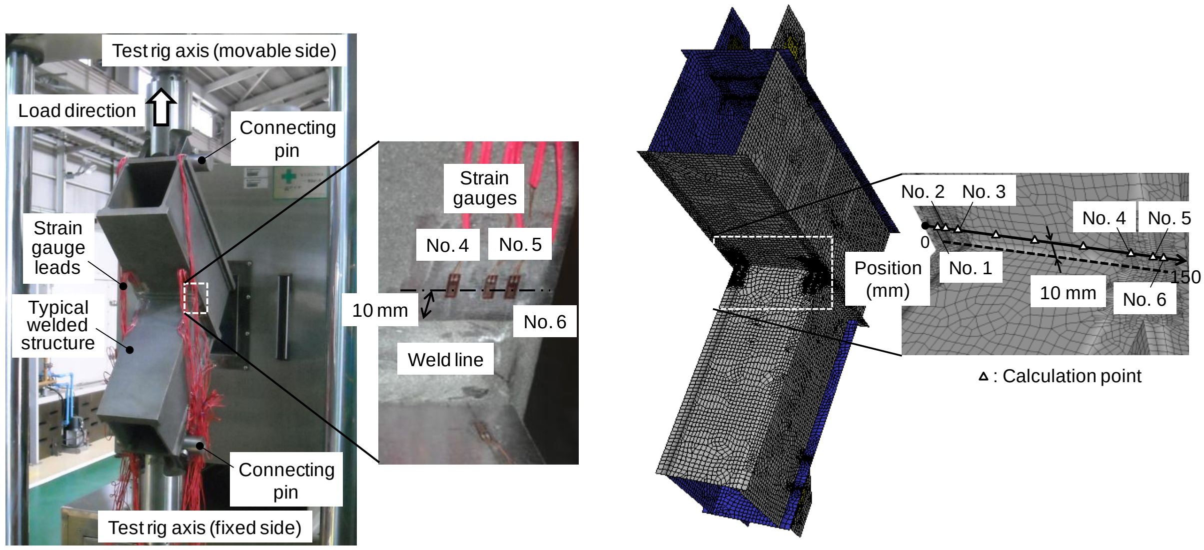
6. Example

6.1. Measurement and simulation of typical welded structure

A welded structure that includes the weld joints typically used in construction machinery was designed and experimentally produced. A load of 100 kN was applied to the produced welded structure, and strains were measured with strain gauges near a weld line as shown in Figure 3(a). This static load test was performed twice, and the strains were accordingly measured twice. Although the strains were measured at a static load of 100 kN, the strains measured at the other applied loads were needed to investigate the prediction accuracy of the bias correction model in Eq. (2). A virtual load history shown in Figure 4 was therefore assumed, and then the measured strain histories caused by the load history were generated by using the liner relationship between the applied load and the strains. In addition, the finite element model of the typical structure was also prepared to calculate strains at the points where the strain gauges were placed. The finite element model shown in Figure 3(b) was composed of shell elements, each of which was rectangular and had four nodes. Numbers 1 to 6 in Figure 3 show the points where strains were measured on the produced structure. The measured and simulated strains were then utilized to predict real strains $y^R(\mathbf{x}, \mathbf{F})$ at the points where strains were not measured by using the procedure described in section 5. The measured and simulated strain values for this prediction are shown in Figure 5. Since the measured strains in Figure 5 were generated using the two test results, two measured values are shown at a position and a load. Notice that there is obviously a model bias caused by incomplete modeling because the simulated strains are larger than the measured ones except for one point. Despite this discrepancy, the finite element model shown in Figure 3(b) was generated in accordance with the International Institute of Welding recommendations.

6.2. Prediction of real strain and bias with GP model

In the case of the GP model, the two strains measured at the same position and load were averaged, and



(a) Experimentally produced structure (b) Finite element model

Figure 3: Illustrative welded structure

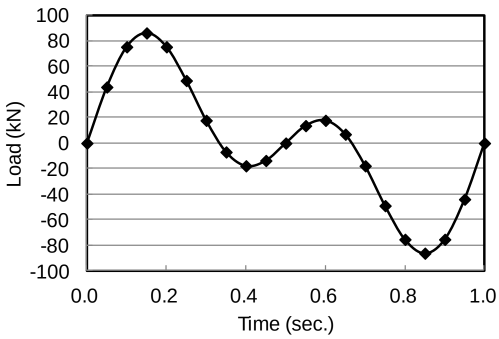
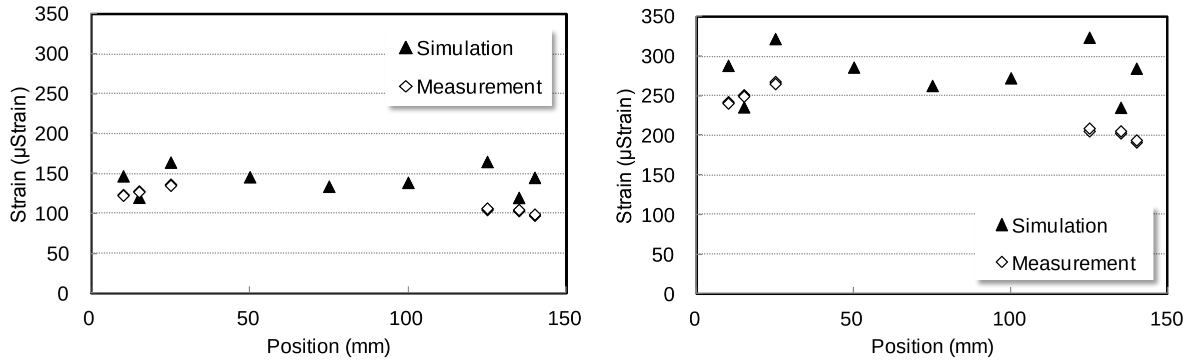


Figure 4: Applied load history



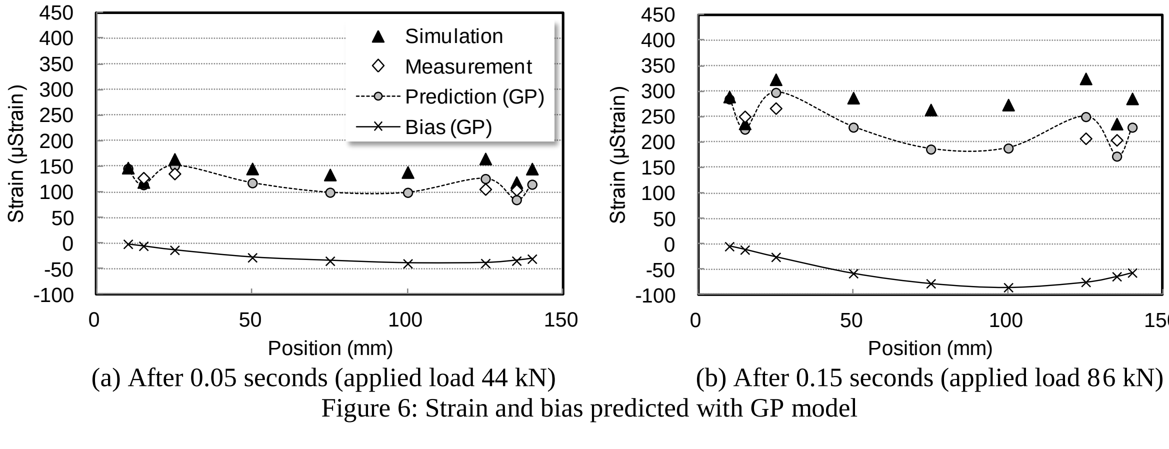
(a) After 0.05 seconds (applied load 44 kN) (b) After 0.15 seconds (applied load 86 kN)

Figure 5: Simulated and measured strains

the averaged value is regarded as a measured strain to predict real strain $y^R(\mathbf{x}, \mathbf{F})$. To validate the prediction accuracy of the bias-corrected model in Eq. (2), not all the measurement points were used to predict the real strains. That is, four out of the six points were selected and used to model the real strain, and then the other two points were used to validate the generated model. Figure 6 shows the expected mean of bias $\delta(\mathbf{x}, \mathbf{F})$ defined in Eq. (7) and the predicted real strain $y^R(\mathbf{x}, \mathbf{F})$ when selecting Nos. 2, 3, 4, and 5 for the modeling. Note that the expected mean of the bias is smooth, hence the predicted real strain represents the feature of the simulated strains, especially for the positions where strains are not measured. By comparing the predicted and measured strains at the points for validation, the predicted strains are respectively 118 % and 119 % of the measured strains for positions Nos. 1 and 6. On the other hand, the simulated strains are respectively 120 % and 148 % of the measured strains for positions Nos. 1 and 6 as shown in Figure 7. It is therefore clear that the bias-corrected model with the GP model works well for predicting the real strains.

6.3. Prediction of real strain and bias with neural network

In the case of the artificial neural network, all the measured values were used to generate the response surface of bias $\delta(\mathbf{x}, \mathbf{F})$. Figure 8 shows the resulting response surfaces of bias $\delta(\mathbf{x}, \mathbf{F})$ and the predicted real strain $y^R(\mathbf{x}, \mathbf{F})$. It can be seen that the response surface of the bias obtained using the neural network is smooth and so is the expected



(a) After 0.05 seconds (applied load 44 kN) (b) After 0.15 seconds (applied load 86 kN)

Figure 6: Strain and bias predicted with GP model

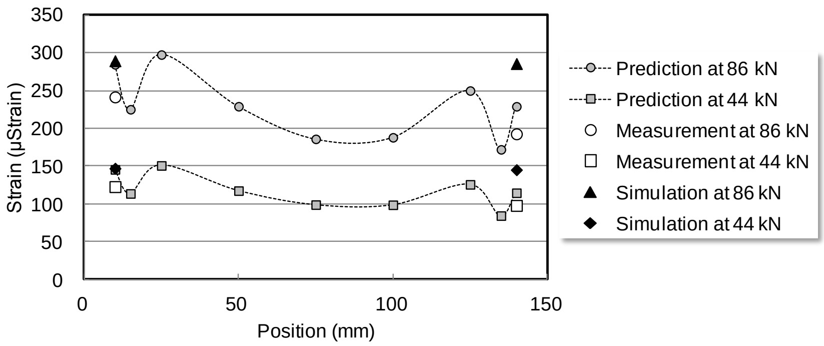
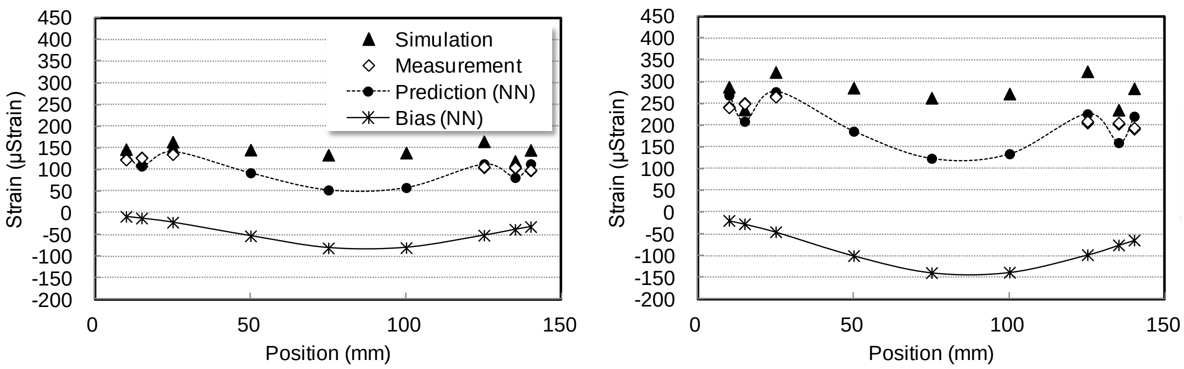


Figure 7: Verification of predicted strains



(a) After 0.05 seconds (applied load 44 kN) (b) After 0.15 seconds (applied load 86 kN)

Figure 8: Strain and bias predicted with neural network

mean of the bias obtained using the GP model. While the absolute value of the bias when using the neural network is larger than that when using the GP model at 50mm to 100mm positions, both response surface models generate almost the same bias near the measurement positions.

7. Conclusion

Surrogate models can be used as virtual sensors for monitoring the operation condition of infrastructures. Methods for generating a surrogate model using measured data and simulation results have been applied to several engineering examples in the last decade. This hybrid surrogate model is accurate enough to be used as virtual sensors and therefore was used to predict structural responses, namely strains, in this study. To investigate the accuracy of prediction in unmeasured strains, a typical welded structure was experimentally produced to obtain the measured strains, and the numerical analysis of the welded structure was also conducted to obtain the simulated strains. When predicting the strains, the discrepancy between the measured and simulated strains was approximated with the response surfaces, namely the GP model and the neural network. In the results of this investigation, the predicted strains agreed well with the measured ones. It can therefore be concluded that the hybrid surrogate model can work as virtual sensors.

8. References

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