Structural Dynamic Topology Optimisation of a Direct-Drive Single Bearing Wind Turbine Generator

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1. Abstract
Reducing weight of off-shore wind turbine nacelles is currently a key driver of innovation within the wind turbine industry. Weight reduction will not only lead to smaller loads and thus smaller towers of the turbine, but also reduce logistic costs during the turbine’s installation. This holds even more so for off-shore turbines, the costs related to installing a turbine is a substantial investment compared to the operational cost. A reduced nacelle weight will, subsequently, lead to reduced cost of wind energy.

For direct-drive turbines, the generator is one of the heaviest parts of the wind turbine nacelle. Due to the low rotational speed of the generator, the loads are especially high in this type of turbine, which increases the necessary structural mass of the rotor. Recently, designed flexibility has been identified as one approach to achieve weight reduction. However, reducing the weight of the support structure has proven difficult, due to the complex pattern of dynamic excitation forces.

Until now, density based topology optimisation has hardly been employed for the design of wind turbine parts. This publication investigates the possible weight reduction which results from applying this method to the support structure of the generator rotor. As a first step, crucial excitation frequencies and spatial force distributions that are generated by the magnetic field are presented. Then the topology optimisation is executed using a modal and a harmonic approach, applying the identified force distributions.

Keywords: density based structural topology optimisation, direct-drive wind turbine generators, modal participation factors

2. Introduction
In multi-megawatt direct-drive wind turbines the operating conditions of the generator are influenced by the design choice to omit the gear box between hub and generator rotor. The rotational speed of the hub, and hence of the generator rotor, is determined by the tip speed ratio and thus by the length of the blades. The huge nominal power that is generated in such a machine leads to a large torque that needs to be transformed into electrical power in the generator. The torque is proportional to the squared of the magnetic flux density, which is limited to a certain magnitude in radial flux electric machines. An increased air gap surface least and increased total flux in the machine and thus also to an increased total torque. This leads to large generator diameters, which requires heavy structures to transfer the electro magnetic forces to the bearing of the rotor. In the past optimisation of the structural elements of a wind turbine with the goal of weight reduction has been at the focus of several publications. [1, 2] investigate direct-drive wind turbine generators and how their weight can be optimised. [3] explores designed flexibility as a solution for weight reduction for this type of generator. [4] optimises a slip ring permanent magnet generator.

Topology optimisation was identified as one technique to compute the optimal topology for the rotor structure that supports the electro-magnetic active parts of the generator. The technique has been used before in combination with forces originating from magnetic fields. [5] uses the technique to minimise vibrations generated by magnetic harmonic forces. [6] uses topology optimisation in a magneto-mechanical coupled system to minimise compliance of the yoke structure.

All of the above studies lack either the focus on electric machines, or the focus on structural dynamical behaviour of the system. This paper will investigate the optimal distribution of material to minimise dynamic resonance for the rotor of a permanent magnet direct-drive wind turbine generator. The forces considered include static as well as dynamic forces that are generated by the magnetic field in the air gap.

The paper follows two approaches for the optimisation process. The first approach uses an harmonic analysis to calculate the dynamic energy within the structure at certain frequencies. The second approach calculates the vibration modes and uses mode sensitivities to optimise the mode shapes to minimise participation factors of modes in a certain frequency range. In both approaches the ‘Method of Moving Asymptotes’ [7] was used.
Figure 1: The XD-115 feature a direct-drive generator (a). The force distribution is depicted in (b) for the static case and in (c) for the dynamic case in $N^2$. The domain for the topology optimisation can be described by a hollow cylinder (d). The red line indicates the edge that is clamped in three directions. The grey surface indicates the barrel surface of the cylinder where the magnetic forces are applied.

3. The XD-115 – The System to Optimise

The XD-115 is a 5MW wind turbine by XEMC-Darwind [8]. It incorporates a direct-drive topology that is commonly found in off-shore wind turbines, and a permanent magnet generator. The diameter of the generator is about 5 m and its axial length is about 1 m. The single bearing supporting the rotor has a diameter of about 2 m less than the diameter of the air gap. The bearing is located between the generator and the hub of the rotor. That location reduces bearing torques and leads thus to small bearing loads. Fig. 1 a shows the current structure. The rotor part that is of interest for this study is highlighted in blue.

3.1. The force distribution

In electric machines the main source of dynamic excitation is the torque ripple, which is created by the interaction of the permanent magnets with the stator slots and the space harmonics which are cause by the distribution of the winding. The force distribution encountered in the air gap only depends on the number of poles and the winding distribution. These parameters do not change during the operation of the machine. The force distribution is, thus, constant for any operating point of the machine. Contrary to this, the magnitude and the frequency of the excitation forces depend on the relative position of the rotor to the stator and does thus depend on the rotation frequency. This is especially important for variable speed wind turbines where the operation rotational speed depends on the wind speed and thus changes significantly during operation. Therefore, the generator should be designed in such a way that the amplitude of the dynamic response does not depend on the frequency of the exciting force.

The magnitude of the dynamic and static forces can be estimated from the rotational speed and the nominal power of the turbine. The rotational speed of the turbine is at maximum power about 18 rpm. The torque that needs to be transmitted from the hub to the electro-magnetic active parts in the generator is thus 2.6 MNm. The force is distributed around the circumference of the rotor. Due to the design of the generator, the force distribution is fixed and rather well known. It can be described by

$$f_{mag,stat}(x_{mag}) = \hat{f}_{mag} \begin{bmatrix} 50(1 - \sin(n_{poles}\theta)) \\ 0 \\ (1 - \sin(n_{poles}\theta + \frac{\pi}{2})) \end{bmatrix}$$

in cylindrical coordinates, where $\theta$ denotes the circumferential coordinate in cylindrical coordinates, $n_{poles}$ the number of poles of the machine and $\hat{f}_{mag}$ the amplitude of the force distribution. At full load, the force in radial direction is about 50 times higher than the tangential force.

Additionally, there is a much smaller dynamic force that excites the structure dynamically. This force acts at the same location as the static force. It can be described by

$$f_{mag,dyn}(x_{mag}) = \frac{\hat{f}_{mag}}{50} \begin{bmatrix} (1 - \sin(n_{poles}\theta))e^{j\theta_{shift}} \\ 0 \\ (1 - \sin(n_{poles}\theta + \frac{\pi}{2}))e^{j\theta_{shift}} \end{bmatrix} e^{j\frac{n_{poles}}{2}n_{sym}\omega_{mech}}$$

with $\theta_{shift} = \theta n_{sym}$

where $j$ is the imaginary unit. Previous research showed that the dynamic force in radial direction was about 50 times smaller that the static force. The same was assumed for the dynamic forces in tangential direction. $\theta_{shift}$ denotes the phase shift of the magnetic force. This is caused by the cyclic symmetry within the machine. The value depends on $n_{sym}$, the number of symmetric sections of the electro-magnetic active part of the generator. The
\( \frac{\pi}{2} \) originates from the fact that the radial force peak has a phase shift with respect to the tangential force peak of a quarter of a pole. \( x_{mag} \) is the subset of \( x \) that includes all points, where the magnetic force \( f_{mag} \) is applied. To make sure that the total torque is equivalent to the 2.6 MNm

\[
\int_{\Omega_{mag}} e_\theta \cdot f_{mag,\text{stat}} \, d\Omega_{mag} = 2.6 \text{MNm}
\]  

(3)

where \( \Omega_{mag} \) denotes the surface of the cylinder barrel, where the magnetic force is applied and \( e_\theta \) a vector pointing in tangential direction. Solving Eq. 3 for \( f_{mag} \) yields the amplitude of the magnetic force distribution.

3.2. The Optimisation Approach

The goal of the topology optimisation is to find a design that shows the smallest displacement of the structure at the air gap and the excitation of the dynamic magnetic forces described in Eq. 2. The excitation frequencies are determined by the operating speed of the turbine. As a result, the optimisation can be restricted to a certain frequency range.

To make the design magnetically feasible, a magnetically conducting connection needs to be established between the magnets. This is usually accomplished by placing the magnets on a magnetically conducting hollow cylinder. To ensure that the solution of the optimisation problem satisfies this, a constraint is introduced that limits the minimal mass of the outer surface of the optimisation domain. Further, it is necessary to ensure that the solution of the optimisation limits the maximal displacement for the statically applied forces described in Eq. 1. This is done by introducing a constraint that limits the strain energy at the barrel surface of the design domain.

Fig. 1d shows the design domain that is defined for the topology optimisation. At the red edge the degrees of freedoms are clamped in all directions. The grey surface indicates the barrel surface of the domain at which the electro magnetic forces defined by Eq. 1 and Eq. 2 are applied.

A penalty function was introduced for the stiffness and density of the structure to ensure regions with full or zero density are more beneficial. Further, the density of the structure was lowered to avoid unphysical local modes with low resonance frequencies. A solid isotropic material with penalisation model is used [9]. Using this model, the young modulus and density of the material can be described by

\[
E = 200 \times 10^9 (0.1 + 0.9 \rho_f^3)
\]  

(4)

\[
\rho = \begin{cases} 
\rho_f & \text{for } \rho_f > 0.1 \\
\rho_f^{10} & \text{for } \rho_f < 0.1
\end{cases}
\]  

(5)

Two approaches are followed in this paper to minimise the displacement in the air gap of the generator due to dynamic excitation. The harmonic approach solves the system of equations in the frequency domain at certain predefined frequencies. The modal approach uses a modal analysis of the system to calculate the participation factors for the modes within the frequency range of interest.

The selection of which method to use is based on the size of the numerical system and the frequency range that is relevant for the optimisation. Larger frequency ranges should be optimised by the ‘modal’ approach while smaller frequency ranges should be optimised using the ‘harmonic’ approach. In this paper both methods are used separately.

3.2.1. Harmonic Approach

For the Harmonic approach a set of frequencies \( \omega_{freq} \) has been chosen for which the displacements of the system as a result of the dynamic forces are calculated. In order to do this, the system of equations is transformed to the frequency domain. The result can be written as

\[
(-M \omega^2 + j \omega C + K)u = f_{mag}
\]  

(6)

For this approach, the optimisation problem can be described by

\[
\min_{\rho_f(x)} \sum_{\omega_{freq}} f_{mag}(x_{mag}) \cdot u(x_{mag}, \omega_{freq})
\]

subject to

\[
\int_{\Omega} \rho_f(x) \, d\Omega < m_{max} ; \quad \int_{\Omega_{mag}} \rho_f(x) \, d\Omega > m_{mag,min}
\]  

(7)

\[
u(x_{mag}) \cdot f_{mag}(x_{mag}) < w_{max}
\]

\[
10^{-2} \leq \rho_f \leq 1
\]
where \( m_{\text{mag}} \) denotes a maximal mass of the rotor structure, \( f_{\text{mag}}(u_{\text{mag}}) \) the force distribution originating from the magnetic field in the air gap and \( \rho \) the density of the structure. \( \Omega_{\text{mag}} \) denotes the barrel surface of the domain where the magnetic forces are applied and \( w_{\text{mag}} \) the maximal strain energy associated with the displacement at the barrel surface. To make sure that mass is placed at the barrel surface to create a ring that connects the magnets, a constraint is introduced that limits the minimal mass at the barrel surface. \( m_{\text{mag},\text{min}} \) denotes this minimal mass.

This approach has the disadvantage that the number of frequencies \( \omega_{\text{freq}} \) has a direct influence on the calculation cost of this method. The more frequencies are included the more often Eq. 6 needs to be solved. The frequencies in \( \omega_{\text{freq}} \) should not be too close to each other, because this increases the computational costs. Choosing the distance between them too large could entail that a resonance frequency is not captured by the algorithm and is thus not included in the optimisation function.

The harmonic optimisation was conducted with 8 frequencies within the frequency range of interest. The distance between the frequencies was about 20 Hz. The model consists of approximately 318000 degrees of freedom it contains 106000 design variables for the topology optimisation, one for each node of the finite element model.

3.2.2. Modal Approach

The modal approach minimises the displacement caused by the harmonic excitation forces by minimising the participation factors of the mode within the frequency range of interest. To do that, it solves the eigenvalue problem to calculate the mode shapes

\[
(K - \omega^2 M)\Phi_r = 0
\]

where \( \omega_r \) denotes the \( r \)th eigenvalue of the system and \( \Phi_r \) the corresponding eigenvector. The optimisation problem can then be written

\[
\min_{\rho_f(x)} \sum_k (f_{\text{mag}}(x_{\text{mag}}) \cdot \Phi_k(x_{\text{mag}})(f_{\text{mag}}^*(x_{\text{mag}}) \cdot \Phi_k(x_{\text{mag}})))
\]

subject to

\[
\int_\Omega \rho_f(x) d\Omega < m_{\text{max}}
\]

\[
\int_{\Omega_{\text{mag}}} \rho_f(x) d\Omega > m_{\text{mag},\text{min}}
\]

\[
u(x_{\text{mag}}) \cdot f_{\text{mag}}(x_{\text{mag}}) < w_{\text{max}}
\]

\[
0 > \rho_f > 1
\]

where the asterix denotes the complex conjugate of the load vector and \( \Phi_k \) denotes a set of eigenvectors that is chosen from all solutions of Eq. 8 by choosing the eigenvectors with the highest values within a certain frequency range. This means that the objective function can change abruptly when a the eigenfrequency of a vibration mode changes and the mode is no longer considered in the objective function. This approach harbours the danger that the optimisation algorithm does not converge but instead is caught in a loop of evaluating the same sets of design parameters. This can happen when a mode is no longer considered for the objective function due to an eigenfrequency that is too low or too high and the optimisation subsequently change the set of design parameters, so that the mode is considered again.

The 'modal method' has the advantage that the system has to be solved less often than for the harmonic method. The eigenvalue solver will identify the resonance frequencies which are not specifically calculated in the harmonic method. The drawback is that the optimisation function is not self adjoint. The adjoint variables need to be calculated for every mode that is included in the objective function. For that a linear system of equations of the size of Eq. 8 is solved.

The model for the modal method consists of about 154000 degrees of freedoms. The density field was described per element, leading to a total number of 44352 design variables.

4. Results

To be able to compare the optimised design with the original design the maximal strain energy resulting from the static forces and the maximal weight of the structure were set to the same values that the original design has. This way the optimisation yields results that exhibit the same displacement and weight as the original design. The participation factors indicate to what extend the new design is better or worse than the old design.

4.1. Harmonic Optimisation Approach

Fig. 2 shows the results of the harmonic approach. The participation factors associated with this design are listed in Tab. 1. It is apparent that the resulting participation factors are higher than the ones of the original design. The
Table 1: The participation factors for selected modes and the two approaches. The participation factor for the current design are stated as reference.

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>harmonic approach</th>
<th>modal approach</th>
<th>original analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>norm freq part factor</td>
<td>norm freq part factor</td>
<td>norm freq part factor</td>
</tr>
<tr>
<td>tilting</td>
<td>0.83 + 0.01i 0.54 + 0.1i</td>
<td>0.58 −0.07 + 0.01i</td>
<td>0.68 0.13 −0.02i</td>
</tr>
<tr>
<td>1st bending</td>
<td>1.61 + 0.02i −0.38 −0.43i</td>
<td>1.17 0.05 −0.04i</td>
<td>1.2 0.02 −0.04i</td>
</tr>
<tr>
<td>axial</td>
<td>1.15 + 0.014i −0.04 −0.0i</td>
<td>1.27 −0.07 −0.02i</td>
<td>1.29 0.01 + 0.01i</td>
</tr>
<tr>
<td>2nd bending</td>
<td>- -</td>
<td>1.83 0.03 + 0.04i</td>
<td>3.18 −0.8 + 1.55i</td>
</tr>
<tr>
<td>torsional</td>
<td>2.3 + 0.03i 0.04 −0.12i</td>
<td>1.47 0.06 + 0.06i</td>
<td>3.24 −0.04 + 0.02i</td>
</tr>
<tr>
<td>shear</td>
<td>1.91 + 0.02i 0.71 + 0.23i</td>
<td>1.61 −0.04 + 0.06i</td>
<td>- -</td>
</tr>
<tr>
<td></td>
<td>1.91 + 0.02i −0.05 + 0.17i</td>
<td>1.66 0.03 + 0.02i</td>
<td>- -</td>
</tr>
</tbody>
</table>

4.2. Modal Optimisation Approach

The results of the modal method are depicted in Fig. 3. The participation factors are given by Tab. 1. The participation factors are for most of the relevant modes lower than for the original design. However it should be noted that the constraint limiting the static compliance is violated for this result. Fig. 3 also shows that the density of the structure is slightly higher than in the original design.
at the barrel surface is not high enough to satisfy the second constraint. The high participation factors and the
not satisfied boundary conditions for the modal approach result can be explained by the coarse mesh. The density
is described element wise. One element has a side length of 10 cm. This yields a minimal wall thickness of 10
cm. The static compliance constraint cannot be satisfied with this coarse mesh. Likewise the constraint that the
outer ring is solid and filled with elements of full density cannot be satisfied without violating the maximum mass
constraint for the whole structure.

5. Discussion
Topography optimisation of such a large structure that requires a high resolution for the density field is difficult. The
results are thus at the lower resolution limit that is barely able to describe the structure and the modes. The results
have to be used with caution as the finite element approximation for the stiffness of thin walled structures might
not be accurate when the structure is only one element wide. This is the case at several location in the solution. A
shape optimisation, whose topology is based on the results of this study could increase the confidence in the results
that are presented here.

Methods to overcome the problem, that the limit of degrees of freedom imposes on the model, include the
application of a cycle symmetric eigenvalue solver, employing flocked theory [10]. This approach requires further
research to what extent the flocked theory can be included in the optimisation algorithm.

The result of the harmonic method also show higher participation factors than the one of the original solution.
This might be caused by a gap that is too large between consecutive frequencies at which the displacement was
 calculated. Better results are expected when a finer frequency resolution is chosen.

6. Conclusion
Topology optimisation was used to analyse and improve the design of a direct drive wind turbine generator. Special
emphasis was given to the dynamic behaviour of the structure, that is excited by the magnetic force in the air gap.
Various methods resulted in similar topologies. However, the optimised results did not perform better than the
original design. Further research is necessary to identify why the performance could not be improved.

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