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Electromagnetic levitation coil design using gradient-based optimization

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1. Abstract

Electromagnetic levitation melting is a containerless technique to obtain material properties of reactive, electrically conductive materials that would otherwise result in sample contamination when in contact with a container at high temperatures. The levitation coil geometry, and the magnitude and frequency of the alternating current determine the sample sizes of a specific material that can be levitated as well as the temperature of the levitated sample.

The levitation cell is modelled using a one-dimensional analytical approach. This model requires the material properties of the sample and surrounding atmosphere as input variables. Since there is a large amount of uncertainty in measuring these properties they are regularized using experimental data from a known coil design and current. The levitation cell model with regularized material properties is then used in a gradient-based optimization scheme to design a coil for the levitation melting of specified sample size and material.

The consequences of using a multistart, gradient-based optimization scheme are reported. Coils are designed to minimize the temperature of the levitated sample or maximize the stability of the sample during levitation.

2. Keywords: gradient-based, multistart, electromagnetic levitation

3. Introduction

Measuring the physical properties of metals at high temperatures present some challenges especially for containing the sample, since many materials have a corrosive nature at high temperatures. Experiments often result in the container being destroyed or the sample being contaminated by container material while in contact with the container at high temperatures. Electromagnetic levitation melting is an alternative non-contact experimental approach to measure physical properties including viscosity and surface tension.

Trial and error methods are common practice for levitation coil design [1], [2]. However, this often results in levitation cell experiments being complicated by coil designs that are very sensitive to sample position in the coil and/or small changes in coil geometry. There also exists a need to use levitation melting for an increased variety of materials while accessing a larger range of temperatures with more precise control [2].

The sample sizes of a specific material that can be levitated and the temperature of the levitated sample are determined by the levitation coil geometry, and the magnitude and frequency of the alternating current. Royer et al. [2] use a genetic-like algorithm to design a levitation coil that would minimize the temperature of the levitated sample for a large range of currents. In this paper we use a gradient-based, multistart optimization algorithm to design similar coils. We specifically investigate formulating levitation coil design as an optimization problem to develop a design tool based on a levitation cell model that would allow the design of robust and reliable coils for the levitation melting of specified sample materials and sizes. In the process we also use optimization to regularize the material properties in the Royer et al. [2] model and to minimize the difference between two functions. In all three instances the constrained optimization function (fmincon) in Matlab's optimization toolbox is utilized.

4. Electromagnetic levitation melting process description

An electromagnetic levitation cell basically consists of a water-cooled coil supplied with high frequency alternating current. The coil has an associated high frequency alternating magnetic field around it since a magnetic field is generated around any current-carrying conductor. Faraday's law states that a changing magnetic field induces an electric field [3]. Therefore the alternating magnetic field will induce eddy currents in any electrically conductive body placed inside the coil.

The magnetic field of the induced eddy currents opposes that of the coil [2]. The interaction of the magnetic field of these eddy currents with the magnetic field of the coil current results in a Lorentz force [1], [2], [4]. If a position exists inside the coil where the Lorentz force is equal and opposite to the weight of the body, the body will be levitated in that position inside the coil. Additionally, the induced eddy currents in the body will cause resistive heating (Ohmic / Joule heating) [1], [4]. If this causes the body to melt, the Lorentz force will cause flow inside the molten droplet [1], [5].

In order to create a region inside the coil where stable levitation is possible, the current in a part of the coil has to flow in the opposite direction to the current in the rest of the coil [1]. This can be achieved by adding a stabilizing part to the coil that is connected in series to the main coil and wound in the opposite direction.

5. Levitation cell model

The analytical one dimensional levitation cell model proposed by Fromm and Jehn [6] and used by Royer et al. [2] has been implemented. The current induced in the sample is modelled using the concept of mutual inductances. This results in a nonlinear expression for the Lorentz force on a sample in terms of the sample position. The sample levitation position is found where the Lorentz force is equal and opposite to the weight of the sample. The temperature of the sample is solved from another nonlinear equation resulting from the energy balance of the induced heating of the sample and the convection and radiation heat losses to the environment. The electromagnetic and thermal parts of the problem are coupled because the heat induced in the sample is a function of its levitated position.

The model approximates the helical coil as a set of axisymmetric circular loops (Fig.1). It is further based on the assumptions that the sample size is small relative to the coil, the sample is spherical, the coil is operating at high frequencies and the alternating magnetic field over the sample position is uniform in space.

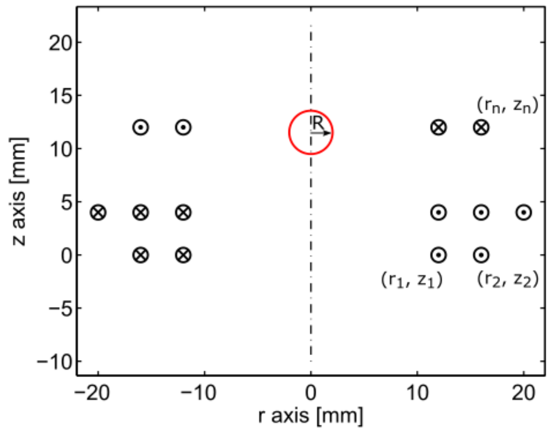


Figure 1: Axisymmetric approximation of a levitation cell coil with the sense of the current in the loop and the sample levitation position indicated. Crosses and dots indicate opposite alternating current senses.

The magnetic field predicted by the Fromm and Jehn model [6] is highly nonlinear around the sample. To compensate for this and the spatial distribution of the sample mass within the coil, Royer et al. [2] suggests discretizing the sample into a number of disks, computing the induced heat in each disk using the method of Fromm and Jehn [6] and then using a volume weighted average to find the total heat induced in the sample. Fromm and Jehn [6] compute the induced heat as

$$\dot{q}_L = 3\left(\frac{\pi^3 \mu f}{\gamma}\right) R^2 H^2 \tag{1}$$

where H is the originally assumed uniform magnetic field strength. In our implementation of the model we replace H with a volume average $\tilde{H}$, computed numerically from the non-uniform magnetic field strength H(z) as

$$\tilde{H} = \frac{1}{V} \int H(z)\, dV. \tag{2}$$

6. Regularization

Royer et al. [2] report uncertainty in the sample emissivity and temperature dependent material properties (the sample conductivity and the properties of the fluid surrounding the sample) as possible sources of error in their model.

We attempt to address this problem with regularization. The experimental results from a known coil design and electrical current is used to adjust the uncertain material properties in such a way that the difference between the model predictions and experimental results are minimized while penalizing large variations in the material properties. The experimental results reported by Royer et al. [2] for their seed coil geometry is used for regularization. An equality constraint requires the Lorentz force to be equal to the sample weight to ensure levitation takes place and the inequality constraint $dF/dz < 0$ ensures that the levitation position is stable. Analytical gradients are supplied to the optimization algorithm to reduce the computational time requirement.

The regularized material properties are given in the table below.

Table 1: Regularization results

Material property	Original value	Regularized value	% change
Sample emissivity, ε	0.1	0.0934	-6.5
Sample electrical conductivity, γ	4252890	4252890	0.0
Fluid thermal conductivity, k_f	0.0177	0.0287	61.8
Fluid density, ρ_f	0.1625	0.1797	10.6
Fluid kinematic viscosity, η	1.9900×10^{-5}	1.9089×10^{-5}	-4.0
Fluid specific heat, C_p	520.3	520.3	0.0

In Fig.2 sample temperature predictions at various currents given by the model with and without the regularized material properties are compared to the experimental results reported by Royer et al. [2]. It can be seen that the model is improved by including the regularized material properties but the model still has difficulty predicting the gradient of the experimental results.

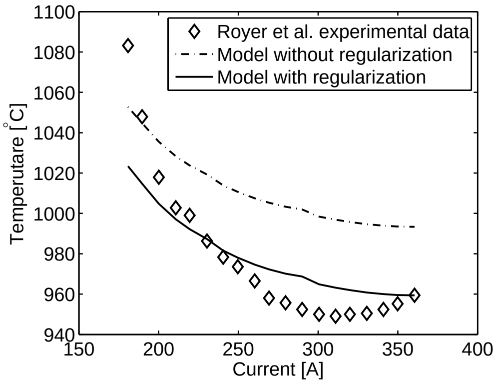


Figure 2: Model results with and without regularization compared to experimental results reported by Royer et al. [2].

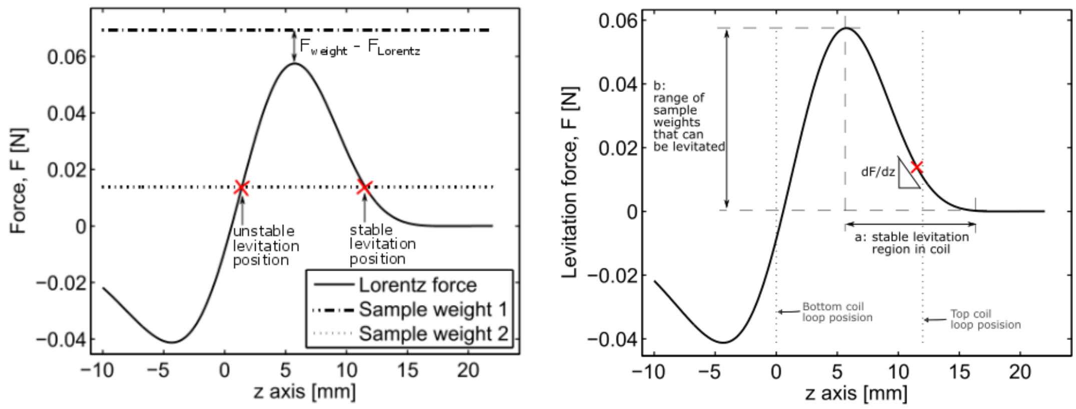
7. Coil design using gradient-based optimization

A gradient-based multistart optimization formulation with a Sequential Quadratic Programming (SQP) algorithm is used to design coils for various experimental requirements. The variables used as design variables are: the total number of loops in the coil, the number of loops in the stabilizing part of the coil, the magnitude and frequency of the current supplied to the coil and the radial and axial positions of each of the coil loops.

The total number of loops in the coil and the number of loops in the stabilizing part of the coil are discrete variables. The optimization algorithm is therefore repeated for every possible permutation of these two variables and the permutation that obtains the lowest value of the objective function at its optimum is selected.

All the design variables are bounded. The number of coil loops are between five and seven of which either two or three loops have the opposite current sense to form the stabilizing part of the coil. Current magnitude can vary between 150 A and 400 A and current frequency is in the range $100-200$ kHz. The maximum radius of the coil is 95 mm and the minimum radius of any coil loop is 10 mm to allow enough space for the sample inside the coil. The axial position of the coil loops can be anywhere between 0 mm and 20 mm. Constraints are applied to prevent any of the 3mm tubes that form the coil loops from overlapping. Further constraints ensure that the Lorentz force is equal to the sample weight for levitation and that the levitation position is stable, $dF/dz < 0$. Fig.3 illustrates the last two constraints. The constraints applied to the coil design optimization is kept at a minimum to allow us to investigate which coil design properties produce specific levitation cell characteristics. This could result in coil designs that are difficult to manufacture and it might later be necessary to add manufacturing constraints to the coil design optimization formulation.

The shape of the Lorentz force curve in Fig.3 is determined by the coil geometry and the electrical current in the coil. Since these are design variables, some combinations will not satisfy the equality constraint that requires the Lorentz force to be equal to the sample weight (e.g. Sample weight 1 in Fig.3a). If this is the case the analysis will minimize the square of the difference between the sample weight and the Lorentz force within the coil and return the corresponding sample position to the optimization algorithm. In this way the optimization algorithm still receives information that will allow it to determine in which direction to search for a solution which would satisfy the equality constraint.



(a) Possibilities of stable, unstable and no levitation.

(b) Effect of dF/dz on stability and the range of sample weights that can be levitated.

Figure 3: Typical variation of the Lorentz force along the z-axis of a coil.

7.1. Minimizing the temperature of the levitated sample

For some comparison with the results obtained by Royer et al. [2] who minimize the temperature of the levitated sample for a large range of currents, the first objective function that is investigated, is to minimize the temperature of the sample during stable levitation.

It is found that the minimum value obtained with this objective function varies significantly depending on the initial design vector supplied to the algorithm. A multistart optimization formulation with ten starts for each of the six discrete variable permutations is therefore used. The initial design vector for each start is a random value within the lower and upper bounds. Fig.4 shows the sorted objective function values obtained at the optima of the various starting points that yielded feasible coil designs.

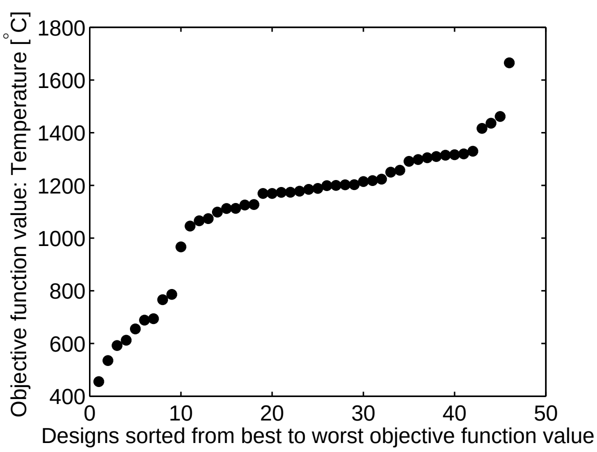
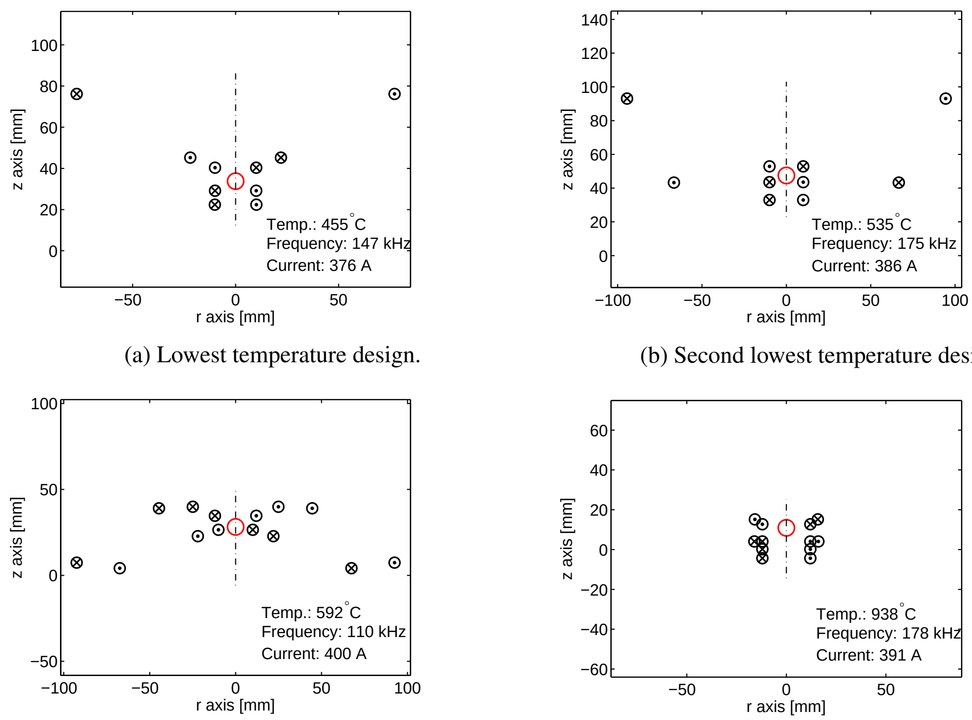


Figure 4: Sorted objective function values for multi-start coil design optimization to minimize temperature.

It can be seen from Fig.4 that there are nine designs that are significantly better than the rest. The three coil designs that yield the lowest sample temperatures as well as the design proposed by Royer et al. [2] for minimizing the sample temperature for a wide range of currents are given in Fig.5.



(a) Lowest temperature design.

(b) Second lowest temperature design.

(c) Third lowest temperature design.

(d) Royer et al. [2] optimized coil for lowest sample temperature over a range of currents.

Figure 5: Sorted objective function values for multi-start coil design optimization to minimize temperature.

These results cannot be directly compared to that of Royer et al. [2], since we include current magnitude and frequency as design variables, which Royer et al. [2] does not, and we are minimizing the sample temperature at any specific current while Royer et al. [2] minimize the sample temperature over a range of currents. However, some similarities can be observed between the first two designs obtained and the design obtained by Royer et al. [2]. In all three cases the bottom three coil loops have a very small equal radius while the top two loops form a conical shape. An interesting difference is that we get a result where the sense of coil current changes twice from the bottom to the top of the coil as opposed to most coils in literature that report only one change in the sense of the current. We also obtain larger maximum coil radii than Royer et al. [2]. These might be trends in the coil designs, but a larger number of different starting points is required to show this definitively.

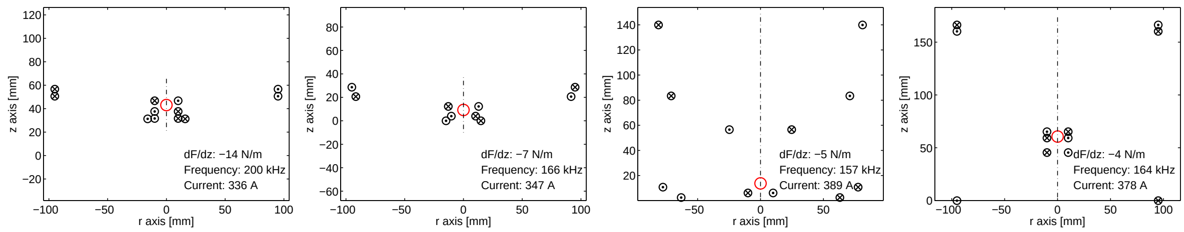
7.2. Maximizing the stability of the sample position

We are interested in designing levitation cell coils with which experimental work would be as easy, reliable and repeatable as possible. One factor that determines this is the stability of the sample levitation position. Therefore the objective function in the optimization problem is now changed to make the slope of the Lorentz force-position curve, which has to be negative for stability, as steep as possible

$$\max \left| \frac{dF}{dz} \right| . \tag{3}$$

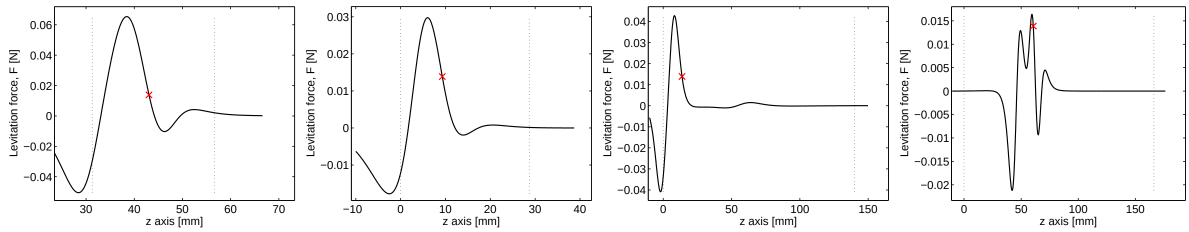
Once again a multistart optimization formulation with ten starts for each permutation of the discrete design variables is used. Of all the results that are obtained, the four designs given in Fig.6 have significantly larger values for $|dF/dz|$ than the rest. These four designs all have a loop with a small radius directly below the sample levitation position. All except the third coil have at least three closely spaced small radius loops around the sample position and two or three large radius loops. The first two coil designs both have small overall heights.

Fig.7 shows the variation of the Lorentz force along the z-axis of the four coils designed for maximum stability. It is evident from this figure that a consequence of maximizing $|dF/dz|$ is that the part of the coil where stable



(a) Largest $|dF/dz|$. (b) 2nd largest $|dF/dz|$. (c) 3rd largest $|dF/dz|$. (d) 4th largest $|dF/dz|$.

Figure 6: Multistart coil design optimization to maximize stability. Coil designs sorted according to objective function value.



(a) Largest $|dF/dz|$. (b) 2nd largest $|dF/dz|$. (c) 3rd largest $|dF/dz|$. (d) 4th largest $|dF/dz|$.

Figure 7: Multi-start coil design optimization to maximize stability. Variation of the Lorentz force along the z-axis of the coil.

levitation is possible becomes small - around 5 *mm* for these four designs. The fourth design also shows multiple peaks in the force curve which is not desirable. Fig.3b shows that by maximizing $|dF/dz|$ we are increasing the range of sample weights that can be levitated, while the part of the coil where stable levitation is possible becomes smaller. It can therefore be recommended to add a constraint to specify the minimum size of the part of the coil where stable levitation is possible.

8. Conclusion

We show that a multistart, gradient-based optimization algorithm can be used to design levitation cell coils for different experimental applications.

We are learning how to develop a design tool for repeatable and reliable levitation cell experiments through experimentation with different formulations of the optimization problem. Future work will include replacing the one dimensional levitation cell model with a more accurate one and experimentally validating the optimized levitation cell designs.

9. References

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