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Surrogate-based Particle Swarm Optimization for Large-scale Wind Farm Layout Design

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1. Abstract
Wind farm layout optimization (WFLO) is the process of optimizing the location of turbines in a wind farm site, with the possible objective of maximizing the energy production or minimizing the average cost of energy. Conventional WFLO methods not only limit themselves to prescribing the site boundaries, they are also generally applicable to designing only small-to-medium scale wind farms (<100 turbines). Large-scale wind farms entail greater wake-induced turbine interactions, thereby increasing the computational complexity and expense by orders of magnitude. In this paper, we further advance the Unrestricted WFLO framework by designing the layout of large-scale wind farms with 500 turbines (where energy production is maximized). First, the high-dimensional layout optimization problem (involving 2N variables for a N turbine wind farm) is reduced to a 6-variable problem through a novel mapping strategy, which allows for both global siting (overall land configuration) and local exploration (turbine micrositing). Secondly, a surrogate model is used to substitute the expensive analytical WF energy production model; the high computational expense of the latter is attributed to the factorial increase in the number of calls to the wake model for evaluating every candidate wind farm layout that involves a large number of turbines. The powerful *Concurrent Surrogate Model Selection (COSMOS)* framework is applied to identify the best surrogate model to represent the wind farm energy production as a function of the reduced variable vector. To accomplish a reliable optimum solution, the surrogate-based optimization (SBO) is performed by implementing the Adaptive Model Refinement (AMR) technique within Particle Swarm Optimization (PSO). In AMR, both local exploitation and global exploration aspects are considered within a single optimization run of PSO, unlike other SBO methods that often either require multiple (potentially misleading) optimizations or are model-dependent. By using the AMR approach in conjunction with PSO and COSMOS, the computational cost of designing very large wind farms is reduced by a remarkable factor of 26, while preserving the reliability of this WFLO within 0.05% of the WFLO performed using the original energy production model.
2. Keywords: Large-scale wind farm; Layout optimization; Surrogate-based optimization; Concurrent surrogate model selection; Predictive estimation of model fidelity; Particle Swarm Optimization

3. Introduction
The wind energy harvested through large utility-scale wind farms can compete with conventional energy resources. A large utility-scale wind farm generally has a 500 MW installed capacity (can consist hundreds of wind turbines). Planning such a large scale wind farm is a complex process and extremely time-consuming. It includes various mutually correlated factors and large-scale effects, especially the energy losses due to the wake effects caused by a large number of turbines. The wind farm layout optimization (WFLO) for large-scale wind farms is desired, however, performing the WFLO of a large scale wind farm is computationally expensive. Surrogate-based optimization approaches can be applied to alleviate the computational burden in this complex design problem.

3.1 Surrogate-base Optimization
Surrogate models are purely mathematical models (i.e., not derived from the system physics) that are used to provide a tractable and inexpensive approximation of the actual system behavior. They are commonly used as an alternative to expensive computational simulations or to the lack of a physical model in the case of experiment-derived data. Major surrogate modeling methods include Polynomial Response Surfaces, Kriging, Moving Least Square, Radial Basis Functions (RBF), Support Vector Regression(SVR), and Neural Networks [1]. These methods have been applied to a wide range of disciplines, from aerospace design and automotive design to chemistry and material science [2].

The major pitfall in using surrogate models in optimization is that they can often mislead the search process, leading to suboptimal or infeasible solutions. To address this issue and provide optimum designs with high fidelity system evaluations, "*Surrogate-Based Optimization*" strategy can be applied. In these approaches, the accuracy and robustness of the surrogate model is improved by adding points where additional evaluations of the high fidelity model / experiment are desired to be performed. Infill points are generally added in (i) the region where the optimum is located (local exploitation); and/or (ii) the entire design space to improve the global accuracy of the surrogate (global exploration)[3, 4]. Infill points can be added in a fully sequential (one-at-a-time), or can be added in a batch sequential manner. There exist various criteria for determining the locations of the infill points including (i) Index-based criteria (e.g., Mean Square Error and Maximum Entropy criteria) and (ii) Distance-based criteria (e.g., Euclidean distance, Mahalonobis distance, and Weighted distance criteria) [5].

Over the last two decades, different Surrogate-Based Optimization strategies have been developed [6, 7]. Trosset and Torczon in 1997 [8] proposed an approach where the balance between exploitation and exploration was considered using the aggregate *merit function*, $\hat{f}(x) - \rho d_{min}(x)$, where, $d_{min}(x) = \underset{x}{\mathrm{Min}}\ \|x - x^i\|,\ \rho > 0$. It is important to note that, this technique is independent of the type of surrogate modeling technique being considered. Jones et al. in 1998 [6] developed a well-known model management strategy that is based on an *Expected Improvement (EI)* criterion, and is called *Efficient Global Optimization (EGO)*. This powerful approach is however generally limited to surrogate models based on Gaussian processes. The surrogate-based optimization presented in this paper has the following characteristics: (i) It is independent of the type of model, (ii) It uses a reliable surrogate model (with a desired level of fidelity) at any given iteration of SBO, and (iii) It determines the optimal batch size for the infill points for the upcoming iteration of SBO.

3.2 Layout Optimization of Large Scale Wind Farms

The current research on solving the large layout optimization problem is mostly limited to quantify the layout using the streamwise and the spanwise spacings between turbines, and assumes that a specified number of turbines are uniformly distributed in pre-defined boundaries. As a result, the optimization is incompletely performed due to the prescribed conditions. Fuglsang et al. [9] defined the wind farm layout as a function of the spacings between rows and columns for a specified number of turbines. Perez et al. [10] used the numbers of rows and columns, the streamwise spacing and the spanwise spacing between neighboring turbines, the turbine rotor diameter, and a specified rectangular boundary to determine the wind farm layout. Wagner et al. [11] developed a framework that can solve the WFLO for up to 1000 turbines. In this framework, the initial location of turbines is restricted to an array-like layout. However, a radial displacement around each turbine is allowed. The approach presented in this paper is capable of optimizing the location of turbines for large wind farms, i.e., 500-turbine scale wind farms, without prescribing the farm boundaries.

4. Mapping of the Layout for a Large Scale Wind Farm

In this section, we present a mapping approach to quantify the layout of a very large wind farm using six mapping factors: (i) the maximum allowable *streamwise* and *spanwise* spacings between neighboring turbines (s_{max} and r_{max}), (ii) the average spacings of rows and columns (d_r and d_c), (iii) the normalized local radial displacement, (iv) the turbine rotor diameter (D), (v) Number of turbines (N_{turb}), (vi) the farm site orientation (ϕ), and (v) the maximum number of rows and/or the maximum side length of the wind farm. The maximum allowable *streamwise* and *spanwise* spacings between neighboring turbines are determined based on the size of turbine rotor diameter. the average spacings of rows and columns are variable for difference rows and columns, and given by

$$
\begin{aligned}
d_r &= (r_{max} - r_{min}) \times \left\{ A\pi \frac{j}{N_{row}} - \lfloor \frac{1}{2} + (A\pi \frac{j}{N_{row}}) \rfloor + 1 \right\} + r_{min} \\
d_c &= (s_{max} - s_{min}) \times \left\{ B\pi \frac{i}{N_{column}} - \lfloor \frac{1}{2} + (B\pi \frac{i}{N_{column}}) \rfloor + 1 \right\} + s_{min}
\end{aligned} \tag{1}
$$

where s_{min} and r_{min} are the minimum *streamwise* and *spanwise*, respectively; A and B are design factors in mapping function; i and j are respectively the row and column indexes; N_{row} and N_{column} are number of rows and column, respectively. By this definition, non-uniform spacings between rows and columns are allowed.

In this paper, the normalized local radial displacement is expressed as the Gaussian distribution with zero mean ($\mu = 0$) and the variable standard deviation (σ) [11]. The actual radial displacement is obtained by multiplying the minimum *streamwise/spanwise* between turbines. This allows the turbines not to be restricted to be in the grid form. The side length of the wind farm can provided based on the average land usage of US commercial wind farms. Therefore, the coordinates of the turbine at the I^{th} row and the J^{th} column are given by

$$\begin{aligned} X_I &= \sum_{j=1}^{J} \left[(r_{max} - r_{min}) \times \{A\pi \frac{j}{N_{row}} - \lfloor \frac{1}{2} + (A\pi \frac{j}{N_{row}}) \rfloor + 1\} + r_{min} \right] + r_{min}\hat{r}(0,\sigma) \\ Y_J &= \sum_{i=1}^{I} \left[(s_{max} - s_{min}) \times \{B\pi \frac{i}{N_{column}} - \lfloor \frac{1}{2} + (B\pi \frac{i}{N_{column}}) \rfloor + 1\} + s_{min} \right] + s_{min}\hat{r}(0,\sigma) \end{aligned} \tag{2}$$

5. Surrogate model selection using COSMOS

In Section 4, the dimensionality reduction is performed. In this section, the *Concurrent Surrogate Model Selection (COSMOS)* framework (developed by Chowdhury et al. [12]) is applied to select the best surrogate model to represent the *average annual energy production of a large-scale wind farm* as a function of the mapping factors and the farm site orientation (ϕ), illustrated in Table 2. In this paper .This automated model selection is based on the error measure given by the Predictive Estimation of Model Fidelity (PEMF) [13] and Mixed Integer Nonlinear Programing (MINLP). Unlike most other model selection methods, the COSMOS framework coherently operates at all the three levels necessary to facilitate optimal selection, i.e., (i) selecting the model type (e.g., RBF or SVR), (ii) selecting the kernel function type (e.g., cubic or multiquadric kernel in RBF), and (iii) determining the optimal values of the typically user-prescribed parameters (e.g., shape parameter in RBF) – thereby allowing selection of globally competitive models. COSMOS offers five different criteria for selection of optimal surrogates. In the current implementation, three of them are selected including: (i) the median error (E^{mo}_{med}), (ii) the maximum error (E^{mo}_{max}), and (iii) the variance of the median error ($E^{\sigma^2}_{med}$).

Next, the proposed approach is implemented in a 500-turbine wind farm. In this problem, the number of sample points for training the surrogate models considered in the COSMOS framework is $N(\{X\}) = 200$ and the average annual energy production in these sample points is estimated using the model incorporated in the Unrestricted Wind Farm Layout Optimization framework [14]. The surrogate models identified using COSMOS for the average annual energy production of a large-scale wind farm under the mentions threes criteria, are listed in Table 1. In this problem, the selected surrogate model will be

Table 1: The set of surrogate models given by COSMOS for the wind farm energy production problem

COSMOS TEST	Pareto Surrogate Model Selected
Test I: $min[E^{mo}_{med},\ E^{mo}_{max}]$	Kriging with Linear correlation , SVR with Sigmoid kernel
Test II: $min[E^{mo}_{med},\ E^{\sigma^2}_{med}]$	Kriging with Linear & Exponential correlation , SVR with Sigmoid kernel

used as an initial model in the surrogate-based optimization process. In the surrogate-based optimization process, the fidelity of surrogate model iteratively should be improved by adding infill points in the optimization process. Therefore, Kriging model with Linear correlation function that is selected in Test I and Test II could be the best choice. The dimensionality reduction and the implementation of the surrogate model reduce the computational cost on a very large wind farm layout optimization more than 95%.

4. Adaptive Model Refinement (AMR)

The major challenge of using surrogate models in optimization is that they can often mislead the search process due to underestimation or overestimation of system behavior and leading to suboptimal or infeasible solutions. To address this issue, surrogate-based optimization with adaptive model refinement (AMR) [15] is applied in this paper. In AMR, reconstruction (or refinement) of the model is performed by sequentially adding a batch of new samples at any given iteration (of surrogate-based optimization (SBO)) when a switching metric is met. The major components of the surrogate-based optimization using AMR method include:

(i) The model switch metric [16]: This metric is perceived as a decision-making tool for the timing of surrogate model refinement. Performing model refinement (by adding infill points) too early in the optimzaiton process can be computationally expensive while wasting resources to explore undesirable regions of the design domain. On the other hand, updating surrogate model too late might mislead the search process early on to suboptimal regions of the design domain, i.e., leading to scenarios where the global optimum is outside of the region spanned by the population of candidate solutions in later iterations.This metric is formulated by comparing (1) the uncertainty associated with the outputs of the current model, and (2) the distribution of the latest fitness function improvement over the population of candidate designs. Whenever the model switching metric is met, the history of the fitness function improvement is used to determine the desired fidelity for the upcoming iterations of SBO.

(ii) The Predictive Estimation of Model Fidelity [13]: In AMR, the PEMF method with certain implementation is used to quantify the surrogate model fidelity, and identify the optimal batch size (e.g., size of infill points). The inputs and outputs of PEMF (in the AMR method) are expressed as:

$$[\mathbb{P}_{(\mu_\varepsilon,\sigma_\varepsilon)},\ \Gamma^{Infill}] = f_{PEMF}(X,\ \varepsilon^*) \tag{3}$$

where the vector X represents the sample data (input and output) used for training the surrogate model; and ε^* is the desired fidelity in the model refinement process. In Eq. 3, $\mathbb{P}_{(\mu_\varepsilon,\sigma_\varepsilon)}$, and Γ^{Infill} respectively represent the distribution of the error in the surrogate model, and the batch size for the infill points to achieve a desired level of fidelity in the AMR method.

(iii) Optimization algorithm: Mixed-Discrete PSO (MDPSO): In the presented surrogate-based optimization methodology, optimization is performed using an advanced implementation of the Particle Swarm Optimization (PSO). In this paper, we use one particular advanced implementation of the PSO algorithm called Mixed-Discrete PSO (MDPSO), which was developed by Chowdhury et al [17].

6. Numerical Experiments

In this section the proposed approach is implemented to design a 500-turbine wind farm for measured site data, in order to illustrate the effectiveness of this unique wind farm layout optimization approach. The objective is to maximize the average annual energy production of the farm through layout optimization ($P_{farm} = \text{annual energy production}/365 \times 24$). In this problem, the farm layout optimization is started using the best surrogate model selected in Sec. 4 (Kriging with Linear correlation function). To reach a reliable optimum solution, a refinement of the surrogate is then performed by sequentially adding batches of new samples using the adaptive model refinement approach. The optimization problem is defined as

$$\begin{aligned} \max_{V}: \quad & f = P_{farm} \\ & V = \{v_1, v_2, \ldots, v_6\} \\ \textit{subject to} \quad & \\ & g_1(v) \leq 0 \\ & g_2(v) \leq 0 \\ & V^{Lower} \leq v_i \leq V^{Upper} \end{aligned}$$

The upper and lower bounds of design variables ($v_i = 1, 2, ..., 6$) are listed in Table 2. These variables include the mapping factors (that defines the coordinates of the wind turbines in the farm), and the farm site orientation (ϕ). In Eq. 4, $g_1(v) = \|A_{MW} - 45(hectare/MW)\|$ is the land area constraints. This constraint is defined based on the average land usage of US commercial wind farms in 2009. In this equation, $g_2(v) = \sum_{i,j=1,i\neq j}^{N=500} max\{[4D - d_{ij}]\}$ is the minimum inter-turbine spacing constraints. In this paper, the GE-1.5MW-XLE turbine is chosen as the specified turbine-type [18]; the minimum *streamwise* (s_{min}) and *spanwise* (r_{min}) are set to the same value: $4D$; and the wind data is obtained from the North Dakota Agricultural Weather Network (NDAWN)[19].

Table 2: Upper and lower bounds of design variables

Design variables	Lower bound	Upper bound
r_{max}	5D	15D
s_{max}	5D	15D
A	-20	20
B	-20	20
σ	0	1
ϕ	0	90

7 Results and Discussion

In this optimzation problem, the AMR is applied at every iteration of PSO. The population size of PSO is pre-specified to $N_{pop} = 300$, and the PSO algorithm converges by satisfying the predefined function tolerance, $\delta f = 1e-6$. In this problem the model refinement will be implemented if the current size of data set is less than $N(\{X\}) = 500$ In this optimization problem, the initial population of particles is generated using the Kriging with Linear correlation function. The AMR approach adaptively improve the fidelity of the surrogate model fifteen times during the optimization process (over a total of 110 iterations), resulting in an optimum design with a reasonable level of fidelity.

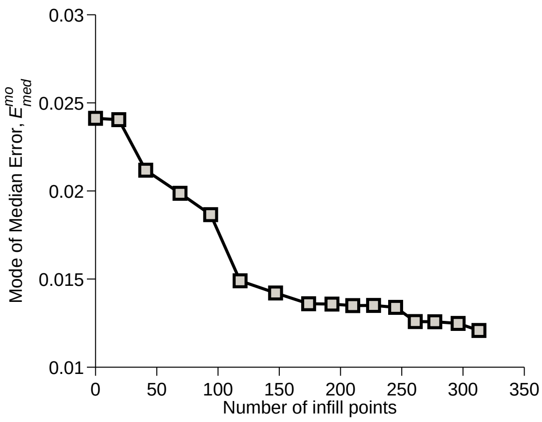

Figure 1: The improvement of the surrogate model fidelity through sequential model refinement process with AMR

Figure 1 represents the improvement of the model fidelity for the wind farm energy production problem through the sequential model refinement process using the AMR method. This figure illustrates that the error (estimated using PEMF) decreases with sequential addition of batches of infill points in SBO, thus showing that the batch size of the infill points and the location of these points determined by the AMR method resulted in lower error in the refined (updated) surrogate model. It should also be noted that the novel use of the modal value in the PEMF method, promotes a decrease of the error measure with infill points, as opposed to the untraceable noisy variation that is often characteristic of mean or RMS error measures [13].

The convergence history of the optimization problem is illustrated in Fig. 2. These figures also indicate: (i) the active surrogate model at each iteration (using different colors), (ii) the size of data set $(N(\{X\}) = N^{Current}(\{X\}) + \Gamma^{Infill})$ used to refine (update) the active surrogate model in the AMR approach, and (iii) the iteration that the model refinement is performed (the AMR metric is met). It is observed that, from the first iteration till the 8^{th} iteration the initial surrogate model with $N(\{X\}) = 200$ is active, before refining the model using additional 19 sample data through the AMR approach. The final switching event, from the surrogate model constructed using $N(\{X\}) = 496$ training points to the surrogate model constructed using $N(\{X\}) = 513$, occurs at the 96^{th} iteration. The optimization progresses using the surrogate model constructed using $N(\{X\}) = 513$ training points for another 14 iterations before reaching convergence. In this case, the algorithm converges by satisfying the predefined function tolerance, $\delta f = 1e-6$.

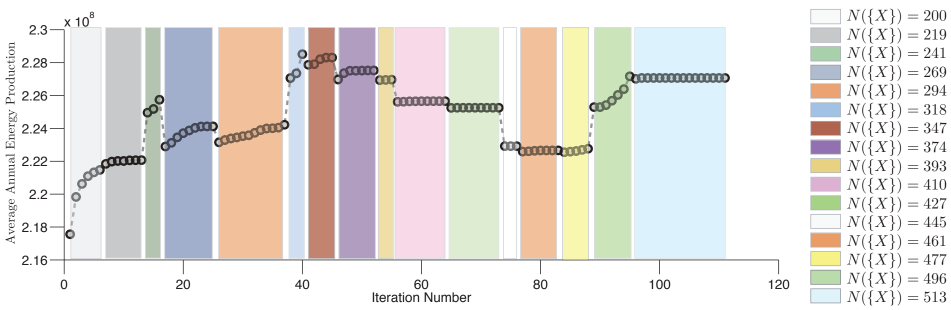

Figure 2: Convergence history of the surrogate-based optimization using AMR

Next, we investigate how the AMR method performs better than simply using the single stage sampling strategy for SBO. The result yielded by the AMR method are therefore compared with the results yielded by running MDPSO using the surrogate model constructed in the one-step method (using all 500 sample points). The optimum results are thus obtained, and the total number of function evaluations in each case are reported in Table 3. The final column of this table shows the actual function estimate for the optimum design obtained under each optimization run. It is observed that the PSO-AMR not only requires 26 times less computing time compared to PSO-HF, it also provides the optimum value that is 14.9 MWh better that simply using the single stage sampling strategy for SBO. It is also observed that, in the PSO-AMR approach, the optimum is located in the region where the SM model has 0.63% error.

Table 3: Optimization results using different optimization approaches

Approach	v_1^* $\times D$	v_2^* $\times D$	v_3^*	v_4^*	v_5^*	v_6^* (degree)	Optimum function P_{farm} ($f^* \times 10^8$)	Model in last iteration	No. of HF function evaluations	HF response at optimum ($f_{HF}(v^*) \times 10^8$)	A_{MW}
PSO-SM	14.03	13.40	15.68	4.72	0.05	70.73	2.3013	SM	500	2.1248	29.7085
PSO-HF	13.70	13.54	16.27	4.92	0.08	69.69	2.2760	HF	300×45	2.2760	43.80
PSO-AMR	12.65	12.57	9.99	-2.53	0.03	72.17	2.2892	SM	513	2.2748	44.74

PSO-SM: optimization performed by MDPSO solely using the surrogate model with all 500 sample points
PSO-HF: optimization performed by MDPSO solely using the high fidelity model
PSO-AMR: optimization performed by MDPSO using AMR approach (surrogate-based optimization using AMR)

8 Conclusion

This paper presented a new approach for layout optimization of very large scale wind farms. A mapping of the wind farm layout to the detailed turbine coordinates was created, allowing the wind farm layout to be represented by a set of 6 parameters. As a result, the design variable space for optimizing the layout of a large scale wind farm is significantly reduced. The COSMOS framework was then applied to select the globally-best surrogate model to represent the energy production of the wind farm as a function of the reduced set of layout variables. Surrogate-based optimization was then preformed using the Adaptive Model Refinement approach, implemented through Particle Swarm Optimization. The objective of wind farm layout optimization was to maximize the average annual energy production of a 500-turbine wind farm. The results indicated that AMR along with Mixed Discrete PSO improved the efficiency of the optimization process by a factor of 26 when compared to optimization using the standard energy production model, while retaining an accuracy of within 0.05% of the latter.

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