Amme 3500: System Dynamics and Control

Block Diagrams

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# Course Outline

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Reminders/Announcements

- Labs start after Easter break
- Tutorials this week: lab group sign-up
- Assignment 2 released today
As we saw in the introductory lecture, a subsystem can be represented with an input, an output and a transfer function:

\[ \frac{Y(s)}{U(s)} = H(s) \]

Input: control surfaces (flap, aileron), wind gust
Aircraft output: pitch, yaw, roll

Block Diagrams

- Many systems are composed of multiple subsystems
- In this lecture we will examine methods for combining subsystems and simplifying block diagrams
Examples of subsystems

- Automobile control:
Examples of Subsystems

- Antenna control

In the time domain, the input-output relationship is usually expressed in terms of a differential equation

\[
\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_0 u(t)
\]

- \((a_{n-1}, \ldots, a_0, b_m, \ldots b_0)\) are the system’s parameters, \(n \geq m\)
- The system is LTI (Linear Time Invariant) iff the parameters are time-invariant
- \(n\) is the order of the system
In the Laplace domain, the input-output relationship is usually expressed in terms of an algebraic equation in terms of $s$

$$s^n Y(s) + a_{n-1}s^{n-1} Y(s) + \cdots + a_0 Y(s) = b_m s^m U(s) + \cdots + b_0 U(s)$$
Cascaded systems

- In time, a cascaded system requires a convolution
  \[ u(t) * h(t) \equiv \int_{-\infty}^{\infty} u(\tau) h(t - \tau) \, d\tau \]

- In the Laplace domain, this is simply a product
  \[ Y(s) = U(s)H(s)H_1(s) \]
\[ y(t) = h(t) \ast u(t) \]

\[ = \int_0^t h(t)u(t - \tau)\,d\tau \]

\[ Y(s) = \int_0^\infty e^{-st}y(t)\,dt \]

\[ Y(s) = \int_0^\infty e^{-st} \int_0^t h(\tau)u(t - \tau)\,d\tau\,dt \]

\[ = \int_0^\infty \int_0^t e^{-s\tau}e^{-s(t-\tau)}h(\tau)u(t - \tau)\,d\tau\,dt \]

\[ = \int_0^\infty \int_0^\infty e^{-s\tau}e^{-s(t-\tau)}h(\tau)u(t - \tau)\,dt\,d\tau \]

\[ v = t - \tau, \quad t = v + \tau, \quad \frac{\partial v}{\partial t} = 1 \]

\[ = \int_0^\infty \int_0^\infty e^{-s\tau}e^{-sv}h(\tau)u(v)\,dv\,d\tau \]

\[ = \int_0^\infty h(\tau)e^{-s\tau}d\tau \int_0^\infty e^{-sv}u(v)\,dv \quad = H(s)U(s) \]
“Frequency Response”

• Why is it called the “Frequency Domain”?
• Take the Laplace transform:
  \[ U(s) = \int_0^\infty u(t)e^{-st} \, dt \]
• Now suppose \( s = j\omega \), then
  \[ e^{j\omega t} = \cos(\omega t) + j \sin(\omega t) \]
• So
  \[ U(j\omega) = \int_0^\infty u(t)e^{-j\omega t} \, dt \]
• i.e. the *Fourier transform* (Frequency content)
So the Transfer Function of a system is:

\[ G(s) = \frac{Y(s)}{U(s)} \]

When looking at the imaginary axis:

\[ G(j\omega) = \frac{Y(j\omega)}{U(j\omega)} \]

Is simply the ratio of particular frequencies in input and output.

It is a complex number, with a magnitude and a phase (much more on this later).
Many control systems can be characterised by these components/subsystems.
Simplifying Block Diagrams

• The objectives of a block diagram are
  – To provide an understanding of the signal flows in the system
  – To enable modelling of complex systems from simple building blocks
  – To allow us to generate the overall system transfer function

• A few rules allow us to simplify complex block diagrams into familiar forms
Components of a Block Diagram

- A block diagram is made up of signals, systems, summing junctions and pickoff points

Typical Block Diagram Elements

- Cascaded Systems

\[ R(s) \xrightarrow{G_1(s)} X_2(s) = G_1(s)R(s) \xrightarrow{G_2(s)} X_1(s) = G_2(s)G_1(s)R(s) \xrightarrow{G_3(s)} C(s) = G_3(s)G_2(s)G_1(s)R(s) \]

\[ R(s) \xrightarrow{G_3(s)G_2(s)G_1(s)} C(s) \]

Typical Block Diagram Elements

- Parallel Systems

\[ \begin{align*}
X_1(s) &= R(s)G_1(s) \\
X_2(s) &= R(s)G_2(s) \\
X_3(s) &= R(s)G_3(s)
\end{align*} \]

\[ C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s) \]

\[ (a) \]

\[ \begin{align*}
\pm G_1(s) \pm G_2(s) \pm G_3(s)
\end{align*} \]

\[ C(s) \]

\[ (b) \]

Typical Block Diagram Elements

• Feedback Form

\[ E(s) = R(s) - H(s)C(s) \]

\[ C(s) = G(s)E(s) \]

\[ C(s) = G(s)[R(s) - H(s)C(s)] = G(s)R(s) - G(s)H(s)C(s) \]

\[ C(s)[1 + G(s)H(s)] = G(s)R(s) \]

\[ \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]
Moving Past A Summation

a) To the left past a summing junction

b) To the right past a summing junction

Moving Past Pick-Off Points

a) To the left past a summing junction

b) To the right past a summing junction

Example 1: Simplifying Block Diagrams

- Consider this block diagram
- We wish to find the transfer function between the input $R(s)$ and $C(s)$

Example I: Simplifying Block Diagrams

a) Collapse the summing junctions
b) Form equivalent cascaded system in the forward path and equivalent parallel system in the feedback path
c) Form equivalent feedback system and multiply by cascaded $G_1(s)$

Example II : Simplifying Block Diagrams

- Form the equivalent system in the forward path
- Move 1/s to the left of the pickoff point
- Combine the parallel feedback paths
- Compute $C(s)/R(s)$ using feedback formula

Example III: Shuttle Pitch Control

- Manipulating block diagrams is important but how do they relate to the real world?
- Here is an example of a real system that incorporates feedback to control the pitch of the vehicle (amongst other things)
Example III: Shuttle Pitch Control

- The control mechanisms available include the body flap, elevons and engines
- Measurements are made by the vehicle’s inertial unit, gyros and accelerometers

Example III: Shuttle Pitch Control

- A simplified model of a pitch controller is shown for the space shuttle.
- Assuming other inputs are zero, can we find the transfer function from commanded to actual pitch?

• Combine $G_1$ and $G_2$.
• Push $K_1$ to the right past the summing junction
Example III: Shuttle Pitch Control

- Push $K_1K_2$ to the right past the summing junction
- Hence

$$T(s) = \frac{K_1K_2G_1(s)G_2(s)}{1 + K_1K_2G_1(s)G_2(s)\left(1 + \frac{s}{K_1} + \frac{s^2}{K_1K_2}\right)}$$
Closed Loop Feedback

• We have suggested that many control systems take on a familiar feedback form
• Intuition tells us that feedback is useful – imagine driving your car with your eyes closed
• Let us examine why this is the case from a mathematical perspective
Single-Loop Feedback System

- Error Signal \[ e(t) = d(t) - f(t) = d(t) - Ky(t) \]
- The goal of the Controller \( K(s) \) is:
  - To produce a control signal \( u(t) \) that drives the ‘error’ \( e(t) \) to zero
Controller Objectives

• Controller cannot drive error to zero instantaneously as the plant G(s) has dynamics
• Clearly a ‘large’ control signal will move the plant more quickly
• The gain of the controller should be large so that even small values of e(t) will produce large values of u(t)
• However, large values of gain will cause instability
Feedforward & Feedback

- Driving a Formula-1 car
- Feedforward control: pre-emptive/predictable action: prediction of car’s direction of travel (assuming the model is accurate enough)
- Feedback control: corrective action since the model is not perfect + noise
Why Use Feedback?

- Perfect feed-forward controller: $G_n$

\[
d(t) \rightarrow \frac{1}{G_n} \rightarrow + \rightarrow l(t) \rightarrow \rightarrow G \rightarrow y(t)
\]

- Output and error:
\[
y = \frac{G}{G_n} d + Gl
\]
\[
e = d - y = \left(1 - \frac{G}{G_n}\right)d - Gl
\]

- Only zero when:
\[
G_n = G \quad \text{(Perfect Knowledge)}
\]
\[
l = 0 \quad \text{(No load or disturbance)}
\]
Why Use Feedback?

\[ e = d - y, \quad y = G(u + l), \quad u = Ke \]

So:

\[ e = d - y = d - G(u + l) = d - G(Ke + l) \]
Closed Loop Equations

\[ e = d - GKe - Gl \]

Collect terms: \[ (1 + KG)e = d - Gl \]

Or:

\[ e = \frac{1}{1 + KG} d - \frac{G}{1 + KG} l \]

Demand to Error Transfer Function
Load to Error Transfer Function
Closed Loop Equations

Similarly

\[ y = \frac{GK}{1 + KG} d - \frac{G}{1 + KG} l \]

Demand to Output Transfer Function
Load to Output Transfer Function

Equivalent Open Loop Block Diagram
Rejecting Loads and Disturbances

\[ y = \frac{G}{1 + KG} l \]

Load to Output Transfer Function

If \( K \) is big: \( KG \gg 1 \)

\[ y \approx \frac{G}{KG} l = \frac{1}{K} l \approx 0 \]

- Perfect Disturbance Rejection
- Independent of knowing \( G \)
- Regardless of \( l \)
Tracking Performance

\[ y = \frac{GK}{1 + KG} d \]

Demand to Output Transfer Function

If \( K \) is big: \( KG \gg 1 \)

\[ y \approx \frac{GK}{KG} d = d \]

Perfect Tracking of Demand
Independent of knowing \( G \)
Regardless of \( l \)
Two Key Problems

Power: \( u = K(d - y) \)  
Large \( K \) requires large actuator power \( u \)
Two Key Problems

Noise:

\[ u = K(d - y + n) \]

Large \( K \) amplifies sensor noise

In practise a Compromise \( K \) is required
Conclusions

- We have examined methods for computing the transfer function by reducing block diagrams to simple form
- We have also presented arguments for using feedback in control systems
- Next week, we will look at more closely on feedback control
Further Reading

• Nise
  – Sections 5.1-5.3

• Franklin & Powell
  – Section 3.2