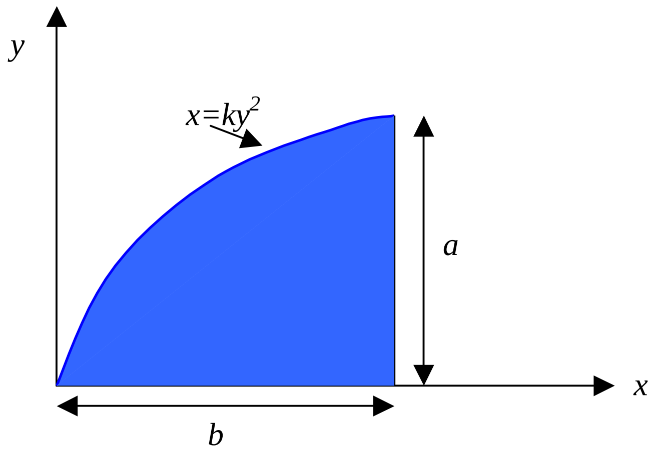
AREA MOMENT OF INERTIA

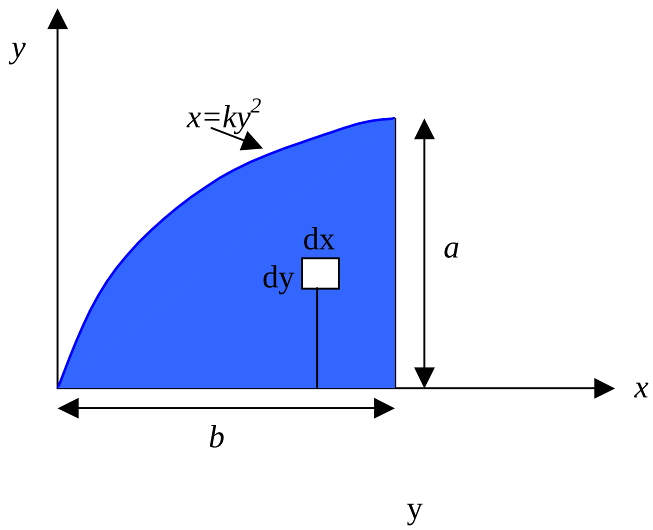
Problem 3:

Determine the moment of inertia of the shaded area about the x-axis:



Solution:

First way: Double integration (choosing to integrate with respect to x first):



$$dA = dxdy$$

$$I_x = \int_A y^2 dA$$

$$I_x = \int_0^a \int_{ky^2}^h y^2 dxdy = \int_0^a y^2\left(b - ky^2\right)dy = \frac{2}{15}a^3 b$$

Since when $y = 0,\ k = 0$ and when $y = a,\ k = b/a^2$

$$I_x = \frac{2}{15}a^3 b$$

Second way: Double integration (choosing to integrate with respect to y first):

$$dA = dxdy$$

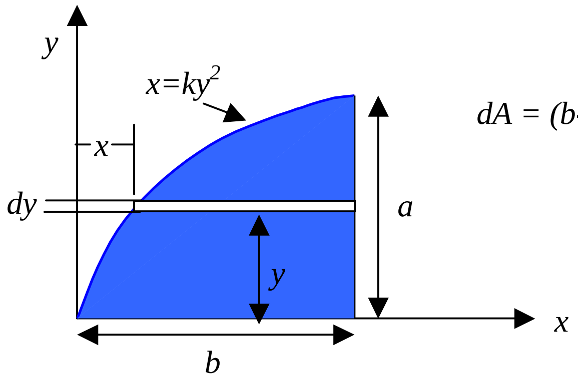
$$I_x = \int_A y^2 dA$$

$$I_x = \int_0^b \int_0^{\sqrt{x/k}} y^2 dydx = \int_0^b \frac{1}{3k^{3/2}} = \frac{2}{15}a^3 b$$

$$I_x = \frac{2}{15}a^3 b$$

Third way (Single integration – Finite length strip):

Case 1: Horizontal strip parallel to the x-axis:

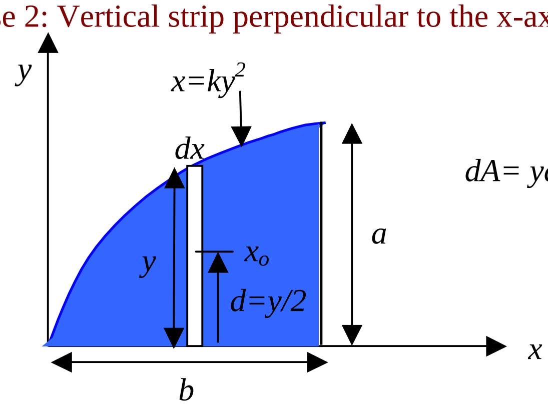


$dA = (b-ky^2)dy$

All parts of the differentia area element are the same distance from the x-axis

$$I_x = \int y^2 dA = \int_0^a y^2 \left(b - ky^2\right)dy = \frac{2}{15}a^3 b$$

Case 2: Vertical strip perpendicular to the x-axis:



$dA = ydx = (x^{1/2}/k^{1/2})dx$

Since all parts of the element area are not at the same distance from the x-axis, we find the moment of inertia by considering the differential area about the x-axis:

$$d\left(I_x\right) = dI_{xo} + dA\left(d\right)^2 = \frac{1}{12}\left(dx\right)y^3 + y\left(dx\right)\left(\frac{y}{2}\right)^2 = \frac{1}{3}\left(dx\right)y^3$$

$$I_x = \int d\left(I_x\right) = \int \frac{1}{3}y^3 dx = \frac{1}{3}\int_0^b \frac{x^{3/2}}{k^{3/2}}dx = \frac{2}{15}a^3 b$$