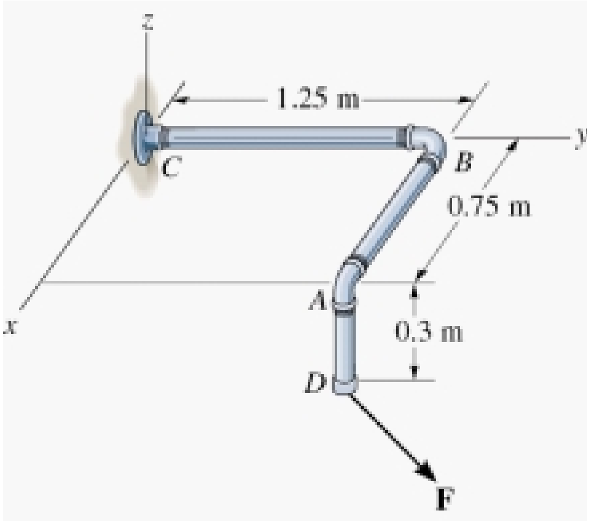
Moment 2

Problem 1:

Determine the moment created by the force $\boldsymbol{F} = \{50\boldsymbol{i} + 100\boldsymbol{j} - 50\boldsymbol{k}\}$ N acting at D, about each of the joints at B and C.



Solution:

B(0,1.25,0); C(0,0,0); D(0.75,1.25,-0.3)

$$\mathbf{F} = \{50\,\mathbf{i} + 100\,\mathbf{j} - 50\,\mathbf{k}\}\text{N}$$

Position vector:

$$\mathbf{r_{BD}} = \{(0.75-0)\,\mathbf{i} + (1.25-1.25)\,\mathbf{j} + (-0.3-0)\,\mathbf{k}\}\text{m}$$

$$\Rightarrow \mathbf{r_{BD}} = \{0.75\,\mathbf{i} + -0.3\,\mathbf{k}\}\text{m}$$

$$\mathbf{r_{CD}} = \{(0.75-0)\,\mathbf{i} + (1.25-0)\,\mathbf{j} + (-0.3-0)\,\mathbf{k}\}\text{m}$$

$$\Rightarrow \mathbf{r_{CD}} = \{0.75\,\mathbf{i} + 1.25\,\mathbf{j} - 0.3\,\mathbf{k}\}\text{m}$$

$$\mathbf{F} = \{50\,\mathbf{i} + 100\,\mathbf{j} - 50\,\mathbf{k}\}\text{N}$$
$$\mathbf{r_{BD}} = \{0.75\,\mathbf{i} + -0.3\,\mathbf{k}\}\text{m}$$
$$\mathbf{r_{CD}} = \{0.75\,\mathbf{i} + 1.25\,\mathbf{j} - 0.3\,\mathbf{k}\}\text{m}$$

Moment of force $\mathbf{F}$ about point B is:

$$\mathbf{M_B} = \mathbf{r_{BD}} \times \mathbf{F}$$
$$\Rightarrow \mathbf{M_B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 0 & -0.3 \\ 50 & 100 & -50 \end{vmatrix}$$
$$\Rightarrow \mathbf{M_B} = \begin{Bmatrix} [(0)(-50) - (-0.3)(100)]\mathbf{i} - [(0.75)(-50) - (-0.3)(50)]\mathbf{j} \\ + [(0.75)(100) - (0)(50)]\mathbf{k} \end{Bmatrix}\text{N.m}$$
$$\Rightarrow \mathbf{M_B} = \{30\,\mathbf{i} + 22.5\,\mathbf{j} + 75\,\mathbf{k}\}\text{N.m}$$

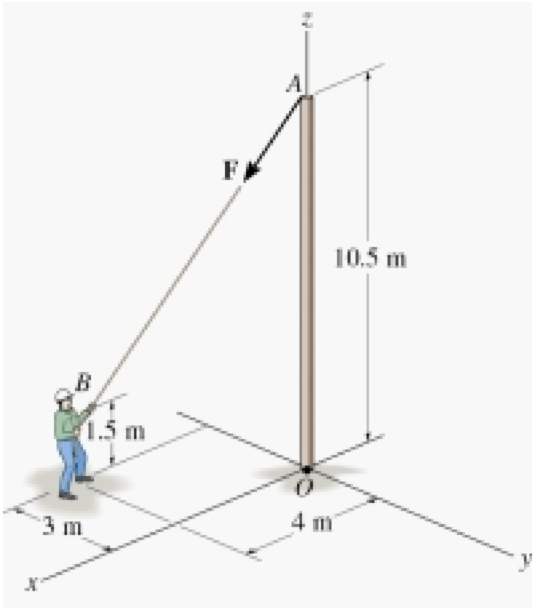
$$\mathbf{F} = \{50\,\mathbf{i} + 100\,\mathbf{j} - 50\,\mathbf{k}\}\text{N}$$
$$\mathbf{r_{BD}} = \{0.75\,\mathbf{i} + -0.3\,\mathbf{k}\}\text{m}$$
$$\mathbf{r_{CD}} = \{0.75\,\mathbf{i} + 1.25\,\mathbf{j} - 0.3\,\mathbf{k}\}\text{m}$$

Moment of force $\mathbf{F}$ about point C is:

$$\mathbf{M_C} = \mathbf{r_{CD}} \times \mathbf{F}$$
$$\Rightarrow \mathbf{M_C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.25 & -0.3 \\ 50 & 100 & -50 \end{vmatrix}$$
$$\Rightarrow \mathbf{M_C} = \begin{Bmatrix} [(1.25)(-50) - (-0.3)(100)]\mathbf{i} - [(0.75)(-50) - (-0.3)(50)]\mathbf{j} \\ + [(0.75)(100) - (1.25)(50)]\mathbf{k} \end{Bmatrix}$$
$$\Rightarrow \mathbf{M_C} = \{-32.5\,\mathbf{i} + 22.5\,\mathbf{j} + 12.5\,\mathbf{k}\}\text{N.m}$$

Problem 2:

Determine Smallest force F that must be applied to the rope, when held in the direction shown, in order to cause the pole to break at its base O. This requires a moment of M = 900 N.m to be developed at O.



Solution:

A(0,0,10.5); B(4,-3,1.5)

$M = 900$ N.m

Position vector:

$$\mathbf{r_{AB}} = \{4\,\mathbf{i} - 3\,\mathbf{j} - 9\,\mathbf{k}\}\text{m}$$

$$r_{AB} = \sqrt{4^2 + (-3)^2 + (-9)^2} = 10.30\text{ m}$$

$$\mathbf{F} = F\frac{\mathbf{r_{AB}}}{r_{AB}} = F\frac{\{4\,\mathbf{i} - 3\,\mathbf{j} - 9\,\mathbf{k}\}\text{m}}{10.30\text{ m}}$$

$$\Rightarrow \mathbf{F} = F\{0.39\,\mathbf{i} - 0.29\,\mathbf{j} - 0.87\,\mathbf{k}\}\text{m}$$

A(0,0,10.5); B(4,-3,1.5)

$M = 900$ N.m

$$\mathbf{F} = F\{0.39\,\mathbf{i} - 0.29\,\mathbf{j} - 0.87\,\mathbf{k}\}\text{m}$$

$$\mathbf{M_O} = \mathbf{r_{OA}} \times \mathbf{F}$$

$$\Rightarrow \mathbf{M_O} = F\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10.5 \\ 0.39 & -0.29 & -0.87 \end{vmatrix}$$

$$\Rightarrow \mathbf{M_O} = F\begin{Bmatrix} [(0)(-0.87) - (10.5)(-0.29)]\,\mathbf{i} \\ -[(0)(-0.87) - (10.5)(0.39)]\,\mathbf{j} \\ +[(0)(-0.29) - (0)(0.39)]\,\mathbf{k} \end{Bmatrix}\text{N.m}$$

$$\Rightarrow \mathbf{M_O} = F\{3.045\,\mathbf{i} + 4.095\,\mathbf{j}\}\text{N.m}$$

$\mathrm{M} = 900\ \mathrm{N.m}$

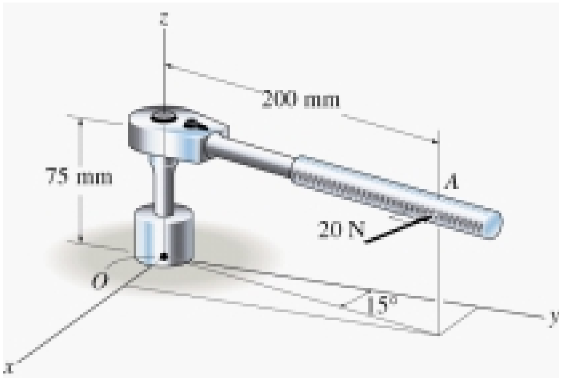
$\mathbf{M_O} = F\{3.045\,\mathbf{i} + 4.095\,\mathbf{j}\}\mathrm{N.m}$

$$\Rightarrow F = \frac{\mathbf{M_O}}{\sqrt{(3.045)^2 + (4.095)^2}} = \frac{900}{5.10}$$

$\Rightarrow F = 176.5\ \mathrm{N}$

Problem 3:

A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and direction of the moment created by this force about point O.



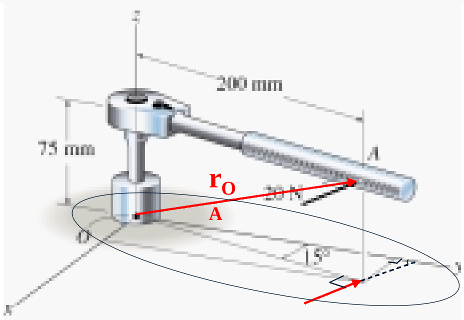
Solution:

$\mathbf{r_{OA}} = \{0.2\,\mathbf{sin}\,15°\,\mathbf{i} + 0.2\,\mathbf{cos}\,15°\,\mathbf{j} + 0.075\,\mathbf{k}\}\mathrm{m}$

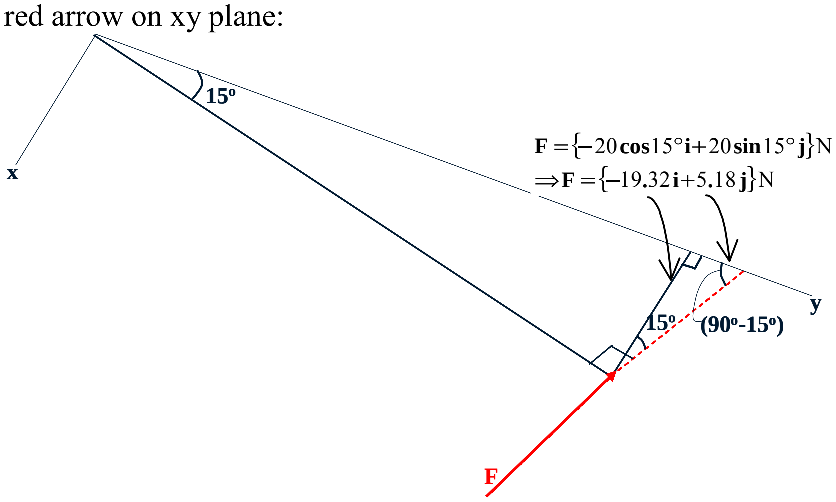
$\Rightarrow \mathbf{r_{OA}} = \{0.0518\,\mathbf{i} + 0.1932\,\mathbf{j} + 0.075\,\mathbf{k}\}\mathrm{m}$

$\mathbf{F} = \{-20\,\mathbf{cos}\,15°\,\mathbf{i} + 20\,\mathbf{sin}\,15°\,\mathbf{j}\}\mathrm{N}$

$\Rightarrow \mathbf{F} = \{-19.32\,\mathbf{i} + 5.18\,\mathbf{j}\}\mathrm{N}$



Look at the red arrow on xy plane:



$$\mathbf{r_{OA}} = \{0.0518\,\mathbf{i} + 0.1932\,\mathbf{j} + 0.075\,\mathbf{k}\}\text{m}$$
$$\mathbf{F} = \{-19.32\,\mathbf{i} + 5.18\,\mathbf{j}\}\text{N}$$

$$\mathbf{M_O} = \mathbf{r_A} \times \mathbf{F}$$
$$\Rightarrow \mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.0518 & 0.1932 & 0.075 \\ -19.32 & 5.18 & 0 \end{vmatrix}$$
$$\Rightarrow \mathbf{M_O} = \begin{Bmatrix} [(0.1932)(0) - (0.075)(5.18)]\mathbf{i} \\ -[(0.0518)(0) - (0.075)(-19.32)]\mathbf{j} \\ +[(0.0518)(5.18) - (0.1932)(-19.32)]\mathbf{k} \end{Bmatrix}\text{N.m}$$
$$\Rightarrow \mathbf{M_O} = \{-0.39\,\mathbf{i} - 1.45\,\mathbf{j} + 4.00\,\mathbf{k}\}\text{N.m}$$

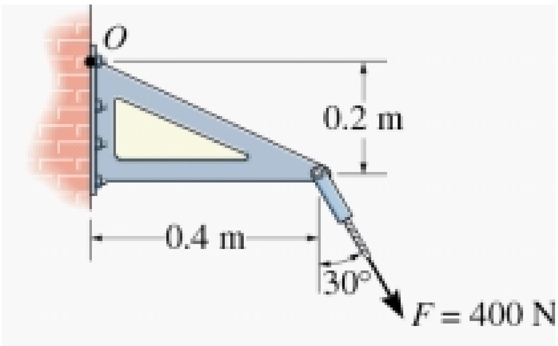
$$\mathbf{M_O} = \{-0.39\,\mathbf{i} - 1.45\,\mathbf{j} + 4.00\,\mathbf{k}\}\text{N.m}$$

$$\Rightarrow M_O = \sqrt{(-0.39)^2 + (-1.45)^2 + (4.00)^2} = 4.27\text{ N.m}$$

$$\alpha = \mathbf{cos}^{-1}\left(\frac{-0.39}{4.27}\right) = 95.2°$$
$$\beta = \mathbf{cos}^{-1}\left(\frac{-1.45}{4.27}\right) = 109.9°$$
$$\gamma = \mathbf{cos}^{-1}\left(\frac{4.00}{4.27}\right) = 20.5°$$

Problem 4:

The force F acts at the end of the angle bracket shown. Determine the moment of the force about point O.



It is noted that in this problem the scalar analysis provides a more convenient method for analysis that the vector analysis.

Solution I (Scalar analysis)

The force is resolved into its x and y components.

Moments of the components are computed about point O :

$$\circlearrowleft + M_O = 400\,\mathbf{sin}\,30°\mathbf{(}0.2\mathbf{)} - 400\,\mathbf{cos}\,30°\mathbf{(0.4)}$$

$$\Rightarrow M_O = -98.6\ \text{N.m} = 98.6\ \text{N.m (clockwise)}$$

Solution II (Vector analysis)

Using a Cartesian vector approach, the force and position vectors shown can be represented as :

$$\mathbf{r} = \{0.4\,\mathbf{i} - 0.2\,\mathbf{j}\}\text{m}$$

$$\mathbf{F} = \{400\,\mathbf{sin}\,30°\,\mathbf{i} - 400\,\mathbf{cos}\,30°\,\mathbf{j}\}\text{N}$$

$$\Rightarrow \mathbf{F} = \{200\,\mathbf{i} - 346.4\,\mathbf{j}\}\text{N}$$

$$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$$

$$\Rightarrow \mathbf{M_O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & -0.2 & 0 \\ 200 & -346.4 & 0 \end{vmatrix}$$

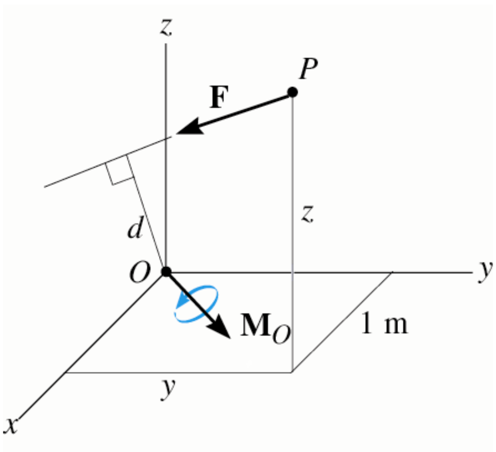
$$\Rightarrow \mathbf{M_O} = 0\,\mathbf{i} + 0\,\mathbf{j} - 98.6\,\mathbf{k}$$

$$\Rightarrow \mathbf{M_O} = \{-98.6\,\mathbf{k}\}\text{N.m}$$

As it is clear, it is recommended to use the scalar analysis for 2-dimensional problems, and the vector method for 3-dimensional problems.

Problem 5:

A force F = {6i –2j +1k} kN produces a moment of M_O = {4i + 5j –14k} kN.m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of x=1 m, determine the y and z coordinates.



Solution:

P(1, y, z)

$\mathbf{F} = \{6\,\mathbf{i} - 2\,\mathbf{j} + 1\,\mathbf{k}\}\text{kN}$

$\mathbf{M_O} = \{4\,\mathbf{i} + 5\,\mathbf{j} - 14\,\mathbf{k}\}\text{kN.m}$

$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$

$$\Rightarrow 4\,\mathbf{i} + 5\,\mathbf{j} - 14\,\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$\Rightarrow 4\,\mathbf{i} + 5\,\mathbf{j} - 14\,\mathbf{k} = (y + 2z)\,\mathbf{i} - (1 - 6z)\,\mathbf{j} + (-2 - 6y)\,\mathbf{k}$

Equate **i**, **j**, and **k** coefficients

$4 = y + 2z \qquad (1)$

$5 = 6z - 1 \qquad (2)$

$-14 = -6y - 2 \qquad (3)$

P(1, y, z)

$4 = y + 2z \qquad (1)$

$5 = 6z - 1 \qquad (2)$

$-14 = -6y - 2 \qquad (3)$

$(2) \Rightarrow 6z = 6$

$\Rightarrow z = 1\,\text{m}$

Substitute in (1)

$\Rightarrow y = 4 - 2z = 4 - 2$

$\Rightarrow y = 2\,\text{m}$

$\Rightarrow P(1,2,1)$