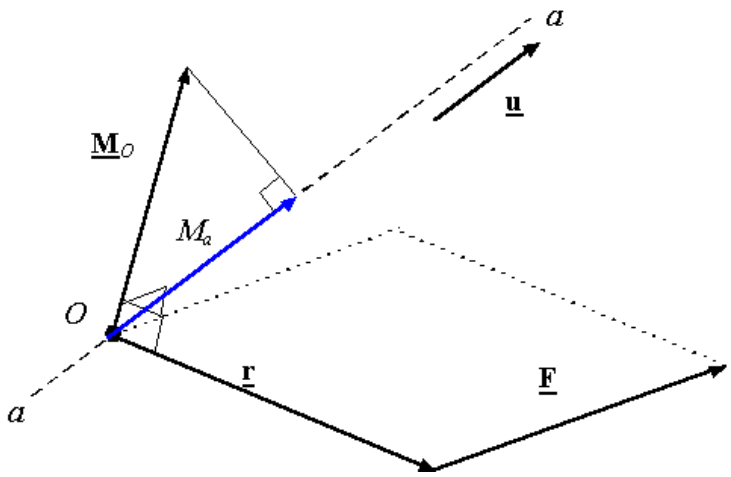
Forces and Moments: Part 3

Moment about an axis:

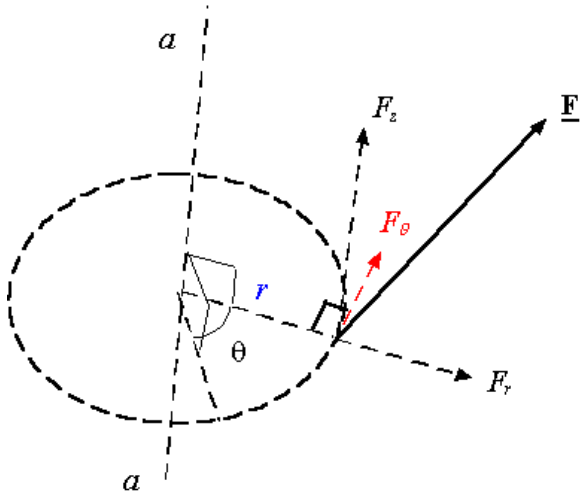
Sometimes the moment about a point is known and you are supposed to calculate its component about an axis. To find the moment, consider the dot product of M_o and unit vector along axis a:

$$M_a = \underline{\mathbf{M}}_O \circ \underline{\mathbf{u}} = (\underline{\mathbf{r}} \times \underline{\mathbf{F}}) \circ \underline{\mathbf{u}}$$

O: any point on *a*-*a*



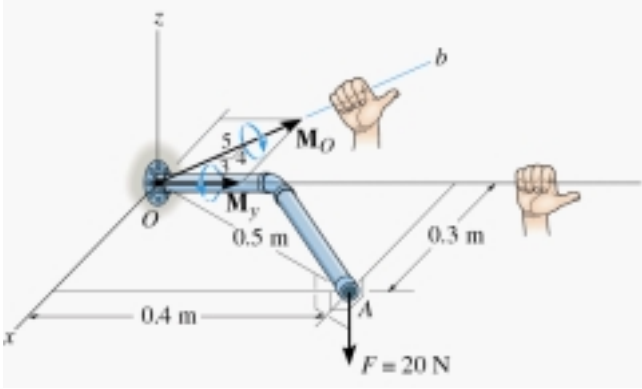
You can also find the tangent force F_θ and then *r* x F_θ is the moment about *aa*:



$$M_a = rF_\theta$$

Let us consider the following example:

Force F causes a moment M_O about point O. What is the component of M_O along axis oy (M_y)?



Solution:

There are 2 methods to find M_y

1. Scalar Analysis
2. Vector Analysis

(1) Scalar Analysis (first way):

M_O=(20)(0.5)=10 N.m

M_O tends to turn the pipe around axis ob. The component of M_O along the y-axis, M_y, tends to unscrew the pipe from the flange at O.

Thus it is important to know its value.

M_y = (3/5)(10) = 6 N.m

Scalar Analysis (second way):

To find M_y directly (not form M_O) it is necessary to determine the moment-arm, knowing that the distance from F to the y-axis is 0.3m:

M_y = (20)(0.3) = 6 N.m

In general, If the line of action of a force F is perpendicular to any specific axis aa thus:

$M_a = F . d_a$

(2) Vector Analysis:

First, use the cross product formula to calculate the moment about O:

$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}$

$\mathbf{M}_O = (0.3\mathbf{i} + 0.4\mathbf{j}) \times (-20\mathbf{k})$

$\mathbf{M}_O = \{-8\mathbf{i} + 6\mathbf{j}\}$ N.m

Then use the dot product of M_O and the unit vector along y-axis to get M_y:

$\mathbf{M}_y = \mathbf{M}_O . \mathbf{u}_a$

$\mathbf{M}_y = (-8\mathbf{i} + 6\mathbf{j}). (\mathbf{j})$

$\mathbf{M}_y$ = 6 N.m