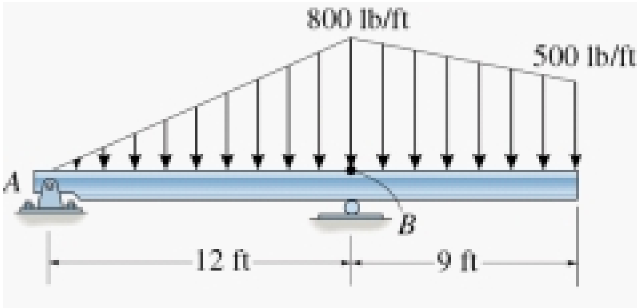
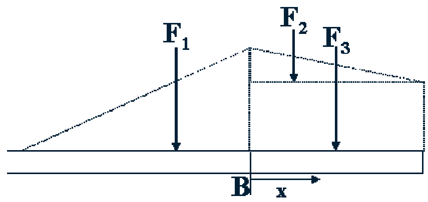
Moments 8:

Problem 1:

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point B.



Solution:



$$F_1 = \frac{1}{2}(800)(12) = 4800 \text{ lb}$$

$$F_2 = \frac{1}{2}(300)(9) = 1350 \text{ lb}$$

$$F_3 = (500)(9) = 4500 \text{ lb}$$

$$+\downarrow F_R = \sum F_y$$

$$\Rightarrow F_R = 4800 + 1350 + 4500 = 10650 \text{ lb}$$

$$\Rightarrow F_R = 10.7 \text{ kip} \downarrow$$

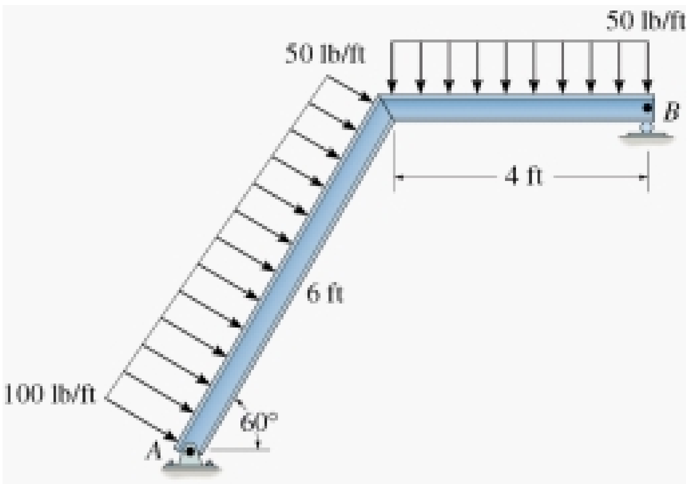
$$\circlearrowleft + M_{R_B} = \sum M_B$$

$$\Rightarrow 10650d = -4800(4) + 1350(3) + 4500(4.5)$$

$$\Rightarrow d = 0.479 \text{ ft}$$

Problem 2:

Replace the loading by an equivalent resultant force and couple moment at point A.

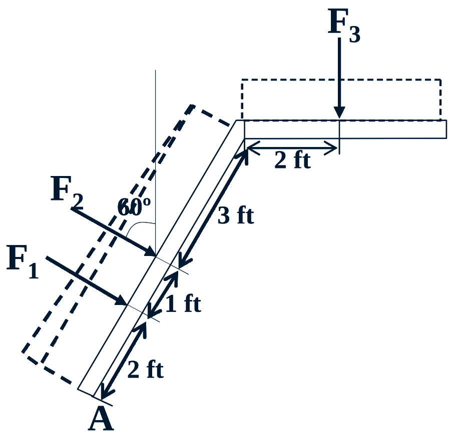


Solution:

$$F_1 = \frac{1}{2}(6)(50) = 150 \text{ lb}$$

$$F_2 = (6)(50) = 300 \text{ lb}$$

$$F_3 = (4)(50) = 200 \text{ lb}$$



$$\xrightarrow{+} F_{R_x} = \sum F_x$$

$$\Rightarrow F_{R_x} = 150 \sin 60° + 300 \sin 60° = 389.71 \text{ lb}$$

$$+\downarrow F_{R_y} = \sum F_y$$

$$\Rightarrow F_{R_y} = 150 \cos 60° + 300 \cos 60° + 200 = 425 \text{ lb}$$

$$F_1 = 150 \text{ lb}; \quad F_2 = 300 \text{ lb}; \quad F_3 = 200 \text{ lb}$$

$$F_{R_x} = 389.71 \text{ lb}; \quad F_{R_y} = 425 \text{ lb}$$

$$F_R = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5°$$

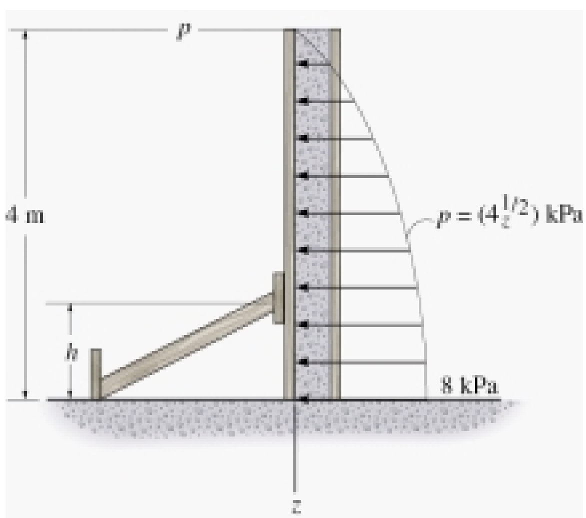
$$(+M_{R_A} = \sum M_A$$

$$\Rightarrow M_{R_A} = 150(2) + 300(3) + 200(6 \cos 60° + 2)$$

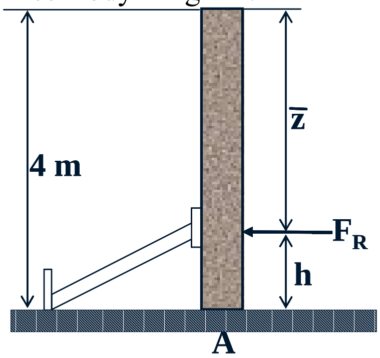
$$\Rightarrow M_{R_A} = 2200 \text{ lb.ft} = 2.2 \text{ kip.ft })$$

Problem 3:

Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



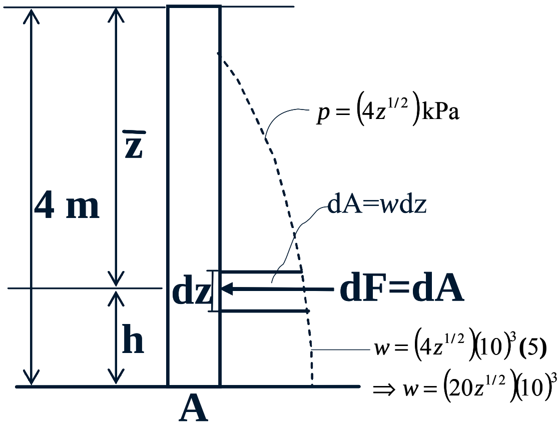
Solution:
Free Body Diagram:



Wall width = 5 m

$p = (4z^{1/2})\,\text{kPa} = (4z^{1/2})(10)^3\ \text{N/m}^2$

$w = (5)\,p = (5)(4z^{1/2})(10)^3 = (20z^{1/2})(10)^3\ \text{N/m}$



Equivalent Resultant Force:

$$\xrightarrow{+} F_R = \sum F_x$$

$$\Rightarrow -F_R = -\int_A dA = -\int_0^z w\,dz$$

$$\Rightarrow F_R = \int_0^{4m} \left(20z^{\frac{1}{2}}\right)(10)^3\,dz$$

$$\Rightarrow F_R = (20)\left(\frac{2}{3}\right)\left(z^{\frac{3}{2}}\right)(10)^3\Bigg|_0^{4m}$$

$$\Rightarrow F_R = \frac{40}{3}\left(4^{\frac{3}{2}}\right)(10)^3$$

$$\Rightarrow F_R = 106.67(10)^3\,N = 107\text{kN} \leftarrow$$

Location of Equivalent Resultant Force:

$$\bar{z}=\frac{\int_A z\,dA}{\int_A dA}=\frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$$

$$\Rightarrow \bar{z}=\frac{\int_0^{4m} z\left[\left(20z^{\frac{1}{2}}\right)(10)^3\right]dz}{\int_0^{4m}\left(20z^{\frac{1}{2}}\right)(10)^3\,dz}$$

$$\Rightarrow \bar{z}=\frac{\int_0^{4m} z\left[\left(20z^{\frac{1}{2}}\right)(10)^3\right]dz}{F_R}$$

Location of Equivalent Resultant Force:

$$\Rightarrow \bar{z}=\frac{\int_0^{4m}\left(20z^{\frac{3}{2}}\right)(10)^3\,dz}{106.67(10)^3\,N}$$

$$\Rightarrow \bar{z}=\frac{(20)\left(\frac{2}{5}\right)\left(z^{\frac{5}{2}}\right)(10)^3\Big|_0^{4m}}{106.67(10)^3\,N}$$

$$\Rightarrow \bar{z}=\frac{\left(\frac{40}{5}\right)\left(4^{\frac{5}{2}}\right)(10)^3}{106.67(10)^3\,N}=\frac{256(10)^3\,N.m}{106.67(10)^3\,N}$$

$$\Rightarrow \bar{z}=2.40\text{ m}$$

Location of Equivalent Resultant Force:

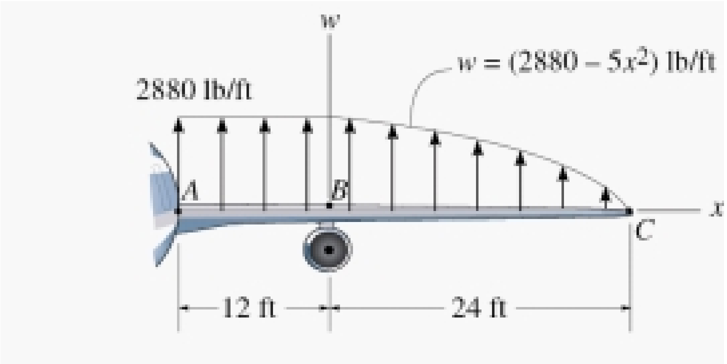
$$\bar{z}=2.40\text{ m}$$

$$\Rightarrow h=4-\bar{z}=4-2.4$$

$$\Rightarrow h=1.6\text{ m}$$

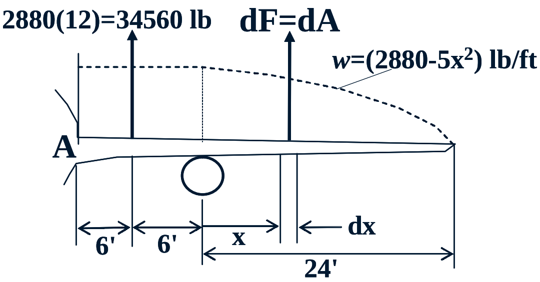
Problem 4:

The lifting force along the wing of a jet aircraft consists of a uniform distribution along AB, and a semi parabolic distribution along BC with origin at B.
Replace this loading by a single resultant force and specify its location measured from point A.



Solution:

FBD:



Equivalent Resultant Force:

$$+\uparrow F_R = \sum F_y$$

$$\Rightarrow F_R = 34560 + \int_0^x w\,dx$$

$$\Rightarrow F_R = 34560 + \int_0^{24\,ft} \left(2880 - 5x^2\right)dx$$

$$\Rightarrow F_R = 34560 + \left(2880x - \frac{5}{3}x^3\right)\Big|_0^{24\,ft}$$

$$\Rightarrow F_R = 34560 + \left(2880(24) - \frac{5}{3}(24)^3\right)$$

$$\Rightarrow F_R = 80640\text{ lb} = 80.6\text{ kip}\uparrow$$

Location of Equivalent Resultant Force:

$$\circlearrowleft + M_{R_A} = \sum M_A$$

$$\Rightarrow 80640\bar{x} = 34560(6) + \int_0^x (x+12)w\,dx$$

$$\Rightarrow 80640\bar{x} = 207360 + \int_0^{24\,ft} (x+12)(2880-5x^2)\,dx$$

$$\Rightarrow 80640\bar{x} = 207360 + \int_0^{24\,ft} (2880x-5x^3+34560-60x^2)\,dx$$

$$\Rightarrow 80640\bar{x} = 207360 + \left(2880\frac{x^2}{2} - 5\frac{x^4}{4} + 34560x - 60\frac{x^3}{3}\right)_0^{24\,ft}$$

Location of Equivalent Resultant Force:

$$80640\bar{x} = 207360 + \left(2880\frac{x^2}{2} - 5\frac{x^4}{4} + 34560x - 60\frac{x^3}{3}\right)_0^{24\,ft}$$

$$\Rightarrow 80640\bar{x} = 207360 + \left(1440(24^2) - 5\frac{(24)^4}{4} + 34560(24) - 20(24)^3\right)$$

$$\Rightarrow 80640\bar{x} = 207360 + 967680$$

$$\Rightarrow \bar{x} = 14.6 \text{ ft}$$