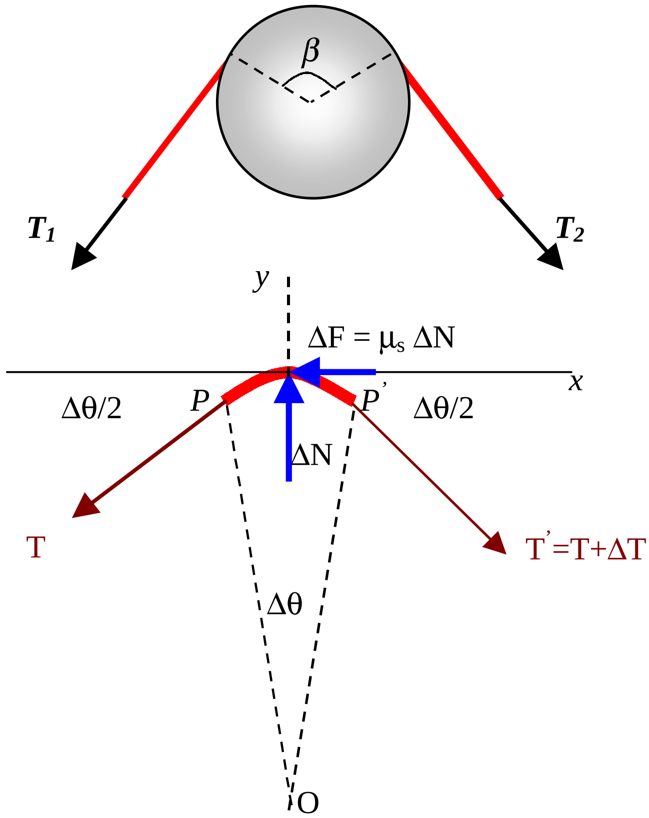
Friction- Part 4:

Belts:

Consider a flat belt passing over a fixed cylindrical drum. We propose to determine the relation existing between the values T_1 and T_2 of the tension in the two parts of the belt when the belt is just about to slide toward the right.



Let us detach from the belt a small element PP' subtending an angle Δθ. We draw the FBD of the element of the belt:

T = the tension at P
T + ΔT = the tension at P'
ΔN = Normal reaction
ΔF = friction force (opposite to the direction of motion)
The motion is assumed to be impending:
$\Delta F = \mu_s \Delta N$

Equations of equilibrium:

$$\sum F_x = 0:$$

$$(T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0:$$

$$\Delta N - (T+\Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

Omitting ΔN:

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T+\frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

When Δθ approaches zero, cosine approaches 1, ΔT approaches 0 and $\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}$ also gets 1.

Also for small Δθ, $\frac{dT}{d\theta} = \frac{\Delta T}{\Delta\theta}$. Therefore:

$$\frac{dT}{d\theta} = \mu_s T \quad \text{or} \quad \frac{dT}{T} = \mu_s d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\beta \mu_s d\theta$$

$$\ln\frac{T_2}{T_1} = \mu_s \beta$$

So:

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$