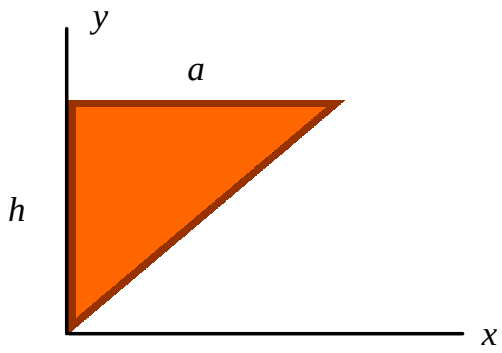
MASS MOMENT OF INERTIA

Problem 2:



Calculate the mass moment of inertia of the triangular plate about the *y*-axis. Assume the plate is made of a uniform material and has a mass of *m*.

Solution:

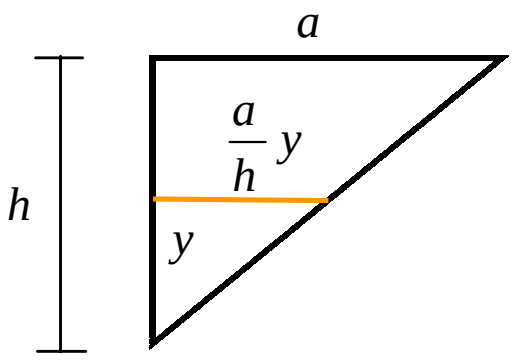
The mass moment of inertia about the *y*-axis is given by

$$I_{zz} = \int_B r^2 dm = \int_A r^2 \rho \, dA$$

The element of area in rectangular coordinate system is given by

$$dA = dxdy = dydx$$

The domain of the triangle is defined by



$$\begin{cases} 0 \le y \le h \\ 0 \le x \le \dfrac{a}{h} z \end{cases}$$

The distance from the *y*-axis is *x*. Therefore, *r*=*x*. The mass moment of inertia about the *y*-axis can be written as

$$I_{zz} = \int_A r^2 \rho \, dA = \int_{y=0}^{y=h} \int_{x=0}^{x=\frac{a}{h}y} x^2 \rho \, dx dy$$

$$= \int_{y=0}^{y=h} \frac{a^3}{3h^3} y^3 \rho \; dy$$

$$= \frac{a^3 h \rho}{12}$$

For a uniform plate the density can be calculated using the total mass and total area of the plate so that

$$\rho = \frac{m}{A} = \frac{m}{\frac{1}{2}ah}$$

Therefore, the moment of inertia in terms of the total mass of the cone can be written as

$$I_{zz} = \frac{a^3 h \rho}{12} = \frac{a^2 m}{6}$$