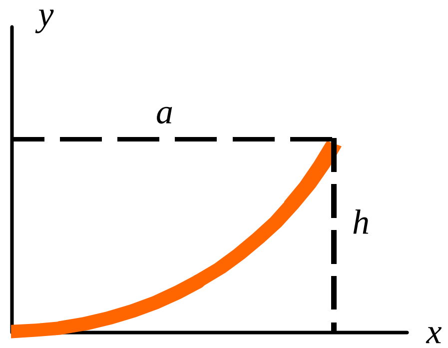
MASS MOMENT OF INERTIA

Problem 3:



Calculate the mass moment of inertia of the parabolic rod about the *y*-axis. Assume the rod is made of a uniform material and has a mass of *m*.

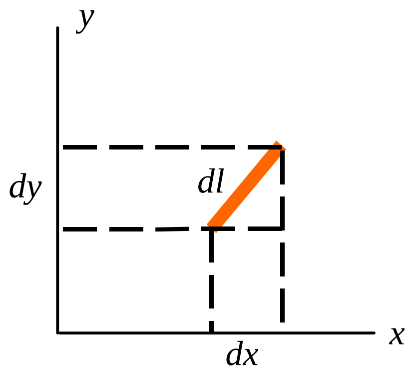
Solution:

The mass moment of inertia about the *y*-axis is given by

$$I_{yy} = \int_B r^2 dm = \int_l r^2 \rho \, dl$$

The length of the bar can be calculated from

$$l = \int_l dl$$



The element of arc length in a rectangular coordinate system can be written as

$$dl = \sqrt{dx^2 + dy^2} = [\sqrt{1 + (\frac{dy}{dx})^2}]dx = [\sqrt{(\frac{dx}{dy})^2 + 1}]dy$$

The equation for the parabola is

$$y = kx^2$$

Substitution of the point (a, h) into this equation givens the equation of the bar as

$$y = \frac{h}{a^2} x^2$$

The length of the bar can, therefore, be calculated as

$$l = \int_l dl = \int_{x=0}^{x=a} \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

$$= \int_{x=0}^{x=a} \sqrt{1 + (\frac{2h}{a^2} x)^2}\, dx$$

The distance from the y-axis is x. Therefore, $r=x$. The mass moment of inertia about the y-axis can be written as

$$I_{zz} = \int_l r^2 \rho \, dl = \int_l x^2 \rho dl$$

$$= \rho \int_{x=0}^{x=a} x^2 \sqrt{1 + (\frac{2h}{a^2} x)^2}\, dx$$

For a uniform bar the density can be calculated using the total mass and total length of the bar so that

$$\rho = \frac{m}{l}$$