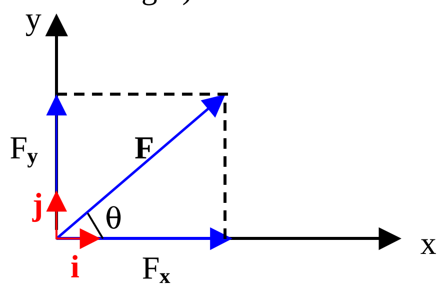
Vectors

Scalar: A quantity like mass or temperature, which only has a magnitude.
Vector: A quantity like heat flux or force which has both a magnitude and a direction; denoted by a bold faced character (***a***), an underlined character (a), or a character with an arrow on it ($\vec{a}$).
Resolution of a Vector: A vector can be resolved along different directions using the parallelogram rule. The figure shows how one resolves vector ***c*** into components ***a*** and ***b*** which are along the given directions (***i*** and ***j*** are the unit vectors; vectors of unit length).

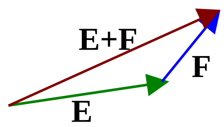
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$$

$$|\boldsymbol{F}| = \sqrt{F_x^2 + F_y^2}$$

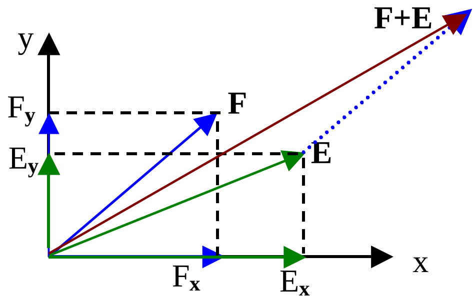
$$\tan q = \frac{F_y}{F_x}$$



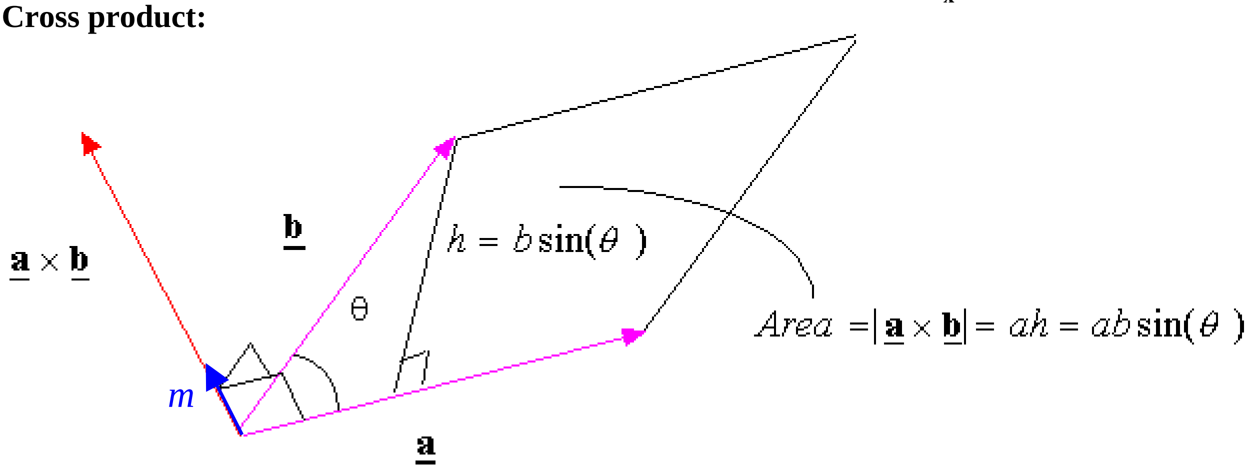
Vector Addition: Vector Addition follows the parallelogram law described in the figure.



$$\boldsymbol{F} + \boldsymbol{E} = (F_x + E_x)\boldsymbol{i} + (F_y + E_y)\boldsymbol{j}$$



Cross product:



The cross product of vectors ***a*** and ***b*** is a vector perpendicular to both ***a*** and ***b*** and has a magnitude equal to area of the parallelogram generated from ***a*** and ***b***. The direction of the cross product is given by the right hand rule (fingers from vector ***a*** to vector ***b*** and thumb is along vector ***c***). Order is important in the cross product:

$$\boldsymbol{a}\times\boldsymbol{b} = -\boldsymbol{b}\times\boldsymbol{a}$$

$$\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k} \quad (\boldsymbol{i}, \boldsymbol{j} \text{ and } \boldsymbol{k} \text{ are the unit vectors})$$

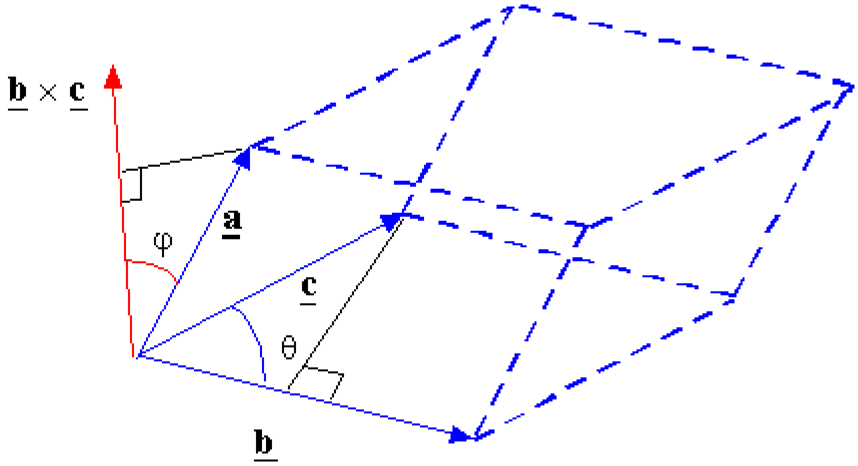
$$\boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$$

$$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_yb_z - a_zb_y)\boldsymbol{i} - (a_xb_z - a_zb_x)\boldsymbol{j} + (a_xb_y - a_yb_x)\boldsymbol{k} \;= \text{Area}\ \boldsymbol{m} = ab\sin q\,\boldsymbol{m}$$

which ***m*** is the unit vector along the line perpendicular to the plane of ***a*** and ***b***

Triple product:

$$\begin{cases} \boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k} \\ \boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k} \\ \boldsymbol{c} = c_x\boldsymbol{i} + c_y\boldsymbol{j} + c_z\boldsymbol{k} \end{cases} \Rightarrow \ \boldsymbol{a}.(\boldsymbol{b}\times\boldsymbol{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_y \end{vmatrix} = (b_y c_z - b_z c_y)a_x - (b_x c_z - b_z c_x)a_y + (b_x c_y - b_y c_x)a_z$$



The volume of the parallelepiped constructed from the vectors ***a***, ***b***, and ***c*** is given by the triple product of the three vectors:

$$\text{volume} = abc\sin q\ \cos j$$

$$\boldsymbol{a}.(\boldsymbol{b}\times\boldsymbol{c}) = a|\boldsymbol{b}\times\boldsymbol{c}|\cos j$$