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Shape Optimization by using Reaction Diffusion Equations

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Abstract

This paper deals with a numerical analysis method based on reaction diffusion equations for solving nonparametric boundary shape optimization problems of domain boundaries in which boundary value problems of partial differential equations are defined. Shape optimization problems are, in most case, formulated as minimization problems of an objective function under the condition that some constraint functions hold. For instance a benchmark problem is the minimization of compliance under a total volume constraint. In this case, the objective function is a compliance and the constraint function is a total volume constraint. To solve this problem, nonparametric boundary variation can be formulated by selecting a one-parameter family of continuous one-to-one mappings from an original domain to variable domains. The shape gradient with respect to a domain variation can be evaluated by the adjoint variable method. The optimized shape is obtained by iterative computation of the domain variation. In other words, to obtain the optimized shape, mathematical programing for iterative computation is needed. However, the direct application of the gradient method for obtaining the domain variation often results in oscillating shapes. It is known that oscillation is caused by a lack of smoothness of the shape gradient.

In this paper, to compensate for the irregularity, we introduce reaction diffusion equations to optimize variable domains. The basic idea was presented in topology optimization [1]. In our approach, the domain variation that minimizes the objective functional is obtained as the solution to reaction diffusion equations defined in the original domain. Neumann boundary condition of reaction diffusion equations is given by the negative shape gradient on the design boundary. A lack of smoothness of the gradient is compensated by the diffusion term of reaction diffusion equations. The optimized shape is obtained by adding time integration of the domain variations to the original domain. In other words, the optimized shape is obtained by the solution to first order ordinary differential equations whose right hand side is the domain variation. The reaction diffusion equations and ordinary differential equations can be solved by the time integration scheme, e.g., BDF method; thus, mathematical programing for iterative computation is not needed for our proposed method. The effectiveness of the proposed method is demonstrated through numerical examples in a structural problem which minimizes a mean compliance of cantilever under a total volume constraint.

References

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