

# Direct and Adjoint Sensitivity Analysis of Nonlinear Magnetostatic System: Application to Shape Optimization of Electrical Machines

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## Abstract

The objective of the paper is to present a direct and an adjoint analytic sensitivity analysis for nonlinear magnetostatic systems excited by permanent magnets and/or inductors, in the context of shape optimization of electric machines. The system is characterized by an interface boundary undergoing shape modification.

The sensitivity approach revisits the material derivative concept of continuum mechanics. The method leads to a set of continuum sensitivity equations that can be solved numerically using the same discretization scheme (Finite Element) as for the original problem. The resulting sensitivity formula can be expressed as either a volume integral or as a boundary integral along the interface where shape modification occurs. It takes into account the potential discontinuity of the state variable across bimaterial boundaries.

A methodology for the calculation of the design velocity field and mesh updating scheme is introduced as well. In this context, the velocity field is determined using a geometrical approach on the boundaries of the domain and is then extended either to the whole domain by a classical Laplacian smoothing technique or to a single layer of elements with the method of unitary boundary layer.

The accuracy of the methodology is investigated on an inductor system, and on a permanent magnet synchronous machine. This investigation suggests that the volume integration technique should be preferred. All methods are freely available for further testing in the open source environment GetDP/Gmsh.