Structural Optimization under Complementarity Constraints

Sawekchai Tangaramvong[1], **Francis Tin-Loi**[2]

[1] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia, sawekchai@unsw.edu.au (corresponding author)

[2] Centre for Infrastructure Engineering and Safety, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia, f.tinloi@unsw.edu.au

Abstract

The paper provides an overview on the formulations and solutions of a challenging "mathematical program with equilibrium constraints" (MPEC) [1] arisen in the applications of structural optimization (see [2,3]). The equilibrium constraints are more precisely the complementarity (the requirement that two nonnegative vectors are orthogonal), which is a typical and recurrent mathematical feature in the nonlinear analysis of structures, e.g. to represent elastoplasticity and contact-like conditions. The resulting state problems lead to instances of mathematical programs known generally as "mixed complementarity problems" [4] for which, under certain conditions (e.g. definiteness of some key matrices), can be efficiently solved. However, the inverse problem that for example arises in the structural optimization under complementarity conditions is far more challenging to process since the underlying mathematical programming problem, known as a MPEC, is disjunctive, nonsmooth and/or nonconvex. Such properties are invariably associated with severe computational difficulties, similar to those in integer programming.

We introduce the concept of complementarity, and review the state problem and its solution before presenting the generic formulation for structural optimization under complementarity constraints [2,3]. We provide an overview of a promising class of solution methods that can be used to solve the resulting MPECs [1]. These all involve application of some regularizing technique followed by conversion of the MPEC into a standard nonlinear programming problem. Three such techniques are penalization, smoothing and relaxation. All perform equally well with the structural optimization described. Numerous examples, two of which are provided herein, attest to their robustness and efficiency.

References

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