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Reliability-Based Microstructural Topology Design with Respect to Vibro-Acoustic Criteria

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1. Abstract
A reliability-based vibro-acoustic microstructural topology optimization model taking into consideration the uncertainty of several selected design-independent parameters, such as the direction of the load, the frequency of the excitation, or their combinations is presented. The design objective is minimization of the sound power radiation from the macro vibrating composite structure that is assumed to be constructed by periodic micro unit cell filled up with two prescribed isotropic materials. A design process consisting of the serial execution of the uncertainty analysis and vibro-acoustic microstructural topology optimization is proposed. Numerical examples show that the uncertainty of the excitation frequency plays more important role in the vibro-acoustic microstructural design in comparison with the uncertainty of the loading direction. It is also shown that the optimum microstructural topology is not so sensitive to perturbation of the loading direction when the normalized variable corresponding to the excitation frequency takes the higher value, i.e. the optimum design is robust for perturbation of both the excitation frequency and the loading direction.

2. Keywords: Microstructural topology optimization; vibro-acoustic criteria; reliability index; uncertainty analysis; bi-material interpolation.

3. Introduction

During the past two decades, several important reliability-based models have been developed and applied to structural optimization, such as the (concurrent) RBDO model, RBSO model and RBTO model [1-4]. In the aspect of the RBTO model, Kharmanda et al. [4] considered the uncertainty of the material elasticity, structural thickness and loading in minimum compliance topology design, and their studies show that the RBTO model normally yields more reliable structures in comparison with the deterministic topology optimization model. Maute et al. [5] applied the first order reliability analysis method to the topology optimization of the compliant micro-electro-mechanical (MEMS) mechanism taking into account the uncertainty of the loading, boundary and material properties. Kang et al. [6] studied the non-probabilistic reliability-based topology optimization problem of the geometrically nonlinear structure. Applications of the RBTO model in thermal system and multi-physics system can be found in the Refs [7-8]. More introduction of the RBTO model may refer to the paper [9]. On the other hand, microstructural topology designs have also drawn a lot of attentions and have been applied to the fields of multi-physics and multi-scales [10-17]. However, up to now there are very few studies concerning the RBTO model combined with the microstructural designs, especially the vibro-acoustic microstructural designs.

The present paper aims at developing a reliability-based vibro-acoustic microstructural topology optimization model taking into account the uncertainty of the load direction, the excitation frequency or their combinations. The paper is organized as follows: Section 4 gives a brief introduction of the probabilistic reliability-based optimization model, and then the reliability-based vibro-acoustic microstructural topology optimization model and the corresponding solution method are presented and discussed in detail in Section 5. Several numerical examples are provided in Section 6 to validate the proposed method and some interesting features are discussed.

4. Optimization Considering Uncertainty

One way of considering the uncertainty of the model is introduction of the stochastic variables described by probability distribution function. A simple way to perform the uncertainty analysis is to introduce a reliability index β and meanwhile transform the random variable $\mathbf{y}$ from the physical space to the normalized variable $\boldsymbol{\mu}$ in the standard space via probabilistic transformation [4, 18], i.e. $\boldsymbol{\mu} = T(\mathbf{x}, \mathbf{y})$, by which the optimization problem under uncertainty may be stated as a nested optimization problem:

$$\begin{aligned} &\min_{\mathbf{x}} \{ f(\mathbf{x}, \mathbf{y}) \} \\ &s.t. \quad \beta(\boldsymbol{\mu}) = -\Phi^{-1}(P_f(\mathbf{y})) \ge \bar{\beta} \\ &\quad\quad \text{where } \ \beta = \min \|\boldsymbol{\mu}\|, \quad s.t. \quad H(\mathbf{x}, \boldsymbol{\mu}) \le 0 \end{aligned} \tag{1}$$

where Φ denotes the standard Gaussian cumulated function and $H(\mathbf{x},\boldsymbol{\mu})$ corresponds to the limit state function in the standard space. If the random variables are dependent on the design variables, which imply that the limit state surface $H(\mathbf{x},\mathbf{y})=0$ may change as the design variables change, solution of problem (1) requires alternating iterations between the reliability analysis of the inner layer and the optimization of the external layer.

5. Reliability-Based Vibro-Acoustic Bi-Material Microstructural Topology Optimization

5.1. Optimization Model

In this Section, the SIMP based vibro-acoustic bi-material microstructural topology optimization model including uncertainty parameters is established to implement the reliability-based zero-one design at the micro-scale. The element material volume density κ_i of the micro unit cell plays the role of the design variable. Each point of the macrostructure is assumed to be constructed by periodically arranged identical microstructure, and hereby the homogenization method may be used to calculate the equivalent material properties of the macrostructure.

Two classes of uncertainty parameters are considered and treated as random variables, i.e. the loading direction angle θ and the excitation frequency ω_p. It is noticed that the random variables here are design-independent (which is normally true in RBTO problem [4]), and thus the alternating iterations between the reliability analysis and the topology optimization may be avoided. Following the similar notations and assumptions as the Refs. [16, 19-21], the Reliability-Based Microstructural Topology Optimization (RBMTO) model for minimization of the sound power Π of the vibrating structure may be formulated in a discrete form as:

$$\begin{aligned} &\min_{\boldsymbol{\kappa}} \left\{ \Pi(\boldsymbol{\kappa},\mathbf{y}(\boldsymbol{\mu})) = \int_S \frac{1}{2}\mathrm{Re}\left(p_f v_n^*\right) dS = \frac{1}{2}\gamma_f c\omega_p^2 \mathbf{U}^* \mathbf{S}_n \mathbf{U} \right\} \\ &s.t.\ \ \beta(\boldsymbol{\mu}) \ge \bar{\beta}, \\ &\quad \left(\mathbf{K} + \mathrm{i}\omega_P \mathbf{C} - \omega_p^2 \mathbf{M}\right)\mathbf{U} = \mathbf{P}, \\ &\quad p_f = \gamma_f c v_n, \ \text{(on S)} \\ &\quad \sum_{i=1}^{n_e} \kappa_i V_i - V^1 \le 0, \ \left(V^1 = \gamma V_0\right), \\ &\quad 0 \le \kappa_i \le 1, \ \left(i = 1, \quad , n_e\right). \end{aligned} \tag{2}$$

where the symbol $\bar{\beta}$ is the prescribed target value of the reliability index, and the symbols **y** and **μ** are the vectors of the random variables and the corresponding normalized variables. The other symbols may refer to [16]. The extended bi-material SIMP model [16, 22-23] is applied to the micro unit cell to implement the zero-one microstructural design. The adjoint method is employed to perform the sensitivity analysis [16] and the MMA method [24] is used to solve the optimization model.

5.2. Reliability Analysis

Under the assumption that the random variables satisfy the Gaussian distribution, the reliability analysis may be performed in a straightforward way [4], where **y** is calculated using the following transformation

$$y_j = E(y_j) + \sigma(y_j)\cdot \mu_j, \quad (j = 1, \quad J) \tag{3}$$

and

$$\beta = \min_{s.t.\ \beta(\boldsymbol{\mu}) \ge \bar{\beta}} \left\{ d(\boldsymbol{\mu}) = \sqrt{\sum_{j=1}^{J} \mu_j^2} \right\}. \tag{4}$$

Here $E(y_j)$ and $\sigma(y_j)$ are the mean value and the standard deviation of the jth random variable y_j. For a prescribed target value $\bar{\beta}$ of the reliability index, the normalized variables may be calculated by Eq. (4), and then the random variable **y** may be evaluated by Eq. (3). The normalized variable μ_j takes the same sign as the derivative of the objective function with respect to the mean value, i.e. $\frac{d\Pi}{dE(y_j)}$ in the present paper, which implies that the optimization will aim at improving the worst case.

5.3. Flow Chart of the Design Process

The design process of the reliability-based vibro-acoustic microstructural topology optimization is given in Fig. 1.

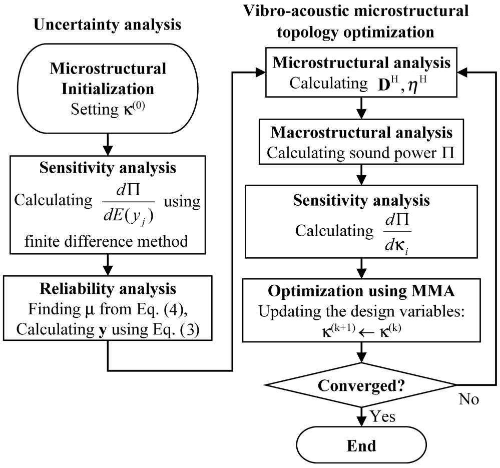


Figure 1: Flow chart of the reliability-based vibro-acoustic microstructural topology design

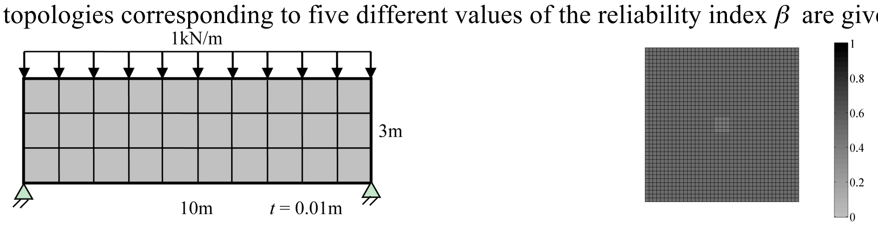
6. Numerical Examples

6.1. Example 1 - Minimization of the Sound Power Flow Considering Single Uncertainty Parameter

The first example concerns design of minimization of the sound power radiated from a simply supported vibrating composite beam-like structure subjected to the uniformly distributed harmonic pressure loading with the amplitude 1kN/m at the upper surface (see Fig. 2). The micro unit cell and the macrostructure are divided by 40×40 and 10×3 mesh using 8-node isoparametric elements respectively. The uncertainty parameter considered here is the excitation frequency ω_p. The mean value of the excitation frequency is $E(\omega_p)$ = 600rad/s and the standard deviation is $\sigma(\omega_p) = E(\omega_p)/10$. The Young's modulus, the Poisson's ratio and the mass density of the two prescribed solid materials are E^1 = 210GPa, $\upsilon^1 = 0.3$, η^1 = 7800kg/m^3, $E^2 = E^1/10$, $\upsilon^2 = \upsilon^1$ and $\eta^2 = \eta^1/10$. The upper limit of the material volume fraction γ of the stiffer material is set as 50%. The damping is ignored here.

Five different values of the reliability index β are tested and the corresponding optimum microstructural topologies are given in Table 1. The iteration histories of the objective function corresponding to different values of the reliability index β are shown in Fig. 3, where the unit of the sound power is transferred from "W" to "dB" by $10\cdot\lg(\Pi/\Pi_0)$, and the reference value of the sound power $\Pi_0 = 10^{-12}\,\mathrm{W}$. It can be seen from Fig. 3 that the optimum value of the sound power becomes higher as the reliability of the design increases, i.e. the design of RBTO makes a balance between the performance and the reliability. The effect of the material volume fraction on the design is also studied. The optimum topologies corresponding to five different values of the material volume fraction α are shown in Table 2, where the reliability index takes the fixed value $\beta = 3$.

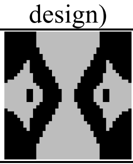
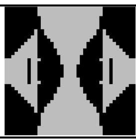
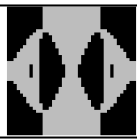
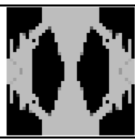
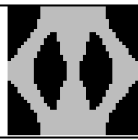
Another design case with different boundary and loading conditions (see Fig. 4) are studied. The configurations of the mesh, materials and uncertainty parameter are the same as Fig. 2. The optimum microstructural topologies corresponding to five different values of the reliability index β are given in Table 3.



(a) (b)

Figure 2: Simply supported beam. (a) Configuration, boundary and loading conditions of the macro beam; (b) Initial material distribution within the micro unit cell.

Table 1: Optimum microstructural topologies corresponding to five different values of reliability index ($\gamma = 0.5$) (Uncertainty parameter: excitation frequency, $E(\omega_p)$ = 600rad/s, $\sigma(\omega_p) = E(\omega_p)/10$)

Optimum topology of the unit cell	$\beta = 0$ (deterministic design)	$\beta = 0.8$	$\beta = 1.5$	$\beta = 2.5$	$\beta = 3$
1 by 1 array					

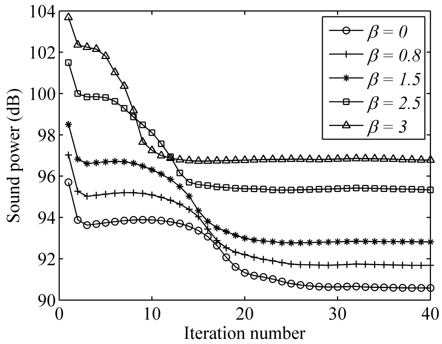
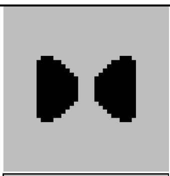
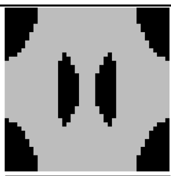
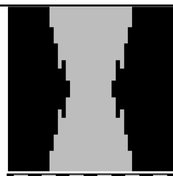
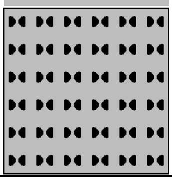
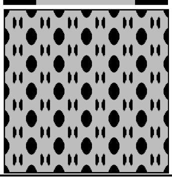
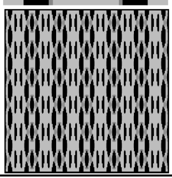
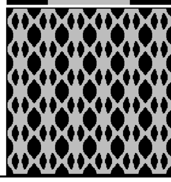
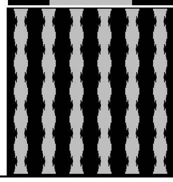


Figure 3: Iteration histories of the objective function corresponding to five different values of reliability index (Uncertainty parameter: excitation frequency, $E(\omega_p)$ = 600rad/s, $\sigma(\omega_p) = E(\omega_p)/10$).

Table 2: Optimum microstructural topologies corresponding to different values of material volume fraction ($\beta = 3$) (Uncertainty parameter: excitation frequency ω_p, $E(\omega_p)$ = 600rad/s, $\sigma(\omega_p) = E(\omega_p)/10$)

Optimum topology of the unit cell	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.6$
1 by 1 array					
6 by 6 array					

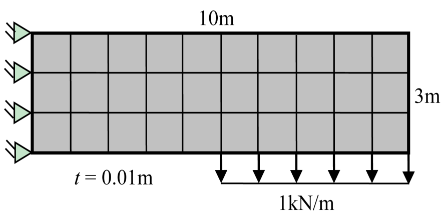
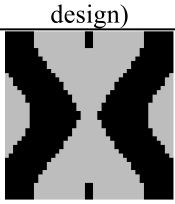
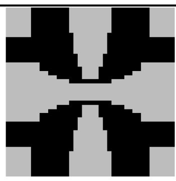
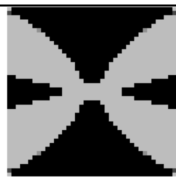
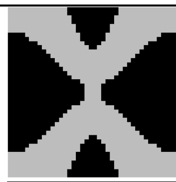
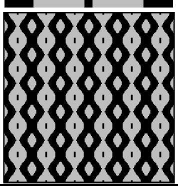
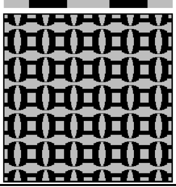
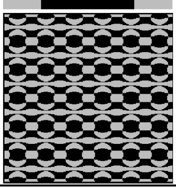
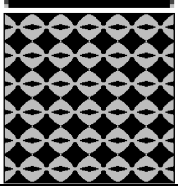
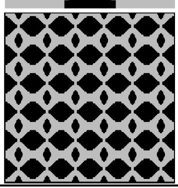


Figure 4: Cantilever beam subjected to bending loads.

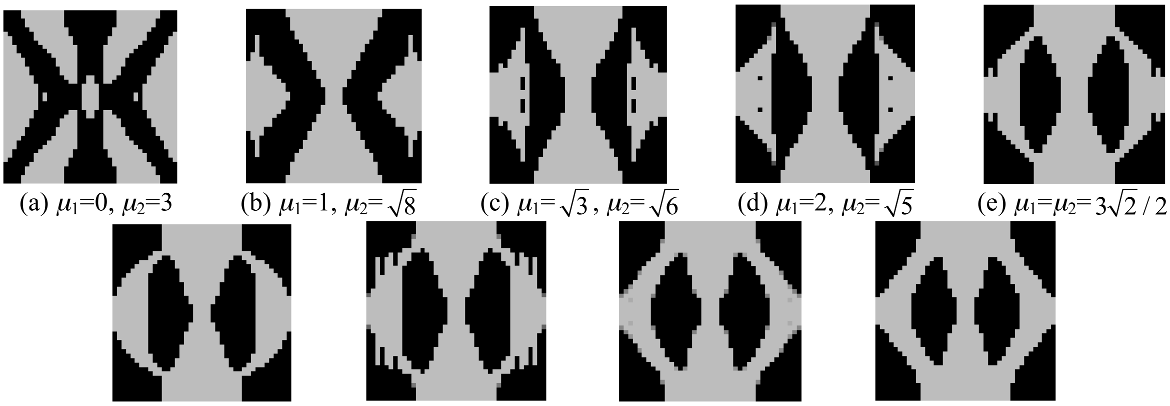
Table 3: Optimum microstructural topologies corresponding to different values of reliability index ($\gamma = 0.5$) (Uncertainty parameter: excitation frequency ω_p, $E(\omega_p)$ = 600rad/s, $\sigma(\omega_p) = E(\omega_p)/10$)

Optimum topology of the unit cell	$\beta = 0$ (deterministic design)	$\beta = 0.8$	$\beta = 2$	$\beta = 2.5$	$\beta = 3$
1 by 1 array					
6 by 6 array					

6.2. Example 2 - Minimization of the Sound Power Flow Considering Multiple Uncertainty Parameters

In this example, two uncertainty parameters, i.e. the excitation frequency ω_p and the loading direction angle θ are considered simultaneously in the design. The normalized variables denoted by μ_1 and μ_2 correspond to the two uncertainty parameters ω_p and θ. The reliability index takes the fixed value $\beta = 3$. The other parameter configurations are the same as those associated with Fig. 2 in the first example of Section 6.1.

The designs with respect to different combination values of the normalized variables but the fixed value 3 of the reliability index are performed, and the corresponding optimum microstructural topologies are shown in Fig. 5. It is seen that different optimum designs may have the same reliability (i.e. the same value of the reliability index) when more than one uncertainty parameters are considered. It can also be seen that when μ_1 takes a higher value, e.g. $\mu_1 > 2$, the change of the optimum microstructural topology is small since the change of the value of μ_1 gets smaller (from 2 to 3), while the interesting thing is, the change of the value of μ_2 is larger (from $\sqrt{5}$ to 0), which implies that the uncertainty design in the present stage is dominated by the excitation frequency and the design is not so sensitive to the uncertainty of the load. The optimum objective function values corresponding to different combination values of the normalized variables are given in Table 4. In order to get an overall sight, the interpolation surface of the optimum objective function with respect to (μ_1, μ_2) is also given in Table 4. It can be seen that the worst case happens at (μ_1=3, μ_2=0). This implies that the uncertainty of the excitation frequency is more important than that of the loading direction for a given value 3 of the reliability index in the vibro-acoustic microstructural topology design.

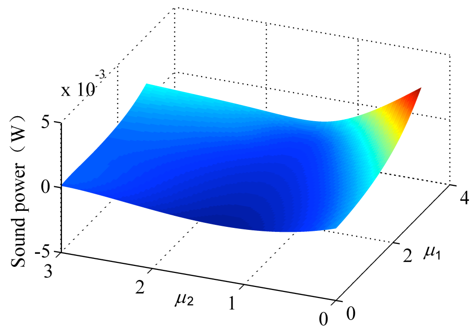


(a) μ_1=0, μ_2=3 (b) μ_1=1, $\mu_2=\sqrt{8}$ (c) $\mu_1=\sqrt{3}$, $\mu_2=\sqrt{6}$ (d) μ_1=2, $\mu_2=\sqrt{5}$ (e) $\mu_1=\mu_2=3\sqrt{2}/2$

(f) $\mu_1=\sqrt{5}$, μ_2=2 (g) $\mu_1=\sqrt{6}$, $\mu_2=\sqrt{3}$ (h) $\mu_1=\sqrt{8}$, μ_2=1 (i) μ_1=3, μ_2=0

Figure 5: Optimum topologies of the unit cells corresponding to different combination values of normalized variables ($\beta = 3$; $\gamma = 0.5$); (Uncertainty parameter 1: ω_p, $E(\omega_p)$ = 600rad/s, $\sigma(\omega_p) = E(\omega_p)/10$; normalized variable 1: μ_1); (Uncertainty parameter 2: θ, $E(\theta)$ = -90°, $\sigma(\theta) = E(\theta)/10$; normalized variable 2: μ_2)

Table 4: Optimum objective function values corresponding to different combination values of normalized variables ($\beta = 3$)

(μ_1, μ_2)	$\prod^{opt}$/W	Interpolation surface of objective function: $\prod^{opt}$ vs. (μ_1, μ_2)
(0, 3)	0.1358×10^{-3}	
(1, $\sqrt{8}$)	0.1779×10^{-3}	
($\sqrt{3}$, $\sqrt{6}$)	0.2227×10^{-3}	
(2, $\sqrt{5}$)	0.2522×10^{-3}	
($3\sqrt{2}/2$, $3\sqrt{2}/2$)	0.2626×10^{-3}	
($\sqrt{5}$, 2)	0.2857×10^{-3}	
($\sqrt{6}$, $\sqrt{3}$)	0.3277×10^{-3}	
($\sqrt{8}$, 1)	0.4121×10^{-3}	
(3, 0)	4.7440×10^{-3}	

7. Conclusions

The reliability-based vibro-acoustic microstructural topology optimization model is developed and solved. The effects of single and multiple uncertainty parameters on the optimum microstructural topologies are studied in detail and several interesting features are revealed. It is found that the uncertainty of the excitation frequency plays more important role in the vibro-acoustic microstructural design in comparison with the uncertainty of the loading direction.

8. Acknowledgements

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