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Topology Optimization for Heat Conduction Using Generative Design Algorithms

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1. Abstract

In this article, generative design algorithms are investigated as a strategy for solving two-dimensional steady-state heat conduction topology optimization problems. The motivation for this study is to investigate alternative numerical strategies for the eventual solution of richer three-dimensional multidisciplinary electro-thermal design problems related to functional electrical power systems. The efficient solution of such problems is critical for future power-dense electronics, where the optimal layout of heat sources (e.g., electrical devices) and heat sinks in combination with heat flow control structures and devices is important. Thus, as a first step toward this greater goal, generative algorithms are explored for their possible benefits, which include enabling a broader variety of objective functions, design constraints, and design variables plus separation of the design description from the computational mesh. Specifically, a new design method based the Space Colonization Algorithm is investigated. The generative algorithm is implemented using two distinct techniques. The first method is to use the generative algorithm to produce a starting topology for the SIMP Method; this will be referred to here as the Hybrid Approach. The second technique is to use the generative algorithm to produce topologies that can be meshed directly and evaluated with a finite element solver; this will be referred to here as the Generative Design Approach. A two-dimensional case study is used to compare the effectiveness of the SIMP, Hybrid, and Generative Design approaches. These initial studies involve a homogeneously heated square design domain where the thermal compliance design objective and computational cost are assessed.

2. Keywords: Topology Optimization, Generative Algorithms, Conduction

3. Introduction

Topology optimization was first explored in structural design. Given a design domain, topology optimization algorithms determine the distribution of material within the domain to achieve the best structural performance. Numerous approaches have been developed to assign material distribution, from density to evolutionary approaches. A recent review describes existing topology optimization approaches in more detail [1]. Beyond structural design, topology optimization algorithms have been tailored to solve heat transfer problems, including two-dimensional heat conduction problems [2, 3, 4] and multiphysics thermo-fluid problems [5, 6]. Extensions to three-dimensional heat conduction problems [7] and transient heat transfer problems [8, 9] have been made.

Generative algorithms involve the iterative application of simple recursive rules to produce sophisticated algorithm outputs. These algorithms have gained popularity in generative art and architecture in recent decades [10]. More recently, they have been used in engineering design, both for their ability to explore novel designs, and for use as design abstractions that allow exploration of high-dimension system designs using a low-dimension set of generative algorithm parameters. More specifically, instead of adjusting system design variables directly, we adjust generative algorithm rule parameters to produce new designs. This indirect representation reduces problem dimension, and can support faster design space search when coupled with optimization algorithms. Applying an optimization algorithm on generative algorithm parameters is analogous to optimizing on a mapping. For some complex design problems, optimization of these indirect system representations have been found to produce meaningful and improved solutions, whereas conventional direct design representations can fail to produce useful designs [11].

In this work, we look to compare three different strategies with respect to their abilities to produce designs for effective heat extraction. The first strategy is the Solid Isotropic Material with Penalization (SIMP) approach, which is a mature topology optimization method that will be used as a comparison baseline. The second strategy is to use a generative algorithm to produce a topology that can be meshed and solved with a finite element solver. The third strategy utilizes the same generative algorithm to create a starting topology for the SIMP approach [12], which is then used to produce an optimal topology. A discussion of findings and suggestions for future work follow.

4. Problem Formulation

Consider a homogeneously heated design domain, as shown in Fig 1. The steady-state conductive heat transfer across the domain can be represented by the following governing equations:

$$\begin{aligned} \nabla \cdot (k\nabla T) + f = 0 \ \ \text{on} \ \ \Omega \\ T = 0 \ \ \text{on} \ \ \Gamma_D \\ (k\nabla T) \cdot \mathbf{n} = 0 \ \ \text{on} \ \ \Gamma_N, \end{aligned} \tag{1}$$

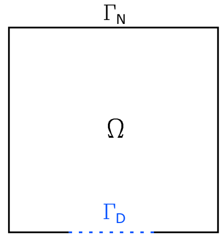


Figure 1: Homogeneously heated design domain.

where T is the temperature state variable, f is the heat generated, and k is the thermal conductivity of the material in the domain, Ω. On the Dirchlet boundary, Γ_D, referred to from here on as the *heat sink*, the temperature is set to zero. The adiabatic Neumann boundary condition, Γ_N, restricts heat flux out of the domain. The thermal compliance of the system is given by the sum of the compliance across the design domain.

$$C = \int \nabla T \cdot \mathbf{q} \, dA = \int \nabla T (k \nabla T) dA \tag{2}$$

The initial design optimization problem considered here is:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & C(\mathbf{x}) \\ \text{subject to} \quad & V(\mathbf{x}) = V_D \\ & R(\mathbf{x}) \geq R_{\min} \end{aligned} \tag{3}$$

where the amount of material, $V(\mathbf{x})$, is constrained to be V_D and the radius of a conductive path, $R(\mathbf{x})$, must be larger than $R_{\min}$. The design variable vector, $\mathbf{x}$, is a general representation of the topology. This representation will change between the different design approaches. The performance of the following three algorithms will be evaluated to assess the use of generative algorithms in topology design optimization for steady-state heat conduction.

4.1 SIMP Approach

The SIMP approach considers a design domain that is discretized into finite elements. Each element is assigned a material amount, γ, which is treated as the design variable. The algorithm is driven by a sensitivity filter that changes the material distribution from element to element. An ideal solution results in a domain consisting only of void, $\gamma = 0$, and fully-dense material, $\gamma = 1$. A detailed description of the implementation of this approach is outlined in [12].

4.2 Generative Design Approach

The generative design approach (GDA) is fundamentally different from SIMP. A gradient-free optimization method is used to adjust generative algorithm parameters to minimize the objective function [11]. Each design candidate considered in the search is represented using an abstract generative algorithm rule parameter vector. This vector is then used to generate a design topology, which is then evaluated via finite element analysis to determine $C(\mathbf{x})$. This vector is much lower in dimension than a design vector based on direct design representation (e.g., γ for each finite element, as in SIMP). Also, in SIMP and related approaches, the design domain discretization is used as the finite element mesh. This can limit the numerical efficiency of thermal analysis. In the generative algorithm approach, the design description is separate from the analysis mesh, allowing us to use a non-uniform mesh that is tailored for each topology, improving analysis accuracy and efficiency. A novel meshing technique is introduced here that is congruent with the unique properties of the GDA. A genetic algorithm (GA) is used here as the gradient-free optimization method.

4.3 Hybrid Approach

The final approach considered here combines the SIMP approach with a generative algorithm. These tools are used in a nested manner to search for an optimal topology. The outer loop GA adjusts generative algorithm parameters to produce a topology that serves as a starting point for the SIMP algorithm (inner loop). The generated topology is mapped to a discretized domain, and the SIMP algorithm uses gradient information to find a local minimum. Using a local improvement strategy for individual designs within a GA population is sometimes referred to as a memetic algorithm [13].

5. Generative Algorithm

Optimal topologies for heat conduction problems typically resemble dendritic structures [2, 4, 5, 7]. Given this tendency, we predict a faster convergence to optimal designs if the search is restricted to dendritic topologies. To perform a targeted search of dendritic structures, a survey of prospective algorithms was completed. Dendritic structures have been a topic of research interest earliest cited in 1976 [14]. In computer graphics, researchers have attempted to efficiently and accurately reproduce dendritic structures [15, 16, 17]. In topology design, researchers have applied different generative algorithms to various heat transfer applications [18, 19, 20, 21, 22]. Table 1 summarizes the primary algorithms of interest, dividing them into three main groups. The L-System and constructal theory algorithms use rules to define components and guide their assembly. The next class of the algorithms may be considered "Interaction Based" where only the rules governing interactions are controlled. The Erosion Model and SIMP techniques can also be looked at as generative algorithms as they evolve designs over time. Where the SIMP procedure moves material, the Erosion Model adds material to the design domain. The algorithms were evaluated on their relative number of design variables, whether or not they have a tendency to create overlapping members, and whether or not the algorithm inherently stays within a prescribed design boundary.

Table 1: Generative algorithm assessment

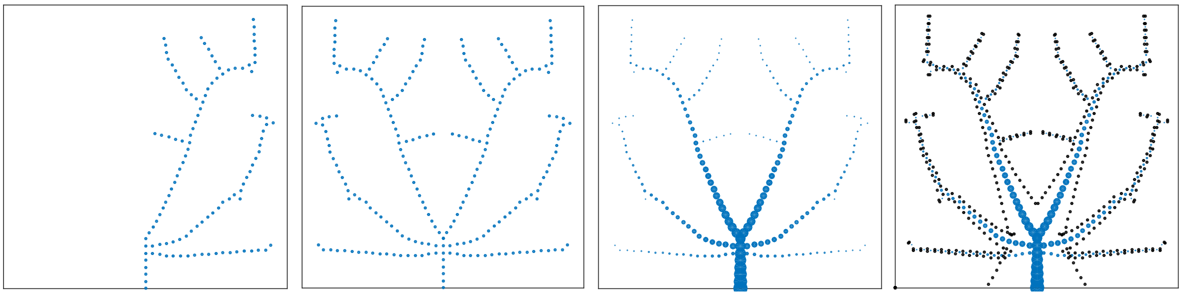
Generative Algorithm		# Design Vars	Overlap	Boundary Adherence
Building Block Based	L-System (LinenMayer)	Med	Yes	No
	Constructal Theory (Bejan)	Med	Yes	No
Interaction Based	Reaction Diffusion	Low	No	Yes
	Particle System (Rodkaew)	Low	No	Yes
	Space Colonization (Runions)	Low	No	Yes
Sensitivity Based	Erosion Model (Bejan)	High	No	Yes
	SIMP (Sigmund)	High	No	Yes

The Reaction Diffusion, Particle System, and Space Colonization algorithms stand out as the best candidates due to their low dimension and ability to handle boundary constraints. The Space Colonization algorithm was ultimately chosen since it has been successfully scaled into three dimensions and its growth procedure intuitively translates to a heat transfer framework. Future work should include the exploration of other candidate generative algorithms.

5.1 Space Colonization Algorithm

The Space Colonization Algorithm (SCA) used here is an adaptation of an algorithm develop by Rodkaew et al. [15]. Hormone centers, called auxins, are strategically placed on the design domain. The algorithm begins from a source node and grows a dendritic structure towards the auxins. This procedure is similar to the accepted theory for vein growth in plant leaves called the *canalization hypothesis* [23]. This concept of growing a topology towards hormone sources may be easily translated to a heat transfer problem where conductive paths can be grown towards heat sources. A rigorous description of implementation in two- and three-dimensions can be followed in [16] and [17], respectively.

Our SCA modifications for conductive heat transfer paths were made to ensure each topology satisfied constraints present when using the SIMP approach (Fig. 2). This supports a direct comparison between the three algorithms presented here. The amount of conductive material is fixed, and the symmetry of boundary conditions is used to reduce the design space by half (Fig. 2a-b). In the SCA, the endpoints of each branch are set to the minimum radius, R_{min}. Path width increases continuously closer to the heat sink (Fig. 2c). A discretized boundary is created to better illustrate the final topology (Fig. 2d).

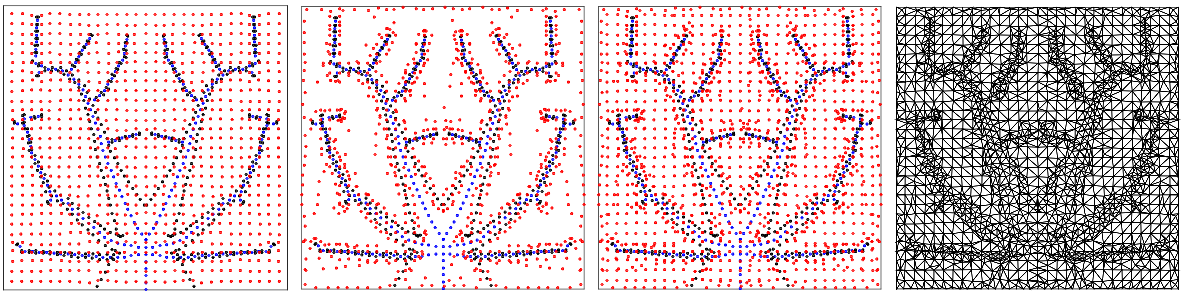


(a) Initial topology (b) Enforced symmetry (c) Thickness distribution (d) Discretized domain

Figure 2: Space colonization modifications applied to a steady-state conductive heat trasnfer problem.

5.2 Application of Generative Design Approach

To use the generated topology with any of the desired approaches, the structure must be converted into a usable form. Given the intricate nature of the generated topologies, standard meshing approaches often result in non-conformal meshes. To handle this issue, a novel technique was developed to mesh the generated designs. This technique was inspired by force-directed graphs [24] and particle generators. An evenly distributed grid of points is generated on the design domain (Fig. 3a). Attractive forces are added between the discretized boundary nodes and the grid nodes. The system is then simulated for a given time to create a point cloud with a dense particle distribution near the boundaries (Fig. 3b). Additional particles can be introduced during the simulation to increase node density. At the final time step, the initial grid is regenerated on the domain to enforce a minimum accuracy for the finite element solver (Fig. 3c). Delaunay triangulation is then used to convert the point cloud into a mesh that can be used with a finite element solver, shown in Fig. 3d.

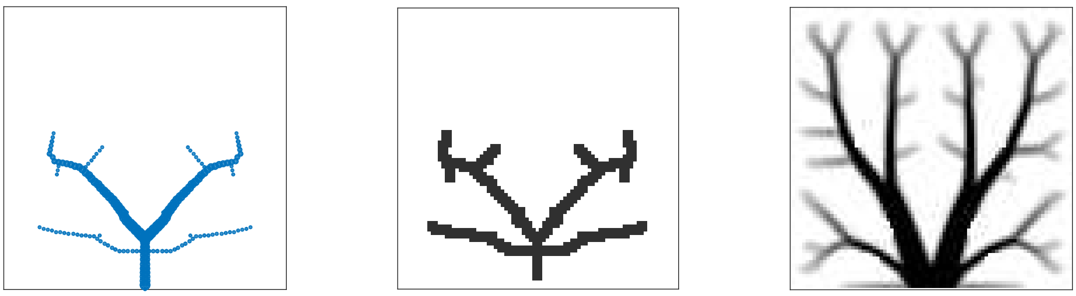


(a) Initial grid (b) Simulate (c) Grid overlay (d) Triangulation

Figure 3: Automated meshing procedure based on force-directed particles and particle generation.

5.3 Application of Hybrid Approach

To use the generated topology with the SIMP approach it must be mapped on to a discretized domain. Starting with a void domain, the parameter γ is set to 1 for every element which contains a node. All of the elements within the minimum radius distance, $R_{\min}$, of each node are also set so $\gamma = 1$. The initial mapping can be seen in Fig. 4b and is used by the SIMP algorithm to produce the optimal topology Fig. 4c.



(a) Initial topology (b) Mapping (c) Solution

Figure 4: Discretized domain mapping for the hybrid optimization approach.

6. Results

The SIMP approach with a sensitivity filter and a homogeneous initial material distribution was used to develop a baseline topology for comparison. The chosen parameters follow: `nelx` = `nely` = 80, `penal` = 3, `volfrac` = 0.2, $R_{\min}$ = 2, [25]. Equivalent parameters were enforced on the generative algorithm. A GA was implemented using the MATLAB® global optimization toolbox. A population size of 100 and 5 generations were used with parallel computing activated. The GA operates on each auxin location. Ten auxins were used to guide the growth of the generative algorithm. The following experimental results were performed using an Intel® Core™i5-4570 CPU @ 3.20 GHz with 8.00 GB (RAM) 64-bit Operating System running Windows 8.1. The results presented in Table 2 are the best of 10 trials.

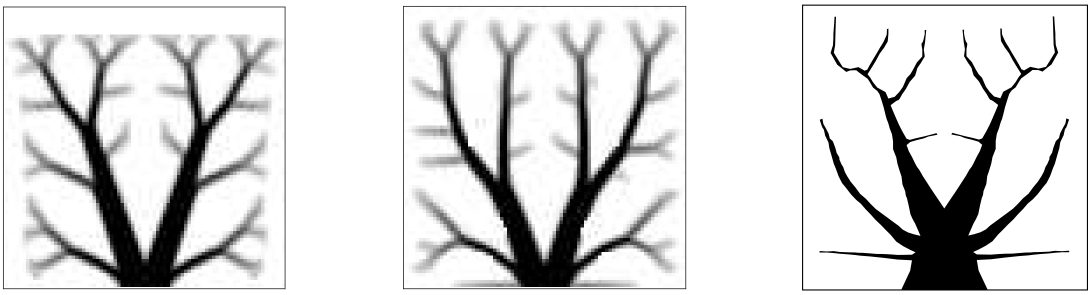
Table 2: Numerical results

	SIMP	GDA	Hybrid
Objective Value	1709	1274	1528
# of Elements	6400	4710	6400
# of Design Variables	6400	40	6440
Total Time (s)	3.19	245.6	1574.0

Table 3: GA variation

	GDA	Hybrid
Best Objective	1274	1528
Worst Objective	1455	1598
Mean Objective	1373	1548
Std. Dev.	$73 \approx 5\%$	$32 \approx 2\%$
Mean Time (s)	220	1579

The SIMP approach was found to converge quickly to a local optimum. The GDA was found to produce the best objective value at an increase in computational cost. The hybrid approach only produces a small improvement in performance, but incurred significant computational expense. Table 3 presents the variation in the GA solutions between the 10 trials. The computational cost of evaluating a topology using the GDA is outlined in Table 4, where the amount of time to complete each task is presented.



(a) SIMP solution (b) Hybrid solution (c) Generative design solution

Figure 5: Best topologies obtained using the three optimization approaches.

Table 4: Generative design approach evaluation time

Section	Generation	Meshing	Finite Element Solver	Total
Time (s)	0.21	0.31	0.17	0.69

7. Conclusion

Given comparable resolution and topological structure, the generative design approach was found to converge on higher performance solutions. Benefits of this approach include the explicit representation of thermally conductive heat transfer paths, the binary assignment of material, and the low design problem dimension. While the GDA design abstraction limits design space coverage, the search is more targeted and finds better performing designs. It is important to note that there is a computational expense increase when comparing the generative design approach to SIMP, yet this increase may be worthwhile due to the improvement in objective value, and for the potential to solve more general problems.

The authors look to increase problem complexity to capitalize on the properties of generative algorithms. Several additional studies are in progress. A concurrent path and layout planning problem that designs the optimal placement of heat sources on a design domain, along with the best topology to extract heat, is being investigated. An alternate formulation to maximize power density with trade-offs between the amount of conductive material and computing elements is also being investigated. Future work will look to embed additional properties in generative algorithms that are important for multi-physics applications, as well extension to three-dimensional problems.

8. Acknowledgments

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9. References

[1] O. Sigmund and K. Maute, "Topology Optimization Approaches," *Structural and Multidisciplinary Optimization*, vol. 48, pp. 1031–1055, December 2013.

[2] A. Gersborg-Hansen, M. P. Bendsoe, and O. Sigmund, "Topology Optimization of Heat Conduction Problems Using the Finite Volume Method," *Structural and Multidisciplinary Optimization*, vol. 31, pp. 251–259, March 2006.

[3] V. Vishwakarma, A. Das, and K. D. P, "Steady State Conduction Through 2D Irregular Bodeis by Smooth Particle Hydrodynamics," *International Journal of Heat and Mass Transfer*, vol. 54, pp. 314–325, January 2013.

[4] C. Zhuang, Z. Xiong, and H. Ding, "A Level Set Method for Topology Optimization of Heat Conduction Problems Under Multiple Loading Cases," *Computer Methods in Applied Mechanics and Engineering*, vol. 196, pp. 1074–1084, January 2007.

[5] E. M. Dede, "Optimization and Design of a Multipass Branching Microchannel Heat Sink for Electronics Cooling," *ASME Journal of Electronic Packaging*, vol. 134, December 2012.

[6] A. A. Koga, E. C. C. Lopes, H. F. Villa Nova, and C. R. de Lima, "Development of Heat Sink Device by Using Topology Optimization," *International Journal of Heat and Mass Transfer*, vol. 64, no. 0, pp. 759–772, 2013.

[7] F. H. Burger, J. Dirker, and J. P. Meyer, "Three-Dimensional Conductive Heat Transfer Topology Optimization in a Cubic Domain for the Volume-To-Surface Problem," *International Journal of Heat and Mass Transfer*, vol. 67, pp. 214–224, December 2013.

[8] H. D. Chungang Zhuang, Zhenhua Xiong, "Topology Optimization of Transient Heat Conduction Problem on a Traingular Mesh," *Numerical Heat Transfer*, vol. 64, pp. 239–262, June 2013.

[9] C. Zhuang and Z. Xiong, "A Global Heat Compliance Measure Based on Topolgy Optimization for the Transient Heat Conduction Problem," *Numerical Heat Transfer, Part B: Fundamentals*, vol. 45, no. 5, pp. 445–471, 2014.

[10] C. Reas and C. McWilliams, *Form+Code, Art, and Architecture*. Princeton Architectural Press, September 2010.

[11] N. Cheney, R. MacCurdy, J. Clune, and H. Lipson, "Unshackling Evolution: Soft Robotics with Multiple Materials and a Powerful Generative Encoding," in *Proceedings of the 15th Annual Conference on Genetic and Evolutionary Computation Conference*, GECCO '13, pp. 167–179, ACM, July 2013.

[12] M. P. Bensoe and O. Sigmund, *Topology Optimization - Theory, Methods, and Application*. Springer, 2nd ed., 2004.

[13] A. E. Eiben and J. E. Smith, *Introduction to Evolutionary Computing*, ch. 10, pp. 173–188. Springer, 2003.

[14] M. Meinhardt, "Morphogenesis of Lines and Nets," *Models and Hypothesis. Differentiation 6*, vol. 6, pp. 117–123, August 1976.

[15] Y. Rodkaew, S. Siripant, C. Lursinsap, and P. Chongstitvatana, "An Algorithm for Generating Vein Images for Realistic Modeling of a Leaf," in *In Proceedings of the International Conference on Computational Mathematics and Modeling*, 2002.

[16] A. Runions, M. Fuhrer, B. Lane, P. Federl, A.-G. Rolland-Lagan, and P. Prusinkiewicz, "Modeling and Visualization of Leaf Venation Patterns," in *ACM Transactions on Graphics*, vol. 24, pp. 702–711, 2005.

[17] A. Runions, B. Lane, and P. Prusinkiewicz, "Modeling Trees with a Space Colonization Algorithm," in *Eurographics Workshop on National Phenomena*, pp. 63–70, 2007.

[18] A. Bejan, I. Dincer, S. Lorente, A. F. Miguel, and A. H. Reis, *Porous and Complex Flow Structures in Modern Technology*. Springer Science, 2004.

[19] A. Bejan and S. Lorente, "Constructal Theory of Generation of Configuration in Nature and Engineering," *Journal of Applied Physics*, vol. 100, no. 4, pp. –, 2006.

[20] A. Bejan and S. Lorente, "Constructal Law of Design and Evolution: Physics, Biology, Technology, and Society," *Journal of Applied Physics*, vol. 113, no. 15, pp. –, 2013.

[21] S. Salakij, J. A. Liburdy, D. V. Pence, and M. Apreotesi, "Modeling in Site Vapor Extraction During Convective Boiling in Fractal-Like Branching Microchannel Networks," *Interantional Journal of Heat and Mass Transfer*, vol. 60, pp. 700–712, May 2013.

[22] D. Heymann, D. Pence, and V. Narayanan, "Optimization of Fractal-Like Branching Microchannel Heat Sink for Single-Phase Flows," *Internation Journal of Thermal Sciences*, vol. 49, no. 8, pp. 1383–1393, 2012.

[23] T. Sachs, "The Control of Patterned Differentiation of Vascular Tissues," *Advances in Botonical Research*, vol. 6, pp. 152–262, 1981.

[24] S. Bowmick and S. M. Shontz, "Towards High-Quality, Untangled Meshes via a Force-Direct Graph Embedding Approach," *Procedia Computer Science*, vol. 1, no. 1, pp. 357–366, 2010.

[25] E. Andreassen, A. Clausen, M. Schevenels, B. S. Lazarov, and O. Sigmund, "Efficient Topology Optimization in MATLAB using 88 Lines of Code," *Structural and Multidisciplinary Optimization*, vol. 43, pp. 1–16, January 2011.