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Efficient Aerodynamic Optimization Using a Multiobjective Optimization Based Framework to Balance the Exploration and Exploitation

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Abstract
In many aerospace engineering design problems, objective function evaluations can be extremely computationally expensive, such as the optimal design of the aerodynamic shape of an airfoil using high-fidelity computational fluid dynamics (CFD) simulation. A widely used approach for dealing with expensive optimization is to use cheap global surrogate (approximation) models to substitute expensive simulation. The effective global optimization (EGO) based on Kriging model is a widely used approach for dealing with the expensive optimization problems. In the standard EGO and most Kriging based aerodynamic optimization application, one sampling point is determined for expensive simulation. To make best use of parallel computing resources, multi-point infill sampling criteria is need to improve the efficiency of aerodynamic shape optimization. In this paper, a recently developed multiobjective optimization based framework balance the global exploration and local exploitation in EGO, called EGO-MO, is introduced. It can generate multiple test solutions simultaneously to take the advantage of parallel computing. The EGO-MO is applied for the aerodynamic shape design of a transonic airfoil to minimize drag maintaining the reference lift. The class/shape transformation (CST) method is employed for the parameterization of airfoil. The open source code SU^2 is adopted to perform the high-fidelity aerodynamic analysis of initial and infill sampling points. The comparison of EGO-MO and standard EGO for the transonic airfoil problem is presented. The investigation shows that the EGO-MO feature less iteration numbers and can give better optimal results.

Keywords: aerodynamic shape optimization; efficient global optimization; multi-point infill sampling criteria; multiobjective optimization; MOEA/D

Introduction

In the optimization design of aerodynamic shape of a flight vehicle [1, 2] or airfoil [3, 4], the objective function evaluation is done via high-fidelity and expensive computational fluid dynamics (CFD) simulation. In spite of great development of computing technology, such as the more accurate and faster simulation code and parallel computing, the efficient optimization method is still an open research area. Modern heuristics are not suitable since these methods often require an unbearable number of function evaluations.

Due to the importance of expensive optimization, much effort has been made for developing methods to produce a reasonably good solution within a given budget on computational cost or time [5, 6]. A widely used approach for dealing with expensive optimization utilizing high-fidelity analysis is to use cheap global surrogate (approximation) models to substitute expensive simulation. Kriging is have been widely applied to many expensive optimization problems [7-10], since it can approximate nonlinear and multi-modal functions, and produce unbiased prediction at untested points. Actually, Kriging has been. Efficient Global Optimization (EGO) is the most popular Kriging based expensive optimization method [5].

The infill sample selection criterion is an important issue for Kriging based optimization, such as EGO. In the original EGO or several aerodynamic optimization problems, one test points for evaluation is determined at each iteration. To make good use of parallel computing resources, a multiobjective optimization based framework has been proposed to balance the exploration and exploitation for expensive optimization problem, called EGO-MO [11]. It treats balancing the local exploitation and global exploration as a multiobjective optimization problem (MOP). Then, a multiobjective optimization algorithm can be used for obtaining the Pareto set, i.e., a set of best trade-off solutions for balancing exploitation and exploration. Several points can be selected from the Pareto set for evaluation in a parallel manner. In such a way, parallel computing techniques can be used for reducing the clock time for optimization. Additionally, the Multiobjective Evolutionary Algorithm based on Decomposition (MOEA/D) [12] is employed to solve the aforementioned MOP. Due to the population nature of MOEA/D, it is able to escape from local optimal solutions and give a set of high quality trade-off candidates for local exploitation and global exploration.

In this paper, the EGO-MO is applied to the aerodynamic shape optimization problem, more exactly the airfoil shape optimization. The remainder of this paper is organized as follows. Section 2 introduces the Kriging and the

EI criterion. Section 3 presents the basic idea and framework of EGO-MO. Section 4 presents the airfoil shape optimization result. The comparison between of original EGO and EGO-MO for airfoil shape optimization problem is also presented in this section. Finally, Section 5 concludes the paper.

1. Kriging and Expected Improvement

1.1 Gaussian Stochastic Process Modeling

Let

$$y = g(x), x \in R^n \tag{1}$$

be the objective to minimize. We assume that the function value evaluation of $g(x)$ is expensive. To construct a Gaussian stochastic process (Kriging) model for $g(x)$, K sample points $x^1,...,x^K \in R^n$ and their function values (responses) $y^1,...,y^K$ are required. Suppose

$$y = \mu + \varepsilon(x) \tag{2}$$

where μ is a constant and $\varepsilon(x)$ is a Gaussian stochastic process with following properties:

$$E[\varepsilon(x)] = 0, \text{Var}[\varepsilon(x)] = \sigma^2, \text{Cov}[\varepsilon(x^i), \varepsilon(x^j)] = \sigma^2 [c_{ij}(x^i, x^j)] \tag{3}$$

where the correlation function $c(x^i, x^j)$ is assumed to depend only on x^i and x^j.

There are some different forms of correlation function such as the Gaussian function, exponential function, spherical function and spline function. The Gaussian function is used in this paper, which has the following form

$$c(x^i, x^j) = \exp\left[-d(x^i, x^j)\right] \tag{4}$$

where

$$d(x^i, x^j) = \sum_{k=1}^{n} \theta_k \left|x_k^i - x_k^j\right|^{p_k} \tag{5}$$

$\theta_i > 0$, and $1 \le p_i \le 2$.

The unknown hyperparameters in the above Gaussian stochastic process modeling are μ, σ, $\theta_i (i = 1,...,n)$, and $p_i (i = 1,...,n)$. Given K points $x^1,...,x^K \in R^n$ and their responses $y^1,...,y^K$, these hyperparameters can be estimated by maximizing the likelihood that $g(x)=y^i$ at sampled points $x = x^i (i = 1,...K)$. The details can be found in [5].

If the hyperparameter estimations $\hat{\mu}$, $\hat{\sigma}^2$, $\hat{\theta}_i$, and $\hat{p}_i (i = 1,...,n)$ are given, one can predict $g(x)$ at any untested point x based on the response values y^i at x^i for $i = 1, \ldots, K$.

The best linear unbiased predictor of $g(x)$ is as follows

$$\hat{y}(x) = \hat{\mu} + r^T C^{-1} (y - \mathbf{1}\hat{\mu}) \tag{6}$$

and its mean squared error is

$$s^2(x) = \hat{\sigma}^2 \left[1 - r^T C^{-1} r + \frac{\left(1 - \mathbf{1}^T C^{-1} r\right)^2}{\mathbf{1}^T C^{-1} r}\right] \tag{7}$$

where $r = \left[c(x, x^1), ..., c(x, x^K)\right]^T$. $N(\hat{y}(x), s^2(x))$ can be regarded as a predictive distribution for $g(x)$ given the response value y^i at x^i for $i = 1,...,K$.

1.2 Expected Improvement

After building a predictive distribution model for the objective function, we then define a metric for measuring the merit of evaluating a new untested point for minimizing $g(x)$.The expected improvement [5] is introduced in the following.

Let $N(\hat{y}(x), s^2(x))$ is a predictive distribution model for $g(x)$, and the minimal value of $g(x)$ over all the evaluated points is $g_{\min}$, then the improvement of $g(x)$ at a untested point x is

$$I(x) = \max\left\{g_{\min} - g(x), 0\right\} \tag{8}$$

Thus, the expected improvement (EI) can be calculated as

$$E\left[I(x)\right]=\begin{cases}\left[\left(g_{\min}-\hat{y}(x)\right)\right]\Phi\left(\dfrac{g_{\min}-\hat{y}}{\hat{s}(x)}\right)+\hat{s}(x)\phi\left(\dfrac{g_{\min}-\hat{y}}{\hat{s}(x)}\right) & \hat{s}>0\\ 0 & \hat{s}=0\end{cases} \tag{9}$$

The above formula has two terms. The first term prefers points whose prediction values are small and have low uncertainty. It reflects the local exploitation. The second term is the product of the error s multiplying the probability density function. It reflects the global exploration. Therefore, the EI can be considered as a balance between exploiting promising areas of the design space and exploring uncertain areas [13].

2. Algorithm Framework for EGO-MO

2.1 The Basic Idea

The basic idea of EGO-MO is to use a multiobjective optimization algorithm for finding a set of test points, which can balance exploitation and exploration in different ways. We set the first and second items of the EI as two objective functions. More specially, the MOP is:

$$\begin{aligned}&\max F(x)=\left(f_1(x),f_2(x)\right)^T\\&\text{subject to}\quad x\in\Omega\end{aligned} \tag{10}$$

where $f_1(x)=\left[\left(g_{\min}-\hat{y}(x)\right)\right]\Phi\left(\dfrac{g_{\min}-\hat{y}}{\hat{s}(x)}\right)$, $f_2(x)=\hat{s}(x)\phi\left(\dfrac{g_{\min}-\hat{y}}{\hat{s}(x)}\right)$.

It should be pointed out that any other two metrics reflect global exploration and local exploitation can also be used as two objectives in our approach. The Pareto optimal solutions of the above problem should be good candidate points for evaluation. Due to its simplicity and efficient, the MOEA/D [12] is used to solve the above MOP.

Let the objective to minimize is $g(x)$, our proposed EGO-MO works as follows.

Algorithm Parameters:

K_I: the number of initial points in **Initialization**;

K_E: the number of function evaluations at each generation.

Step 1 **Initialization**: Generate K_I points $x^1,\dots,x^{K_I}$ from the search space by using an experiment design method and evaluate the function values of these K_I point. Set $P_{\text{eval}}=\left\{x^1,\dots,x^{K_I}\right\}$.

Step 2 **Stopping Condition**: If a predetermined stopping condition is met, output the minimum function value in P_{eval} as an approximation to the optimum and stop.

Step 3 **Model Building**: By using the function values of the points in P_{eval}, build a predictive distribution model for objective function $g(x)$.

Step 4 **Locating Candidate Points**: Using MOEA/D, solve Problem (10) and obtain the Pareto optimal solutions $x^1,\dots,x^N$.

Step 5 **Selecting Points for Function Evaluation**: Select K_E points from $x^1,\dots,x^N$ using a selection scheme.

Step 6 **Function Evaluation**: Evaluate the function values of all the K_E selected points in **Step 5**, then add all these points to P_{eval} and go to **Step 2**.

In **Step 2**, the stopping condition is:

$$\max_{x\in x^1,\dots,x^N} E\left[I(x)\right]/\left(Y_{\max}-Y_{\min}\right)\le\varepsilon_r \tag{11}$$

where $Y_{\max}$ and $Y_{\min}$ are the maximum and minimum of evaluated function values, $\max_{x\in x^1,\dots,x^N} E\left[I(x)\right]$ is the maximum of expected improvement of the Pareto optimal points $x^1,\dots,x^N$, ε_r is a predefined convergence threshold.

2.2 MOEA/D for Locating Candidate Points

To solve Problem (10), MOEA/D first decomposes it into N single objective optimization subproblems. The objective in each subproblem is a weighted linear or nonlinear aggregation function of . f_1 and f_2. Thus, each subproblem is associated with a weight and its optimal solution is a Pareto optimal solution to Problem (10). If the decomposition is done properly, then the optimal solutions to these subproblems will provide a good approximation to the Pareto front of (10). MOEA/D makes use of the neighbourhood relationship among these subproblems and optimizes all the subproblems simultaneously.

2.3 Selecting Points for Function Evaluation

In order to select K_E points from N candidate solutions obtained by MOEA/D, we have the following considerations.

1) The point to be evaluated should be different from the points already evaluated.

2) The selected points should be evenly distributed on the PF. Therefore, if two solutions x^i and x^j whose weight vectors are close, they cannot both be selected.

3) The selected points should be uniformly filling the design space with the evaluated points. The points maximizing the minimum distance to the evaluated points are preferred.

3. Airfoil Shape Optimization

3.1 Optimization Problem

In the airfoil shape optimization problem, the drag is minimized while the lift is maintain the same level of the base airfoil (RAE2822[14]).

$$\begin{aligned} &\text{minimize} \quad C_d \\ &\text{subject to} \quad C_l \geq C_{l_{\text{Base}}} \end{aligned} \tag{1.12}$$

The flight condition used for design optimization is set as follows: (1) The Mach number is 0.729; (2) The angle of attack of 2.31 degree; (3) The freestream temperature is 288.15K; (4) The Reynolds number is 6.5 million.

3.2 Design variable

The airfoil shape is deformed using the class/shape transformation (CST) method [15]. In the CST method, the airfoil shape is represented by the combination of the class function and the shape function, as follows

$$\xi(\psi) = C_{N_1}^{N_2}(\psi) S(\psi) + \psi \Delta\xi_{te} \tag{1.13}$$

where $\xi = \frac{z}{c}$, $\psi = \frac{x}{c}$, $C_{N_1}^{N_2}(\psi) = \psi^{N_1}(1-\psi)^{N_2}$ is the class function, and $S(\psi) = \sum_{i=0}^{N}\left[A_i \psi^i\right]$ is the shape function, $\Delta\xi_{te}$ is the trailing-edge thickness ratio. For general airfoils, the class parameters N_1 and N_2 are set to 0.5 and 1.0, respectively. The Bernstein polynomial is employed as the shape function to describe the detailed shape. The airfoil shape can be represented using the Bernstein polynomial with different weight coefficients. These weight coefficients are then employed as design variables in optimization.

Fourteen weights are used in this paper, i.e., seven weights are placed on each lower and upper surface. The weights are allowed to vary in range of not distorting or disturbing the grid deformation as described in Table 1. The slopes of airfoil described by the upper and lower bounds of variables are presented in Figure 1.

Table 1: The bound of design variable

No.		1	2	3	4	5	6	7
Lower surface	Upper	-0.10	-0.10	-0.10	-0.15	-0.01	-0.10	0.10
	Lower	-0.16	-0.16	-0.17	-0.30	-0.05	-0.20	-0.02
Upper surface	Upper	0.16	0.16	0.20	0.20	0.30	0.20	0.30
	Lower	0.10	0.10	0.10	0.15	0.16	0.12	0.18

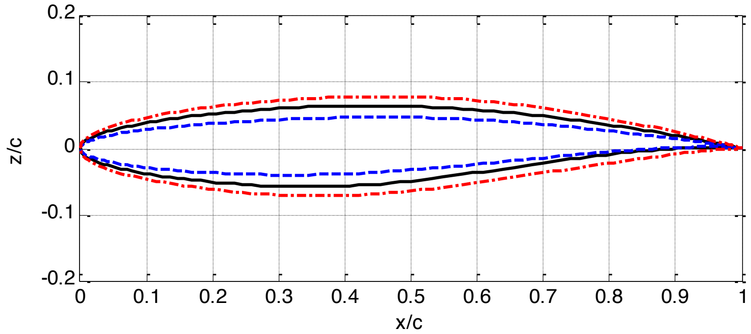

Figure 1: The bounds of the airfoil shape

3.3 Aerodynamic analysis

Although the expensive simulation is replaced with the Kriging model, the numerical experiments on sampling data are still required. The design condition is considered to be compressible flow. In the airfoil shape optimization problem, the Stanford University Unstructured (SU^2) software suite [16] is employed as the high-fidelity computational fluid dynamics (CFD) simulation tools. The governing equation of viscous compressible flow is adopted for aerodynamic analysis. Spatial numerical flux is discretized using JST scheme. Viscous flux is discretized using the 1^{st} upwind differencing method with SA turbulence modeling. The mesh used is an unstructured, O-grid that wraps around the RAE 2822 airfoil. It has 22,842 elements in total with 192 edges making up the airfoil boundary and 40 edges along the far-field boundary. The first grid point of the airfoil surface is at a distance of 1E-5 chords, and the far-field boundary is located approximately one hundred chord lengths away from the airfoil. The grid deformation module in SU^2 is used to deform the grid system, which employs the linear elasticity equations. The computational capacity of SU^2 for the viscous analysis of RAE2822 has been demonstrated in[14]. It is appropriate for being used in design procedure.

3.4 Settings for optimization

The first Kriging model is built on 105 initial sampling test points, which are generated using the optimized Latin hypercube sampling method [17]. The parameter for stopping condition is set as $\varepsilon_r = 1\times10^{-6}$. The multiobjective evolutionary algorithm based on decomposition (MOEA/D) is employed to solve the multiobjective problem that balance exploitation and exploration and locate the candidate points. The number of function evaluation at each generation is 3. Parameters used in MOEA/D are set as follows: a) the number of subproblems *N* is 300; b) the number of generation is 300; c) the number of neighborhood is 20; d) Aggregation method: Tchebycheff approach.

3.5 Design Result

The initial Kriging model is built by using 105 initial points, the optimization stopped by 11 iterations for EGO-MO, and 23 iterations for standard EGO. Although the iteration number of EGO-MO is less than the standard EGO, its function evaluation number, 33, is larger than standard EGO.

The optimum geometry found by EGO-MO and standard EGO is depicted in Figure 2. The wall pressure distribution and pressure field contour is illustrated in Figure 3 and 4. As can be seen in Figure 3 and 4, airfoil obtained by standard EGO is optimized such that shape from the stagnation point to the maximum thickness point is changed gradually on the upper surface; and the maximum thickness point is moved toward the trailing edge. However, the EGO-MO found a more slender shape than the base.

The aerodynamic performance of optimum airfoil is presented in Table 2.

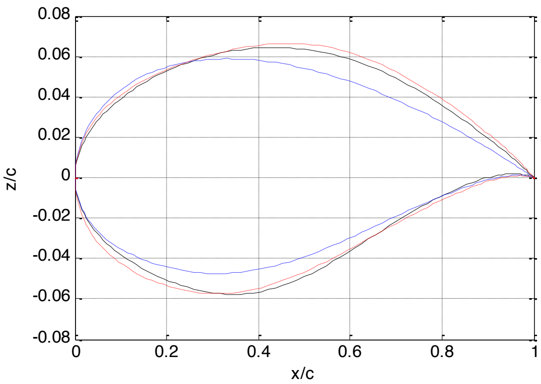

Figure 2: Optimum airfoil shape

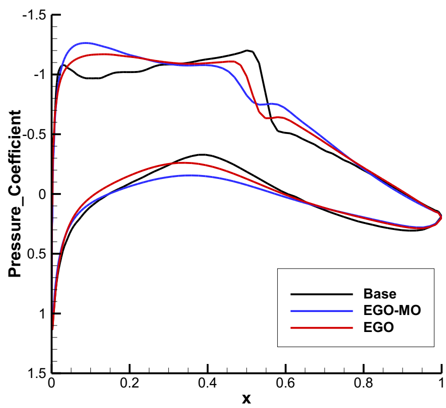

Figure 3: Wall pressure distribution

Table 2: The aerodynamic performance of optimum airfoil

		Lift Coefficient	Drag Coefficient	L/D
Base		0.72377	0.013453	53.8
EGO-MO	Optimum	0.784424	0.0114207	68.6843
	Improvement	+8.38%	-15.11%	+27.67%
Standard EGO	Optimum	0.735569	0.0118942	61.8425
	Improvement	+1.63%	-11.59%	+14.95%

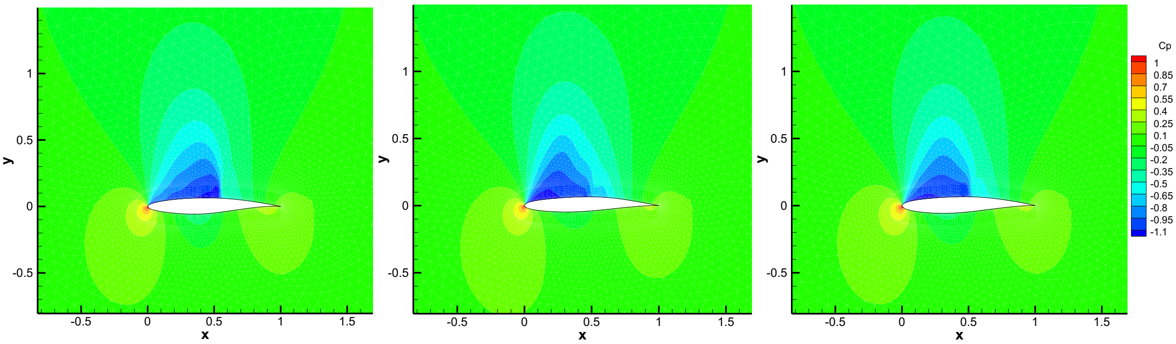

Figure 4: Pressure field comparison (left: baseline, middle: optimum of EGO-MO, right: optimum of standard EGO)

4. Conclusions

In the optimal design of the aerodynamic shape of an airfoil using high-fidelity computational fluid dynamics (CFD) simulation, objective function evaluations can be extremely computationally expensive. In this paper, a recently developed multi-point infill sampling criteria for EGO, called EGO-MO, is applied for the optimization of the transonic airfoil shape. In the EGO-MO, the multiobjective optimization based framework is employed for multi-point infill sampling criteria to enhance the efficiency of the standard EGO. The Kriging and EI infill sampling criteria is introduced firstly. Afterwards, the basic idea and the algorithm framework of EGO-MO are briefly presented. In the airfoil shape optimization problem, the objective is to minimize drag maintaining the reference lift for the transonic airfoil rae2822. The class/shape transformation (CST) function is employed for the parameterization of the airfoil. The open source code SU^2 is adopted to perform the high-fidelity aerodynamic analysis of initial and infill sampling points. The EGO-MO and standard EGO is applied to solve the aforementioned airfoil optimization problem. The two methods are compared for the function evaluation number and the iteration number. The optimum results are also analyzed for the mechanism of the drag reduction.

Acknowledgements

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