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Damage Detection Method in Non-Destructive Testing Based on Topology Optimization and Eigenvalue Analysis

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Abstract

Non-destructive testing detects damage according to a difference in a physical phenomenon between a normal structure and damaged structure. However, the accuracy of such damage detection typically depends on the skill of the engineer. As a solution, a numerical method of detecting damage to a structure based on a dynamical numerical model such as a finite element model was proposed . This method automatically derives a structure with a response that is equal to that of a damaged structure employing an optimization algorithm. The procedure can be applied to not only non-destructive testing but also automatic structural health monitoring. Among structural optimization methods, topology optimization can optimize the structure fundamentally by changing the topology and not just the shape of a structure. We thus employ topology optimization for structural optimization . Damage detection using topology optimization based on frequency response analysis has been suggested. However, the eigenvalue-based technique that is traditionally used in damage detection has not been integrated with topology optimization. The present study thus examines a damage detection method using topology optimization based on eigenvalue analysis. Our method derives a structure that has the same eigenvalues as a damaged structure employing topology optimization and can identify a damaged structure.
Keywords: topology optimization, eigenvalue analysis, non-destructive testing, sensitivity analysis

1 Introduction

Non-destructive testing is important for improving the life span of structures. Different non-destructive testing methods employ ultrasonic waves, eddy current, piezoelectric sensors, lightwave fibers and other phenomena and devices [1]. In non-destructive testing, damage is detected according to the difference between responses for a normal structure and damaged structure. Because the process of specifying damage is usually performed by an engineer, the accuracy of damage detection depends on the skill of the engineer. To establish an identification method that does not depend on the skill of the testing engineer, methods based on a database of damaged structures have been proposed. However, such methods cannot be applied to structures with an innovative shape or in cases of unexpected damage.

In contrast, analytical methods of damage specification have been proposed [2, 3]. Such methods specify a damaged structure by non-destructive testing employing a numerical calculation based on a dynamical model and optimization algorithm.

However, there has been a recent focus on topology optimization as a novel structural optimization method [4], which fundamentally optimizes the structural shape[5, 6, 7]. Moreover, two studies on topology optimization in terms of damage detection methods of non-destructive testing have been published. Lee et al. [8] developed a damage identification method based on the difference in the frequency response between damaged and undamaged structures. Niemann et al. [9] verified this approach experimentally in the damage detection of a composite. These methods derive the damaged structure by matching the frequency response of the optimized structure to that of the damaged structure corresponding to the detected response in actual non-destructive testing. However, only these two studies have reported an analytical damage detection method employing topology optimization. Other vibration characteristics such as the structural eigenfrequency, which is the most fundamental vibration characteristic, have not been used in such a way.

The present study establishes a damage detection method using eigenfrequency analysis and topology optimization. Specifically, this study derives an approximated shape of the damaged structure by matching the eigenfrequencies of the optimized structure with those of the damaged structure corresponding to the detected eigenfrequency in actual non-destructive testing through a structural topology optimization procedure. In chapter 2, we formulate a topology optimization problem to minimize the difference in eigenvalues between a damaged structure and the current structure. In chapter 3, an optimization algorithm based on sensitivity analysis and a method of moving asymptotes (MMA)[10] is constructed. In chapter 4, we clarify the validity and utility of the proposed methodology using numerical examples.

2 Formulation

2.1 Eigenvalue Analysis Employing the Finite Element Method

This study employs the finite element method for the eigenvalue analysis of structures. A free vibration problem of a structure is first discretized by finite element analysis. A discretized vibration equation is obtained by assuming that the solution is a periodic displacement $\mathbf{\Phi}e^{i\omega t}$:

$$(\mathbf{K}-\omega_n^2\mathbf{M})\mathbf{\Phi}=0, \tag{1}$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, ω_n denotes the natural angular frequencies and $\mathbf{\Phi}$ is the amplitude vector. $\mathbf{\Phi}$ is obtained as eigenvectors by solving the above equation as an eigenvalue problem.

2.2 Topology Optimization

A fundamental idea of topology optimization is the introduction of the design domain D and characteristic function χ_Ω. That is, the optimization problem is replaced with a material distribution problem of the characteristic function χ_Ω on the design domain D:

$$\chi_\Omega(\mathbf{x})=\begin{cases}1 & \text{if} \quad \mathbf{x}\in\Omega_d\\ 0 & \text{if} \quad \mathbf{x}\in D\setminus\Omega_d\end{cases} \tag{2}$$

Through this process, the set of χ_Ω in the fixed design domain D is regarded as an optimal structure.
However, optimization based on the above equation evaluates the discontinuous function χ_Ω in the fixed design domain D. This means that a discontinuous value is evaluated about an infinite design variable. Unfortunately, an optimum solution does not exist in this case. This issue is resolved by replacing the optimization problem with respect to a characteristic function with an optimization problem with respect to a continuous density function. As this relaxation method, a homogenization method and the solid isotropic material with penalization (SIMP) method have been proposed [11]. These methods regard the approximated optimization problem to be a problem of the volume fraction of a composite material comprising the host material and a very weak material that imitates a hole. A gray domain in which it is difficult to identify a structure or hole is often observed in this optimization problem. Because the SIMP method can adjust the nonlinearity of the relationship between the density function and physical property by penalization parameters, it is used for topology optimization in the present study. Young's modulus and material density are correlated with the density function $\mathbf{d}=[d_1,d_2,...d_l]$, where l is the number of optimized elements:

$$E_i^*=d_i^3E \text{ for } i=1,...,l, \tag{3}$$
$$\rho_i^*=d_i\rho \text{ for } i=1,...,l, \tag{4}$$

where E and ρ are Young's modulus and the mass density respectively. The superscript suffix * indicates a physical property that is expressed by the density function $\mathbf{d}$ in optimization. The exponent of the design variable is freely set in the SIMP method. We use a Young's modulus of 3 and material density of 1, because these values have been effective in past work [5][6].

2.3 Objective Function and Optimization Problem

We premise that the eigenvalue and eigenmode of the damage structure are obtained, and we detect damage by deriving a structure that has the same eigenvalue employing optimization. This study supposes a simple structure that has non-repeating lower eigenvalues and eigenmodes. To avoid repeating eigenvalues during optimization, we set an objective function that is the square of the rate of change between the designated eigenvalue and current eigenvalue [5]:

$$J(\mathbf{d}) = \sum_{i=1}^{m} w_i \frac{(\lambda_{\text{target}i} - \lambda_i)^2}{\lambda_{\text{target}i}^2}, \tag{5}$$

where $\lambda = \omega_n^2$ is the eigenvalue, $\lambda_{\text{target}i}$ is the target of the i-th eigenvalue, λ_i is the i-th eigenvalue, m is the number of eigenvalues used in optimization and w_i denotes the weighting factor of each eigenvalue. The topology optimization problem in this study is then expressed as

$$\underset{\mathbf{d}}{\text{minimize}}\, J(\mathbf{d}) \tag{6}$$

Subject to

$$0 < d_i \leq 1 \text{ for } i = 1, \ldots, l. \tag{7}$$

3 Optimization Procedure

3.1 Optimization Algorithm

Figure 1 is a flowchart of the optimization procedure used in the present study. This study first sets a damaged structure as the target structure and calculates the eigenvalue using the vibration equation. The obtained eigenvalues are then set as the target eigenvalues in the objective function of Eq.(5). These values imitate the eigenvalues of the damaged structure obtained in actual non-destructive testing. The optimization calculation is then repeated until the objective function converges. We calculate the eigenvalues and eigenvectors by solving the vibration equation of the optimized structure. This method needs to check whether the order and shapes of eigenmodes of the optimized structure correspond to those of the damaged structure. When the order of eigenvalues and eigenmodes of the optimized structure does not correspond to that of the damaged structure, eigenvalues of appropriate order are selected in the objective function. After calculating the objective function, the optimization finishes when the objective function converges. When the objective function does not converge, the method calculates the sensitivities of the objective function and updates the design variable using the MMA.

4 Numerical Examples

We demonstrate the utility and validity of the proposed method in numerical example. Example model is of structures that support a heavy load because their eigenvalues are readily affected by damage. The heavy load is modeled as a non-structural mass. Physical properties and size are handled as dimensionless quantities and all example models are of virtual material with Young's modulus $E = 1$ and mass density $\rho = 1$. The mechanical models of the example is two-dimensional plane stress model. We use the commercially available finite element analysis software Comsol Multiphysics for FEM. Each element is formulated using a first-order isoparametric element. Design variables are defined on each finite element.

This example is the optimization of a 2-by-1 rectangular plate supporting a distributed mass on the right side with in-plane stiffness. The left side is fixed and the distributed non-structural mass having total mass of 1 is set on the right side. The domain is discretized by square elements in an 80-by-40 configuration. The target damaged structure is set to have a semicircular hole at the center of the left side as shown in fig. 3.

After preliminary analysis and optimization of the target structure, we decided to include the first to fifth eigenvalues in the objective function because they are at least required for the smooth convergence of the optimization.

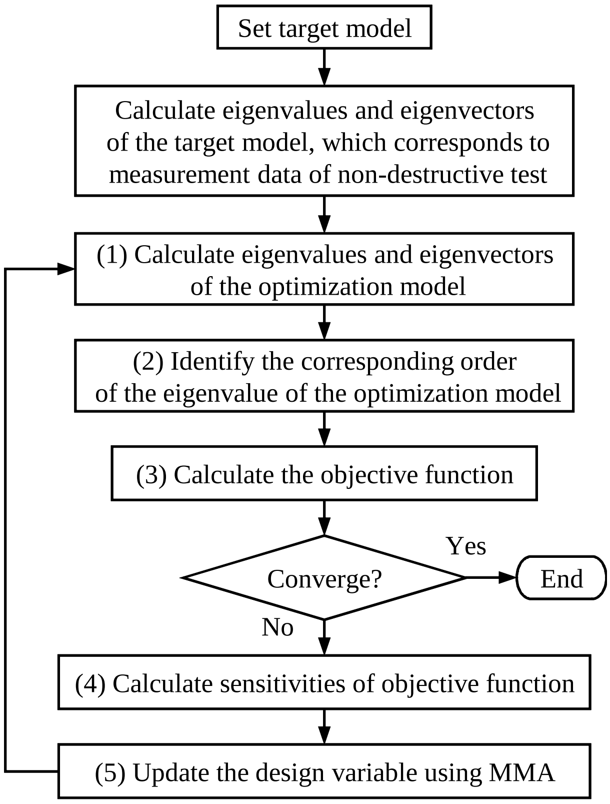

Figure 1: Flowchart of the optimization procedure.

Figure 4 shows the first to fifth eigenmode shapes. The damage at the center of the left side might affect tensile vibration more strongly than bending vibration. All weighting coefficients of eigenvalues in Eq.(5) are 1.
Figure 5 shows the convergence history of optimization. We regard that the objective function has converged after 500 cycles . Figure 6 shows the resulting structure of optimization. The eigenvalues are given in Table 1. During optimization, there was no eigenmode switching and correspondence of the mode shape between the optimized and target structures were maintained. The optimal structure has a slit-like void at the center of the left side. Although the identification of the damage shape is not perfect, it can be said that the damage position was detected from Fig. 6. We confirm that the optimized and damaged structures have almost identical eigenvalues in Table 1 and can say the damage identification was achieved through eigenvalue matching.

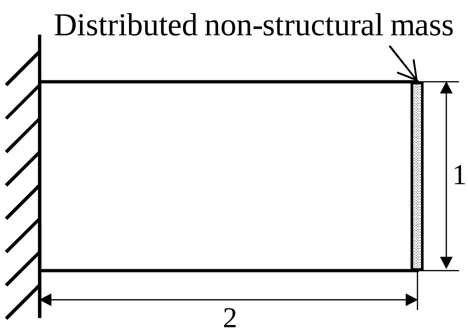

Figure 2: Analysis model of the cantilever example.

Figure 3: Damaged model of the cantilever example.

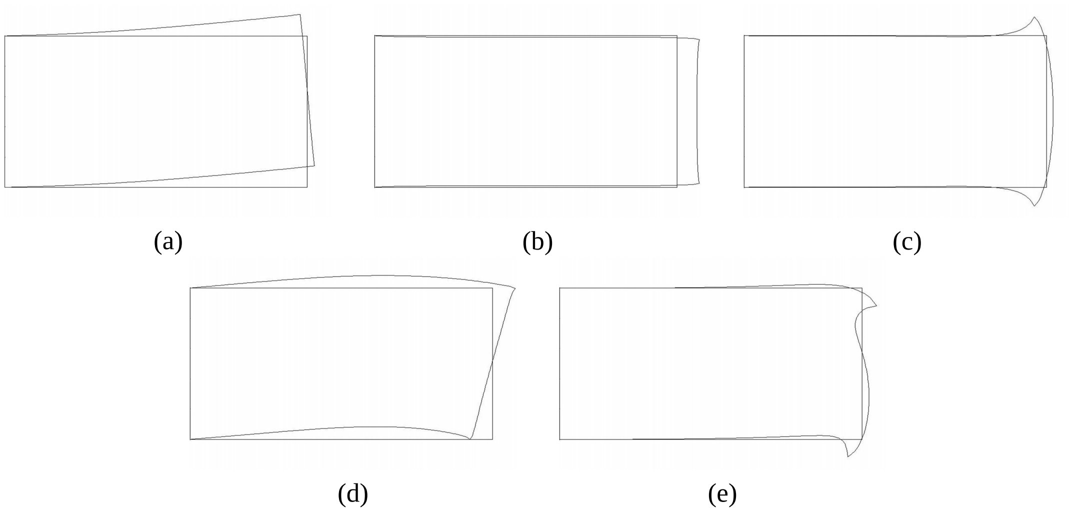

Figure 4: Eigenmode shapes of the damaged structure of the cantilever example. (a) First mode, (b) second mode, (c) third mode, (d) fourth mode, (e) fifth mode.

Table 1: Eigenfrequencies of optimal and damaged structures of the cantilever example (Hz).

Order	1st	2nd	3rd	4th	5th
Damaged structure	2.01×10^{-2}	4.20×10^{-2}	4.98×10^{-2}	5.39×10^{-2}	6.00×10^{-2}
Optimal structure	2.01×10^{-2}	4.20×10^{-2}	4.98×10^{-2}	5.39×10^{-2}	6.00×10^{-2}
Error(%)	0.08	0.01	0.01	0.01	0.01

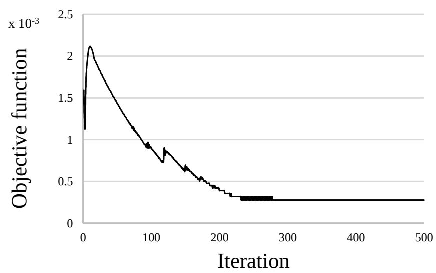

Figure 5: Convergence history of the cantilever example.

5 Conclusions

We developed a damage detection method that designates the eigenvalue of a damaged structure by non-destructive testing and derives a structure having eigenvalues identical to those of the damaged structure automatically through topology optimization. For this method, we set a minimization problem of the difference in eigenvalues between the damaged structure and optimized structure, and construct an optimization algorithm employing sensitivity anal-

Figure 6: Optimal configuration of the cantilever example.

ysis and the MMA. Although identification of the damage shape was not perfect, it was demonstrated that detection of the damage position using the proposed method is possible in several numerical example.

We believe that the application of the proposed method requires only the measuring of the eigenvalue by hammering when eigenmodes are the same for the damaged structure and undamaged structure as in the first example. However, when eigenmodes of the undamaged structure are different from those of the damaged structure, it is necessary to solve not only the eigenvalue but also the eigenmode. In that case, eigenmode optimization should be introduced in damage identification. This approach can contribute also to specification of the damage shape. Of course, effective methods of measuring the eigenmode shape must be studied for practical use in that case.

Because the proposed method is based on eigenvalue analysis, target structures are limited to simple examples. However, various techniques of non-destructive damage detection such as detection using ultrasonic waves or eddy current could be integrated with the topology optimization framework. We will investigate such an extension in future work.

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