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Optimization of structural topology using unstructured Cellular Automata

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1. Abstract
For a few decades topology optimization has been one of the most important aspects of structural design. One of the most important issues stimulating permanent development of this research area is implementation of efficient and versatile methods for generation of optimal topologies. Many modern computational techniques are nowadays invented so as to perform similarly to biological systems. They have gained widespread popularity among researchers because they are easy for numerical implementation, do not require gradient information, and one can easily combine this type of algorithms with any finite element structural analysis code. Among biologically inspired methods, which have recently aroused interest of designers one can find also Cellular Automata (CA). The idea of Cellular Automata is to replace a complex problem by a sequence of relatively simple decision making steps. In engineering implementation of Cellular Automaton the design domain is decomposed into a lattice of cells, and a particular cell together with cells to which it is connected form neighborhood. It is assumed that the interaction between cells takes place only within the neighborhood, and the states of cells are updated synchronously in subsequent time steps according to some local rules. In recent years the Cellular Automata concept has been successfully applied to structural topology optimization problems. The majority of results that have been obtained so far were based on regular lattices of cells. Practical engineering analysis and design require however using, in many cases, highly irregular meshes for complicated geometries and/or stress concentration regions. The aim of the present paper is to extend the concept of Cellular Automata towards implementation of unstructured grid of cells related to non-regular mesh of finite elements. Introducing irregular lattice of cells allows to reduce number of design variables without loosing accuracy of results and without excessive increase of number of elements caused by using fine mesh for a whole structure. It is worth noting that the non-uniform density of finite elements can be, but not necessary is, directly related to design variables which are related to cells of Cellular Automaton. The implementation of non-uniform cells of Cellular Automaton requires a reformulation of standard local rules, for which the influence of neighborhood on current cell is independent of sizes of neighboring cells.
2 . Keywords: topology optimization, Cellular Automata, unstructured mesh.

3. Introduction
Topology optimization of structures is a permanently developing research area. Since the early paper by Bendsoe and Kikuchi [1] one can find in the literature numerous approaches to generating optimal topologies based both on optimality criteria and evolutionary methods. A general overview as well as a broad discussion on topology optimization concepts are provided by many survey papers e.g. [10], [12]. At the same time hundreds of papers present numerous solutions including classic Michell examples as well as complicated spatial engineering structures, implementing specific methods ranging from gradient based approaches to evolutionary structural optimization, biologically inspired algorithms, material cloud method, spline based topology optimization and level set method. It is a permanently developing area and one of the most important issues stimulating this progress nowadays is implementation of efficient and versatile methods to generation of optimal topologies for engineering structural elements. In recent years the Cellular Automata paradigm has been successfully applied to topology optimization problems. In engineering implementation of Cellular Automaton the design domain is decomposed into a lattice of cells, and a particular cell together with cells to which it is connected form neighborhood. It is assumed that the interaction between cells takes place only within the neighborhood, and the states of cells are updated synchronously according to some local rules. The first application of CA to optimal structural design, and to topology optimization in particular, was proposed by Inou et al. [5]. The idea of implementation of CA to optimal design was described also by Kita and Toyoda [6]. During the last two decades implementation of CA in structural design has been under permanent development, and numerous papers related to application of CA to topology optimization, see e.g. [2], [3], [4], [8], [11] or [14], have been published.

The majority of structural topology optimization results that have been obtained so far were based on regular lattices of cells, among which the most common choice is a rectangular grid. One can find only isolated examples of implementation of triangular or hexagonal lattices. Practical engineering analysis and design require however using, in many cases, highly irregular meshes for complicated geometries and/or stress concentration regions. The aim of the present paper is to extend the concept of Cellular Automata lattice towards irregular grid of cells related to non-regular mesh of finite elements. The strategy which consists of resizing of traditional uniform grid of cells allows to obtain more flexible solutions. The advantage of using of non-uniform lattice of cells is the most evident, when the design domain is extremely irregular and it is even impossible to cover design domain with uniform e.g. rectangular cells. On the other hand, it is well known, that holes and sharp edges indicate stress concentration, and the regions of such intensity should be covered with a more fine mesh, what is not necessary for structure as a whole. In other words, a non-uniform density of cells is used in order to achieve a more accurate solution without excessive increase of number of elements caused by using fine mesh for a whole structure. It is worth noting that the non-uniform density of finite elements can be, but not necessary is, directly related to desity of cells of Cellular Automaton. The implementation of non-uniform cells of Cellular Automaton requires a reformulation of standard local rules, for which the influence of neighborhood on current cell is independent of sizes of neighboring cells and neglects for example the length of mutual boundaries. This paper proposes therefore new local update rules dedicated to implemented irregular lattices of cells. The novel concept is discussed in detail and the performance of the numerical algorithm based on the introduced idea is presented.

4. Unstructured Cellular Automata

Most of to date applications of Cellular Automata in structural optimization are conventionally based on regularly spaced, structured meshes. On the other hand using unstructured computational meshes provides more flexibility for fitting complicated geometries and allows local mesh refinement. Some attempts to implement unstructured Cellular Automata have been already reported in the literature e.g. [7], [9], but application to topology optimization is rather incidental (see [13]).

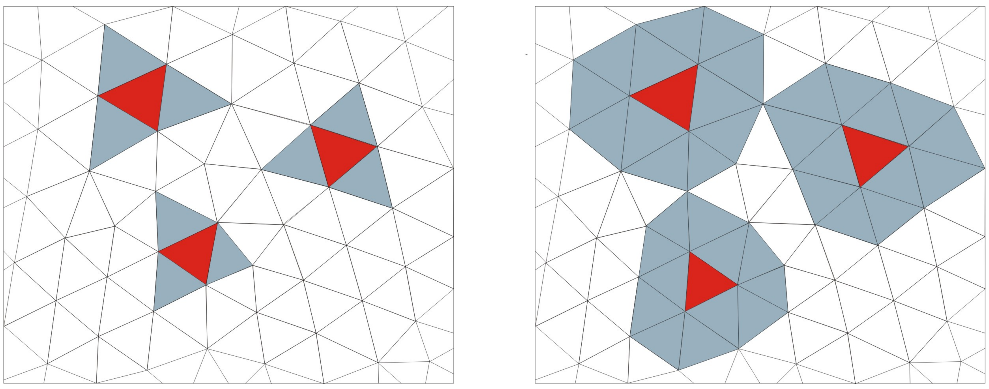

Figure 1: Unstructured triangular mesh. The von Neuman type neighborhood (left) and the Moore type neighborhood (right)

In this paper the concept of topology generator based on Cellular Automata rules is extended to unstructured meshes. Similar to structured (regular) Cellular Automata, several neighborhood schemes can be identified. The two most common ones are the von Neumann type and the Moore type. As can be seen in the Fig.1 in case of the von Neumann configuration only three immediate neighbors are taken into account. These neighboring cells share common edges with the central cell. In the Moore type neighborhood any triangle that has common edges or common vertices with the central cell can be considered as a neighbor of the central triangle. It is worth noting that this type of neighborhood involves more neighbors around the central cell, and the number of neighbors can vary since it depends on particular unstructured mesh arrangement.

5. The algorithm

The performance of Cellular Automata algorithms, reported in literature, is often based on heuristic local rules. Similarly, in the present paper the efficient heuristic algorithm, being extension of the one introduced by Bochenek and Tajs-Zielińska [2], [3], has been implemented. The power law approach defining solid isotropic material with penalization (SIMP) with design variables being relative densities of a material has been utilized. The elastic modulus of each cell element is modelled as a function of relative density d_i using power law, according to Eq.(1). This power p penalizes intermediate densities and drives design to a solid/void structure.

$$E_i = d_i^p E_0, \qquad d_{min} \leq d_i \leq 1 \tag{1}$$

The local update rule applied to design variables d_i associated with central cells is now constructed based on the information gathered from adjacent cells forming the Moore or von Neumann type neighborhood. It is set up as linear combination of design variables corrections with coefficients, the values of which are influenced by the states of the neighborhood surrounding each cell, as presented in Eq.(2):

$$d_i^{(t+1)} = d_i^{(t)} + \delta d_i, \qquad \delta d_i = (\alpha_0 + \sum_{k=1}^{N} \alpha_k) m = \tilde{\alpha} m \tag{2}$$

The compliance values calculated for central cell U_i and N neighboring cells U_{ik} are compared to a selected threshold value U^*. The quantities A_i and A_{ik} stand for areas of central and neighboring cells, respectively. Based on relations Eq.(3) and Eq.(4) specially selected positive or negative coefficients C_{α_0} for central cell and C_α for surrounding cells are transferred to the design variable update.

$$\alpha_0 = \begin{cases} -C_{\alpha_0} & \text{if} \quad U_i \leq U^* \\ C_{\alpha_0} & \text{if} \quad U_i > U^* \end{cases} \tag{3}$$

$$\alpha_k = \begin{cases} -C_\alpha & \text{if} \quad U_{ik} \dfrac{A_{ik}}{A_i} \leq U^* \\ C_\alpha & \text{if} \quad U_{ik} \dfrac{A_{ik}}{A_i} > U^* \end{cases} \tag{4}$$

The move limit m implemented in the above algorithm controls the allowable changes of the design variables values. The numerical algorithm has been build in order to implement the above proposed design rule. As to the optimization procedure the sequential approach, has been adapted, meaning that for each iteration, the structural analysis performed for the optimized element is followed by the local updating process. Simultaneously a global volume constraint can be applied for specified volume fraction. If so the generated optimal topology preserves a specified volume fraction of a solid material.

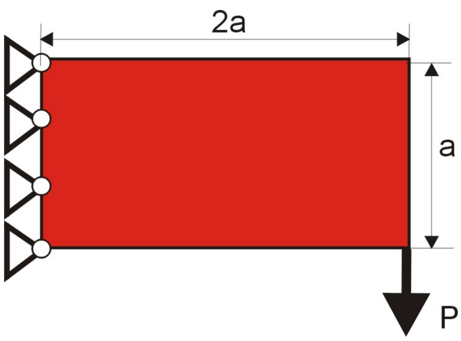

Figure 2: The rectangular Michell-type structure

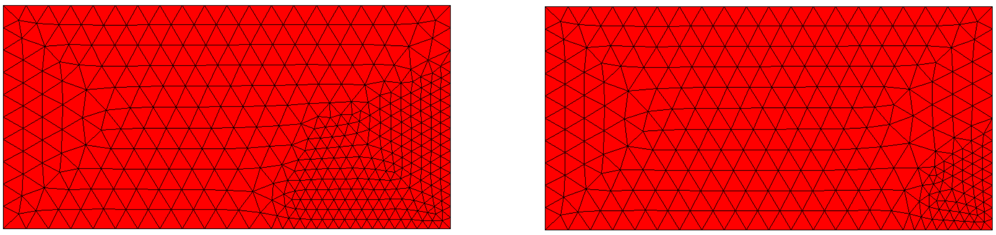

Figure 3: The rectangular Michell-type structure. Irregular meshing

6. Generation of optimal topologies

Selected examples of compliance-based topologies generated using the approach presented in this article are discussed in this section. The first one it is a rectangular Michell-type structure , clamped at the left edge and loaded

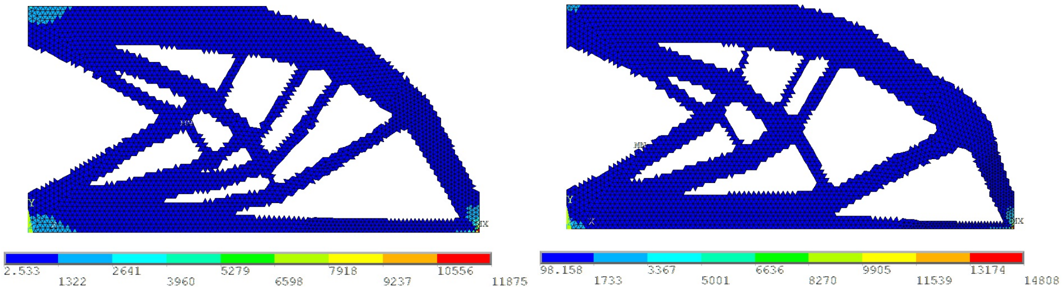

Figure 4: The rectangular Michell-type structure, force P=500 N, a=5m, material data E=200 GPa, ν=0.3. Final compliance: 8.58 10^{-5} Nm (left) and 8.62 10^{-5} Nm (right).

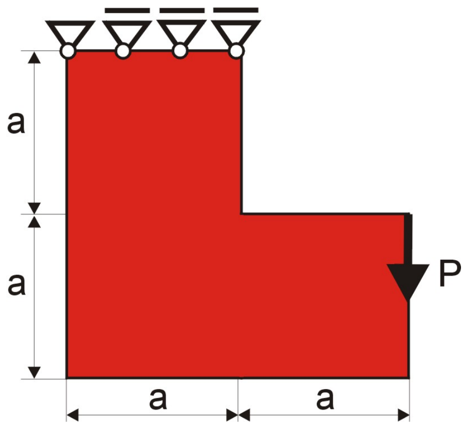

Figure 5: The L-shaped structure

by a vertical force applied at the bottom right corner, see Fig.2. The irregular mesh that consists of triangular elements/cells has been applied. The more dense mesh surrounds right bottom corner of the rectangle, as shown in the Fig.3. The two cases are considered, namely larger and smaller area of mesh concentration. The topology optimization has been performed and the obtained results are presented in the Fig.4, with final compliance 8.58 10^{-5} Nm for total number of cells 10594 and 8.62 10^{-5} Nm for 10439 cells, respectively. The latter case represents more dense mesh surrounding stress concentration region. Calculating von Mises stress gives maximal values of 11.9 kPa for less concentrated and 14.8 kPa for more concentrated mesh. It is worth noting that in order to reflect such stress values with regular meshes it is necessary to use 28056 and 42230 cells, respectively.

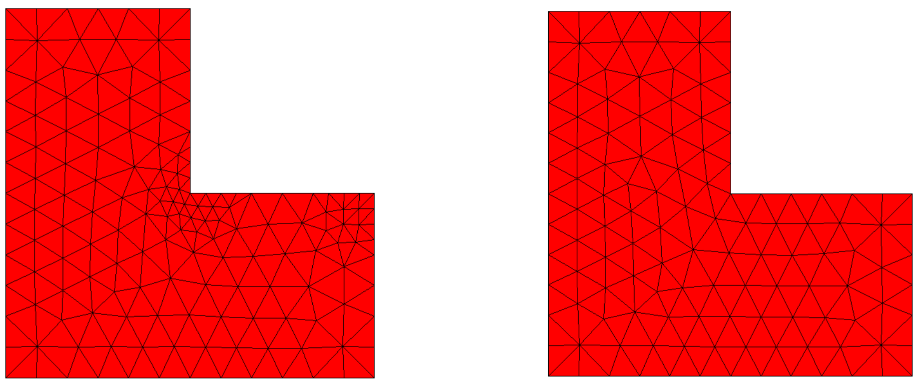

Figure 6: The L-shaped structure. Irregular (left) and regular (right) lattice of cells

The next example it is the L-shaped structure shown in the Fig.5. The unstructured mesh that consists of triangular elements/cells has been applied. The more dense mesh surrounds two corners within stress concentration

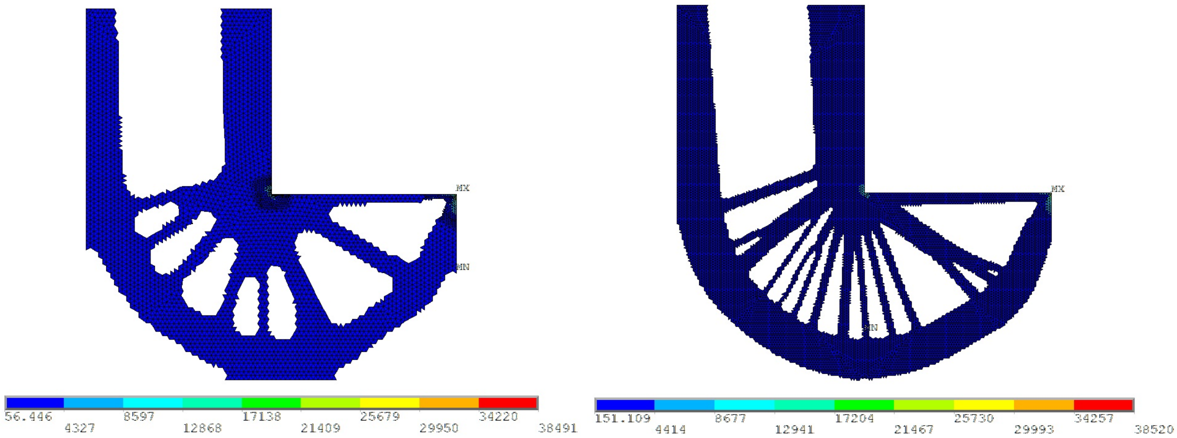

Figure 7: The L-shaped structure, force P=100 N, a=0.8 m, material data E=200 GPa, ν=0.3. Final compliance: Irregular lattice of 11396 cells 4.1 10^{-6} Nm (left) and regular lattice of 36490 cells 4.0 10^{-6} Nm (right), volume fraction 0.5

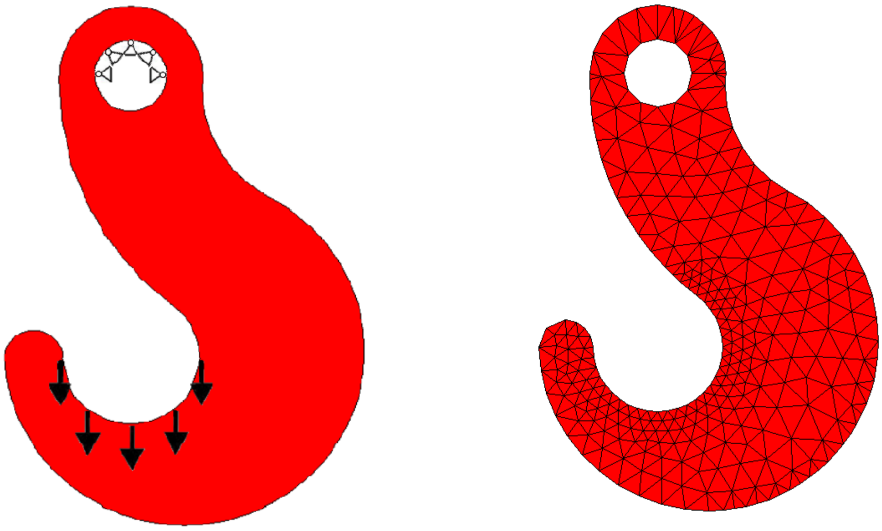

Figure 8: The Hook structure. Loading and support (left), irregular lattice (right)

regions as shown in the Fig.6. The exemplary regular mesh is presented as well. The topology optimization has been performed and the obtained results are presented in the Fig.7, with final compliance 4.1 10^{-6} Nm for irregular lattice of 11396 cells and 4.0 10^{-6} Nm for regular lattice of 36490 cells, respectively. Calculating von Mises stress gives maximal value of 38.5 kPa for both unstructured and regular mesh. It is worth noting that in order to obtain comparable results more than 3 times more cells for regular mesh were required.

The final example it is the Hook structure shown in the Fig.8. As for the previous cases, the irregular/unstructured mesh that consists of triangular elements/cells has been applied. The more dense mesh surrounds region of loading application. The minimal compliance topologies have been found for the considered structure and the obtained results are presented in the Fig.9 for two cases of volume fraction.

7. Concluding remark

The proposal of extension of Cellular Automata concept towards unstructured/irregular grid of cells related to non-regular mesh of finite elements has been presented. The subject is still under development but it seems that the approach presented in this paper demonstrates a significant potential of application to problems which cannot be adequately represented by regular grids. The use of unstructured meshes may be helpful while modelling a domain geometry, accurately specify design loads or supports and compute structure response.

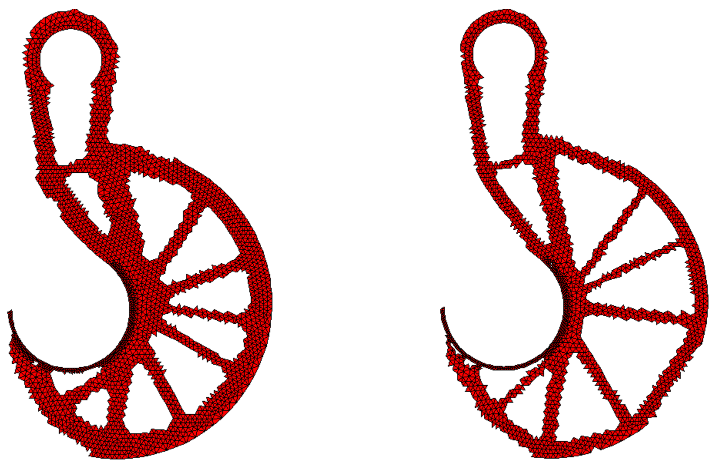

Figure 9: The Hook structure, loading q=1.67 N/mm, material data E=200 GPa, ν=0.25. Final topology, volume fracture 0.5 (left), volume fracture 0.35 (right)

8. References

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