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Topology optimisation of passive coolers for light-emitting diode lamps

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1. Abstract

This work applies topology optimisation to the design of passive coolers for light-emitting diode (LED) lamps. The heat sinks are cooled by the natural convection currents arising from the temperature difference between the LED lamp and the surrounding air. A large scale parallel computational framework is used to perform topology optimisation for minimising the temperature of the LED package subjected to highly convection-dominated heat transfer.

The governing equations are the steady-state incompressible Navier-Stokes equations coupled to the thermal convection-diffusion equation through the Bousinessq approximation. The fully coupled non-linear multiphysics system is discretised using stabilised trilinear equal-order finite elements and solved using Newtons method and a multigrid-preconditioned iterative method. Topology optimisation is carried out using the density-based approach.

The optimisation results show interesting features that are currently being incorporated into industrial designs for enhanced passive cooling abilities.

2. Keywords: topology optimisation, passive cooler, LED lamp, heat sink design, natural convection.

3. Introduction and motivation

The motivation for this work is the design of efficient and visually-pleasing passive coolers for LED lamps. LED lamps are a highly energy-efficient light source, however, it remains a problem to adequately cool them. This is a problem since around 70% of the energy supplied to an LED is converted to heat, which severely affects their lifespan unless effectively cooled. From an industrial design perspective, LEDs offer a large degree of design freedom since LED units are generally quite small and the passive cooling elements have the opportunity to make up the majority of the full lamp design as illustrated by figure 1.

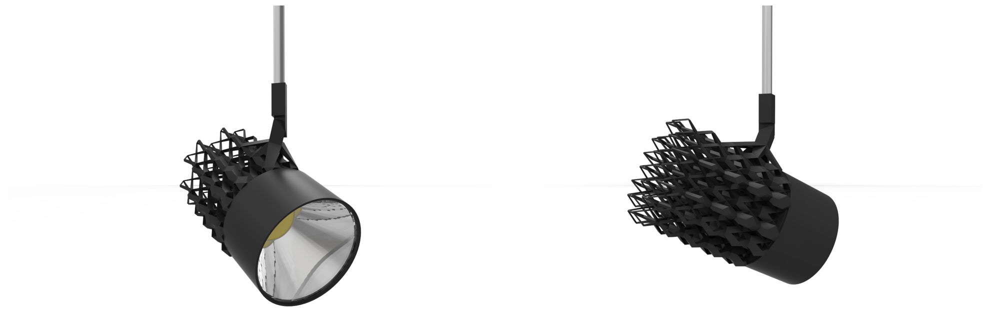

Figure 1: Design concept of a high-power LED spot with a 3D printed aluminium heat sink for passive cooling. Pictures are courtesy of AT Lightning Aps.

In order to fully utilise the design freedom and to allow for the appearance of non-intuitive designs, topology optimisation [1] is used. This is done using the density-based approach as detailed in [2] for two-dimensional natural convection problems. Despite the methodology being the same, the extension to three-dimensions has been far from trivial in the sense of the vast growth in computational workload. Topology optimisation for fluid systems began with the treatment of Stokes flow [3] and has since been applied to Navier-Stokes, as well as scalar transport problems. However, to the authors knowledge, this work is the first to treat a real-life application using a correct and coupled physical model.

4. Theory

4.1. Governing equations

The incompressible Navier-Stokes equations are coupled to the convection-diffusion equation through the Boussinesq assumption. The dimensionless equations are:

$$u_j \frac{\partial u_i}{\partial x_j} - Pr \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial p}{\partial x_i} = -\alpha(\mathbf{x}) u_i - Gr Pr^2 e_i^g T \tag{1}$$

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{2}$$

$$u_j \frac{\partial T}{\partial x_j} - \frac{\partial}{\partial x_j}\left(K(\mathbf{x}) \frac{\partial T}{\partial x_j}\right) = s(\mathbf{x}) \tag{3}$$

where u_i is the velocity field, p is the pressure field, T is the temperature field, x_i denotes the spatial coordinates, e_i^g is the unit vector in the gravitational direction, $\alpha(\mathbf{x})$ is the spatially-varying effective impermeability, $K(\mathbf{x})$ is the spatially-varying effective thermal conductivity, $s(\mathbf{x})$ is the spatially-varying volumetric heat source term, Pr is the Prandtl number, and Gr is the Grashof number.

The effective impermeability is set to 0 in the fluid subdomain and α_{max} in the solid subdomain. Likewise, the effective thermal conductivity is set to 1 in the fluid subdomain and $\frac{1}{C_k} = \frac{k_s}{k_f}$ in the solid subdomain. The volumetric heat source is only active within a specified subdomain, within which it has a constant value.

4.2. Optimisation problem

In order to perform topology optimisation, continuous variables, γ_e, varying between 0 and 1 are introduced in each finite element, e, of the discrete system. Fluid is represented by $\gamma_e = 1$ and solid by $\gamma_e = 0$. For values between 0 and 1, the effective material properties, impermeability and conductivity, are interpolated as described in [2].

$$\begin{aligned}
&\underset{\gamma \in \mathbb{R}^n}{\text{minimise:}} f(\gamma, \mathbf{s}) = \mathbf{f}_t{}^{\mathrm{T}} \mathbf{t} \\
&\text{subject to: } g(\gamma) = \frac{\sum_{e \in \mathbb{E}_d} (1-\gamma_e) v_e}{v_f \sum_{e \in \mathbb{E}_d} v_e} - 1 \leq 0 \\
&\qquad \mathscr{R}(\gamma, \mathbf{s}) = \mathbf{0} \\
&\qquad 0 \leq \gamma_i \leq 1 \quad \text{for } i = 1, \ldots, n
\end{aligned} \tag{4}$$

where γ is a vector of n design variables, $\mathbf{s} = \{\mathbf{u}, \mathbf{p}, \mathbf{t}\}^{\mathrm{T}}$ is the vector of state field variables, f is the objective functional, g is the volume constraint functional and $\mathscr{R}(\gamma, \mathbf{s}) = \mathbf{M}(\gamma, \mathbf{s})\,\mathbf{s} - \mathbf{b}(\gamma, \mathbf{s})$ is the residual of the discretised system of equations.

The objective functional, f, is chosen as the thermal compliance, where $\mathbf{f}_t$ is the vector arising from the discretised volumetric flux and $\mathbf{t}$ is the vector of nodal temperatures. Since the flux load is constant, the optimisation problem essentially becomes to minimise the temperature of the heat source. The constraint functional, g, is a volume constraint on the solid material usage, where $\mathbb{E}_d$ is the set of elements belonging to the design domain and v_e is the volume of element e. Although not always necessary for convection-dominated problems, the volume constraint helps the design to converge to well-defined topologies.

The optimisation problem is solved using the nested formulation, where the discretised system of equations for the state field is solved separately from the design problem. The design sensitivities are found using the adjoint method.

5. Numerical implementation

The governing equations are discretised using stabilised trilinear finite elements as described in [2] and have been implemented in a large scale parallel topology optimisation framework based on PETSc [4, 5].

The resulting fully-coupled non-linear system of equations is solved using a damped Newton method. For the initial design iteration, a slow ramping strategy on the heat flux is applied in order to reach convergence from a zero initial vector. For subsequent design iterations, the state solution from the previous design iteration is used as the initial vector.

The linearised systems of equations is solved using an iterative solver, more specifically F-GMRES with a Galerkin-projection geometric multigrid (GMG) preconditioner. For the GMG smoother and coarse grid solver,

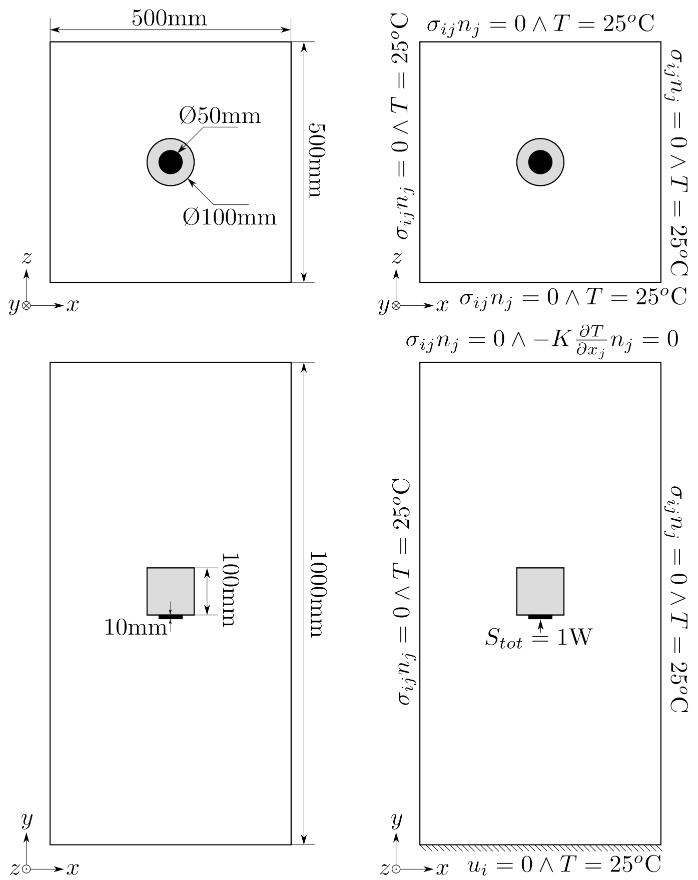

Figure 2: Illustrations of the problem setup showing the dimensions and the prescribed boundary conditions.

GMRES with a Jacobi preconditioner is used. Although multigrid is known not to be an optimal solver for non-elliptic systems of equations, the performance is very good and allows for the optimisation of the presented problem in a reasonable time.

The design field is regularised using the partial differential equation (PDE) filter [6] and the optimisation problem is solved using the method of moving asymptotes (MMA) [4, 7].

6. Problem setup

The problem setup is presented in figure 2. As an initial investigation, the lamp is oriented vertically downwards allowing for quarter symmetry to be imposed. The lamp is sought modelled suspended in free space, so all boundaries should ideally be left as open, that is $\sigma_{ij}n_j = 0$. However, as all rooms are finite, a floor (no-slip condition) is added at the bottom of the domain. This has the added advantage of stabilising the solution process and to ensure the Newton solver converges to a correct physical solution. The temperature field is imposed to be equal to the reference room temperature at all boundaries except the top-most boundary which acts as an outflow.

The reference temperature is assumed to be 25^oC and properties of air and aluminium for this reference temperature have been used. The resulting dimensionless numbers for the presented problem are:

$$\begin{aligned} Pr &= 0.74 \\ Gr &= 1.60 \times 10^5 \\ C_k &= 1.08 \times 10^{-4} \end{aligned} \tag{5}$$

where the Grashof number, Gr, is defined using the diameter of the LED package as the reference length.

The computational domain is truncated around the lamp to a rectangle $10 \times 20 \times 10$ times larger than the diameter of the LED package. Based on initial investigations of the flow field, this is assumed to be large enough for the open boundary conditions not to affect the solution around the lamp significantly. The LED package is modelled as a solid slab of aluminium with a uniform volumetric heat source totalling a power of $S_{tot} = 1$W. This

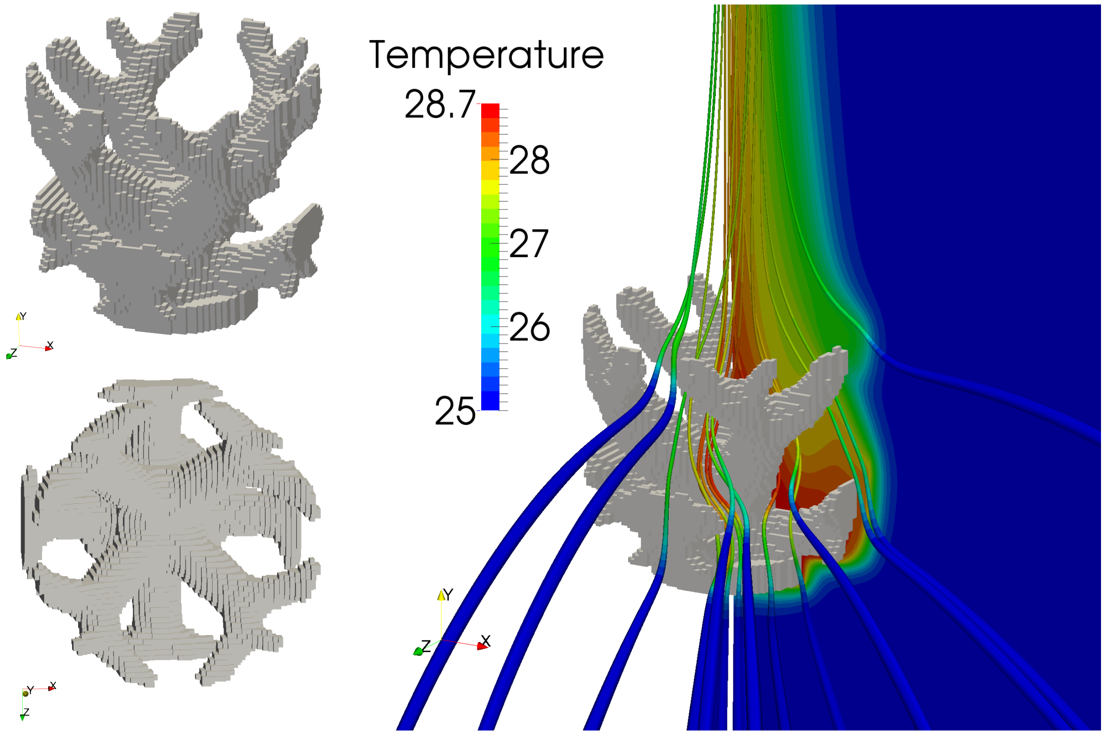

Figure 3: Optimised design from various angles and shown with a slice of the temperature field and streamtubes.

is at the lower end of the intended application range, but is chosen for numerical stability and to demonstrate a proof-of-concept.

Initial investigations using the full domain showed that the flow, temperature and design fields remained quarter-symmetric throughout the optimisation and the computational domain has thus been reduced to a quarter with symmetry boundary conditions. The computational mesh used is $160 \times 640 \times 160$ elements yielding a total number of $16,384,000$ elements and $83,076,805$ degrees of freedom (velocity, pressure and temperature). The design domain consists of $51,776$ elements and the filter radius is set to 4mm.

7. Optimisation results

Figure 3 shows the optimised design from various angles and also with a slice of the temperature field and streamtubes. The design exhibits tree-like branches extending out from the centre of the LED package. This intuitively makes sense as the thermal hotspot is located in the centre of the LED package. The branches conduct the heat away from the LED package and transfer it to the moving air by allowing the flow to move between the members. It can clearly be seen that the offset of the members enables the flow to zig-zag through the cooler in the vertical direction.

Figure 4 shows the optimised design together with slices showing the global velocity and temperature fields. It can be seen that the highest velocity is found some distance above the lamp, as expected and observed from experiments. The air is generally moving slowly far from the lamp and is accelerated above it, when it has been drawn in from the surroundings. The temperature field shows that globally the heat transfer is highly convection-dominated. The ambient temperature is observed in the entire computational domain, except for close to the lamp where a plume forms above it.

The computations have been performed using 2560 cores (128 nodes with two Intel Xeon e5-2680v2 10-core 2.8GHz processors) and the total computational time was approximately 8.5 hours for 200 design iterations. After the first design iteration, the number of Newton steps is around 2-4 and the number of F-GMRES iterations per linear solve is 20-30 for the state problem and 30-40 for the adjoint problem.

8. Conclusions and future work

Initial results for the application of topology optimisation to the design of passive coolers for LED lamps have1 been presented. A computational model problem has been set up to model the natural convection flow around a freely hanging LED lamp. Topology optimisation has been successfully applied to the highly convection-dominated heat transfer problem using a large scale parallel computational framework. The initial optimisation

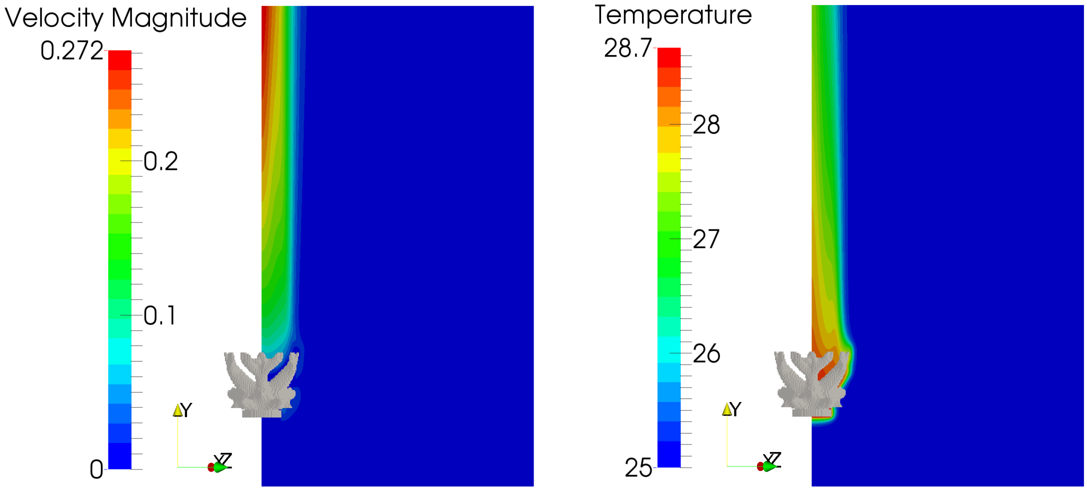

Figure 4: Optimised design with slices showing the global velocity and temperature fields. Please note that the bottom of the domain is not shown due to lack of interesting details.

results show interesting features, such as organic tree-like structures with offset members, that are currently being incorporated into industrial designs for enhanced passive cooling abilities.

Further developments will be presented in a journal paper in the near future. This includes comparison of the numerical model to experimental results in order to validate modelling assumptions and design performance. Initial results show promise and that the modelling assumptions are satisfied at the settings of interest. However, in order to treat higher power LED packages, it may be necessary to extend the analysis and optimisation to handle time-dependent flows. Also, investigations into the modelling accuracy of the fluid and thermal boundary layers is necessary to ensure physically accurate optimisation.

9. Acknowledgements

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