11th World Congress on Structural and Multidisciplinary Optimization
7th - 12th, June 2015, Sydney Australia

Robust shape optimization under vibroacoustic criteria and uncertain parameters

Frédéric Gillot[1], Renata Troain[1], Koji Shimoyama[2], Sébastien Besset[1]

[1] DySCo/LTDS, Ecole Centrale de Lyon, Lyon, France, frederic.gillot@ec-lyon.fr
[2] Institute of Fluid Science, Tohoku University, Sendai, Japan

1. Abstract
Our paper adresses the noise reduction level in acoustic cavities subject to uncertain parameters. Such issue is nowadays of paramount importance when treating inflight conditions of commercial planes or boats. The noise level is represented by an energy density in the cavity. This objective function is provided through an energy method called Simplified Energy Method. We use a transformation function mapping a given 3D cavity surface on a 2D domain. The optimization process directly relies on this function and thus avoids remeshing of the initial geometry. We consider geometrical and material uncertainties during the shape optimization process. Such uncertainties are usually generated by involved manufacturing processes. Robust optimization is performed using the non-dominated sorting genetic algorithm (NSGA-II) together with the Kriging surrogate model. We will show in our presentation the influence of geometrical and material characteristics on the optimal solution.
2. Keywords: Shape optimization, Simplified energy method, Robust optimization, Kriging, Genetic algorithm .

3. Introduction
Reducing noise level in cavities enable human transport means to increase their attractively and wellness. From a research point of view the noise level can be considered as a design objective, when proceeding with robust shape optimization of such cavities. This noise level can be described with the Simplified Energy Method (here and after referred as MES). This method has been fully validated for transient and stationary cases [1, 2], and for various elastic media such as membranes and plates [3].
We already demonstrated that such method can be efficiently used in an iso-geometric like description of a cavity, enabling efficient optimization loop scheme [4] We focus here on the robust shape optimization with regards to geometrical and material uncertainties.

4. MES in curvilinear coordinates

4.1. Short description of the MES
Detailed description of the MES can be found in [5, 6, 7].

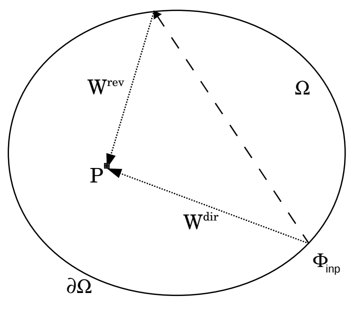


Figure 1: MES formulation: direct and reverberated fields

Since W is a quadratic variable made of partial energy quantities corresponding to both direct and reverberated fields (see Fig. 1), the superposition principle can be applied:

$$W = W_{dir} + W_{rev}. \tag{1}$$

The energy density inside the cavity can then be expressed as a function of the primary sources and fictitious sources (reverberated field sources) located on the boundaries:

$$W(P) = \int_{\partial\Omega} \Phi(M)\, \overline{\mathbf{u}}_{PM} \cdot \overline{\mathbf{n}}(M)\, G(M)\, d\partial\Omega + \int_{\partial\Omega} \sigma(M)\, \overline{\mathbf{u}}_{PM} \cdot \overline{\mathbf{n}}(M)\, G(M)\, d\partial\Omega, \tag{2}$$

where P is a point inside the cavity where W is measured, M is a point of integration on the cavity surface, $G(r) = 1/(4r^2\Pi c)$, $\overline{\mathbf{u}}_{PM} = \overline{\mathbf{PM}}/\|\overline{\mathbf{PM}}\|$, $\overline{\mathbf{n}}(M)$ the unit normal at the point M, $\Phi(M)$ are the acoustic boundary sources and $\sigma(M)$ are the fictitious boundary sources. We will use the term "boundary source" to denote the sources located on the cavity boundary (which may be due, for example, to external excitations).
For every point M_0 of the boundary $\partial\Omega$, $\sigma(M_0)$ depends on the absorption coefficient α, acoustic boundary sources of the system Φ and fictitious boundary sources in all other points of $\partial\Omega$:

$$\begin{aligned} \sigma(M_0) = (1-\alpha) \int_{\partial\Omega} \sigma(M)\, \overline{\mathbf{u}}_{M_0M} \cdot \overline{\mathbf{n}}(M)\, G(M)\, d\partial\Omega \\ +(1-\alpha) \int_{\partial\Omega} \Phi(M)\, \overline{\mathbf{u}}_{M_0M} \cdot \overline{\mathbf{n}}(M)\, G(M)\, d\partial\Omega. \end{aligned} \tag{3}$$

Energy variables are given as a solution of a Fredholm equation, corresponding to an energy balance at the boundary of the domain.

4.2. Curvilinear coordinates
We choose to described 3D cavity by parametrized functions of two variables. This approach is fully described in [11]. Such approach enable the used of complex geometries to described the cavity, such as Bezier curves, Splines, NURBS and so on. Moreover the discritization is conducted on a 2D domain, while design variables are coefficient of transformation function thus avoiding the remitting during the optimization procedure. The matrix formulation of our proposed approach is then

$$\overline{W} = [S]\, \overline{\sigma} + [R]\, \overline{\Phi}, \tag{4}$$

where $[S]$ and $[R]$ are matrices corresponding to the discretization of the integral formulations of MES.
$\overline{\sigma}$ is expressed as follows:

$$\overline{\sigma} = \big([Id] - [\alpha]\big)\, [T]\, \overline{\sigma} + \big([Id] - [\alpha]\big)\, [Q]\, \overline{\Phi}, \tag{5}$$

where $[Id]$ is the identity matrix and $[\alpha]$ the diagonal matrix of the absorption coefficients. Expressing σ as a function of Φ gives:

$$\overline{\sigma} = \big([Id] - [T] + [\alpha]\, [T]\big)^{-1} \big([Id] - [\alpha]\big)\, [Q]\, \overline{\Phi}. \tag{6}$$

Using (4) and (6) we obtain:

$$\overline{W} = \Big([R] + [S]\, \big([Id] - [T] + [\alpha]\, [T]\big)^{-1} \big([Id] - [\alpha]\big)\, [Q]\Big)\, \overline{\Phi} = [M]\, \overline{\Phi}. \tag{7}$$

Such matrix formulation gives an advantage when computing the robustness towards material absorption coefficients. These coefficients are given in matrix $[\alpha]$, while other matrices depend only on geometrical properties of the cavity. That is why calculation of W distribution for given geometry and α following normal law doesn't demand high computational cost. We have to obtain geometry matrices once and after calculate W for every α changing just the values of $[\alpha]$ matrix.

6. Robustness problem
The quantity f to be minimized can be formulated as follows:

$$f = \|\overline{W}(x_i, \alpha)\|. \tag{8}$$

After specifying the geometry of the cavity Ω with bounding surface $\partial\Omega$ and the function of transformation $\bar{r}(\xi_1, \xi_2) = [x(\xi_1, \xi_2),\ y(\xi_1, \xi_2),\ z(\xi_1, \xi_2)]$ the geometrical design variables x_i can be defined as characteristics of the transformation function, *i.e*. the parameters of the functions $x(\xi)$, $y(\xi)$ and/or $z(\xi)$. Absorption coefficients α are chosen to be material design variables. Geometric uncertainties due to cavity manufacturing process are modeled by considering normally distributed design variables with the standard deviation σ_1 around its nominal value μ_1, i.e., it is represented as a normal random variable N(μ_1, σ_1^2). Absorption coefficients are assigned to each panel, and these values are assumed to be independent, represented as a normal random variables N(μ_2, σ_2^2)

6.1. Optimization method
The first step samples values of objective function at several different values of design variables, and approximates a response of objective function using the Kriging surrogate model [8]. It enables an optimizer to promptly *estimate* objective function values at other points where the values of objective function are not given.

The next step performs robust optimization using the non-dominated sorting genetic algorithm (NSGA)-II [9] directly on the Kriging surrogate model. The present robust optimization considers minimizing both the mean and standard deviation of objective function against uncertainties.

6.2. Application exemple

We applied the robust optimization scheme to a parallelepiped acoustic cavity. It takes an area $\Omega = \{x \in [0;4]; y \in [0;2]; z \in [0;2]\}$ (Fig. 2). The cavity surface is considered to be assembled with six patches of Bezier surfaces. Every patch is determined by (4×4) control vertices as presented in Fig. 2.

An acoustic source is applied on the surface (Fig. 2, marked with cross); the test point inside the cavity with

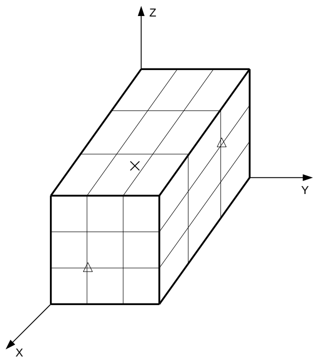


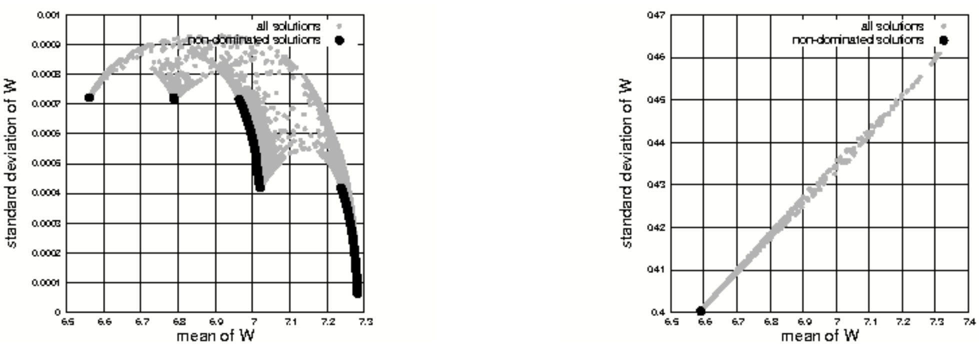
Figure 2: Cavity shape characteristics: control vertices distribution for the Bezier surface definition; position of acoustic source (cross); control points chosen to be optimization variables (triangles)

coordinates (1.33; 0.6; 0.6) was chosen to compute energy density vector. Two control vertices were chosen to be design parameters (Fig. 2, marked with triangles). The coordinate of these vertices perpendicular to the patch plane are under consideration, so the optimization problem depends on two design variable x_1 and x_2 .

6.3. Results and discussion

Results of the optimization process performed by the NSGA-II algorithm are given in Fig. 3. In Fig. (3, a), we can see these non-dominated solutions only appear for a few ranges of the criterion corresponding to the mean of W, which shows the importance of the design: for example, two points are near from .0004 in term of standard deviation of W; nevertheless, the first one leads to $W = 7\ dB$ and the second one leads to $W = 7.25\ dB$. If we want to obtain this kind of value for the standard deviation, it is obvious that the first point should be chosen. Hence, the optimization problem under geometric uncertainties appears to lead to several optimal solutions, and the choice of the design should be done carefully to favor one criterion or the other. Fig. (3, b) is radically different since the optimal solutions are the same for both W and its standard deviation. This means that the two considered criteria are changing in the same direction: thus, it is not useful to consider the material uncertainties for this kind of problem.

In



a) b)

Figure 3: Solutions searched by the robust optimization of cavity shape a) under geometric uncertainties b) under material uncertainties

In Fig. 4 an optimal cavity shape for the non dominated solution is presented, where $\mu(F) = 7.02\, dB$; $\sigma(F) = 0.00041$; $x_1 = 10\, m$; $x_2 = 2.58\, m$. Changing of the color reflects the change in the coordinate normal to the parallelepiped side. This solution corresponds to the compromise between mean and standard deviation of the objective function toward geometrical uncertainties.

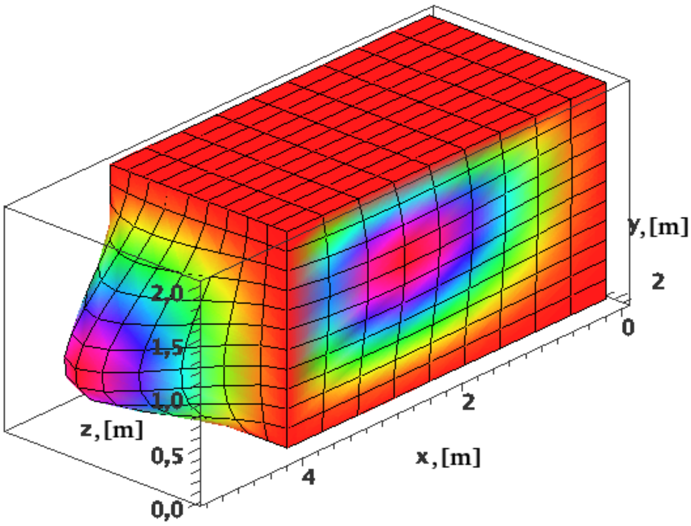


Figure 4: The resulting shape of the cavity after the optimization process. Compromise solution. $\mu(F) = 7.02\, dB$; $\sigma(F) = 0.00041$; $x_1 = 10\, m$; $x_2 = 2.58\, m$.

7. Conclusion

In this paper we introduced a robust shape optimization of cavities under vibroacoustic criteria and uncertain parameters. We used MES approach combined with a projection function to reach, for any cavity described by parametric functions, the energy density value W. We investigated the effect of geometrical as well as material parameters uncertainties on such genetic algorithm (NSGA II) based optimization loop on a Kriging meta model representing the first and second moment order of W with regard to uncertain parameters variation. We applied our approach on an exemple and conclude that material uncertainties can be ignored in such optimization scheme, but not geometrical uncertainties which lead to antagonist behavior of the average and standard deviation of the objective function W.

6. Acknowledgements

The authors would like to thank the financial support provided by the French National Research Agency through the framework of its project ANR-12-JS09-0009. This work was also partly supported by the JSPS Core-to-Core Program, A. Advanced Research Networks, ”International research core on smart layered materials and structures for energy saving”.

7. References

[1] F. Sui, “Prediction of Vibroacoustics Energy Using a Discretized Transient Local Energy Approach and Comparison with Tsea,” J. Sound Vibr. **251**, 163–180 (2002).

[2] A. Wang, N. Vlahopoulos, and K. Wu, “Development of an energy boundary element formulation for computing high-frequency sound radiation from incoherent intensity boundary conditions,” J. Sound and Vibr. **278**(1–2), 413 – 436 (2004).

[3] O. Bouthier and R. Bernhard, “Simple models of energy flow in vibrating membranes,” Journal of Sound and Vibration **182**(1), 129 – 147 (1995).

[4] R. Troian, S. Besset, and F. Gillot, “Shape optimization under vibroacoustic criteria in the mid-high frequency range,” Journal of Computational Acoustics **22**(02), 1450003 (2014).

[5] S. Beset, M. N. Ichchou, and L. Jezequel, “A coupled BEM and energy flow method for mid-high frequency internal acoustic,” J. Comp. Acoust. **18**(01), 69–85 (2010).

[6] S. Besset and M. Ichchou, “Acoustic absorption material optimisation in the mid-high frequency range,” Appl. Acoust. **72**(9), 632 – 638 (2011).

[7] J. Sacks, W. J. Welch, T. J. Mitchell, and H. P. Wynn, “Design and analysis of computer experiments,” Stat. Science **4**(4), 409–435 (1989).

[8] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multi-objective genetic algorithm: NSGA-II,” IEEE Transac. on Evolut. Comput. **6**(2), 182–197 (2002).