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Conceptual Design of Box Girder Based on Three-dimensional Topology Optimization

Dongchen Qin[1] , Peng Du[1] , Qiang Zhu[1], Junjie Yang[1]

[1] Zhengzhou University , Zhengzhou , China , dcqin@zzu.edu.cn

1 Abstract
The box girder is widely used in mechanical equipments. Since the design of box girder mainly relies on traditional experience , the structure is very heavy. This paper applies the topology optimization to the design of box girder to reduce weight. First, an optimization model for the three-dimensional box girder, in which the compliance is minimized and volume is constrained, is established based on SIMP method, and different filter functions are used to avoid numerical instabilities that will occur in the optimization process. By loading the box girder in three different positions respectively and using the Optimality Criteria to solve each of load conditions , three kinds of structures are obtained. Finally, an optimal topology configuration is obtained by combining the three structures, and its validity is tested by FEM. In this paper , by using three-dimensional model, the internal structure of box girder can be obtained ,which is useful for the layout of the ribs. The results provide a reference for the conceptual design of box girder ,and lay some foundation for the multi-load cases problem.
2 Keywords: Topology Optimization; Box girder; Conceptual Design

3 Introduction
To industrial equipments, the box girder , one kind of the fundamental pieces of manufacturing , is not only very common but also very important. Many parts that can be found in the factories, such as the girder of bridge crane, the bed of machine tool , belong to the box girder. The weight of box girder is usually accounted for a large proportion of the total weight of the device. The box girder directly or indirectly determines the precision, carrying capacity and other basic performance of equipment. And of course, the internal structure of box girder is more complex and difficult to reach a rational design.

Currently, when designing box girder structure, traditional design methods include experience design and analogy design are used, and the results is particularly heavy. Bulky parts use more material in the manufacturing process, expend more energy during transport and adversely affect the device performance due to its excessive inertia. Therefore, the design by reducing weight of box-beams is urgent. In structural optimization, topology optimization that is innovative in design is a hot research field. With the method of topology optimization, Huayang Xu[1] has optimized the arm of flight simulator under inertial load , Lunjie Xie[2] has obtained a rational load bearing structure of electric car body in conceptual design phase, Bret[3] has designed a structure for flapping mechanism under multiple load cases, etc.

This paper focuses on applying topology optimization to the box girder design. First, the box girder is abstracted as a simply supported beam, and an optimization model is established of which the objective function is compliance and the constraint is the volume, then, problems in the model solution process are studied and the method to solve them is found. Second, this model and method is used to design some kinds of new structures for the bridge crane girder with the loads position changed. Based on those structures, a new structure that can be applied to different load conditions is designed.

4 topology optimization theory

4.1 SIMP method

SIMP method is widely used among the topology optimization methods. It has been proposed by Mlejnek[4] in 1993, which builds the relationship between the material density and elastic modulus by the following formula:

$$E_i\left(x_i\right)=x_i^p E_0\ , \qquad x_i\in\left[0,1\right] \tag{1}$$

where E_0 is the elastic modulus of material, x_i is the i th element's density, and p is the penalization power($p>1$) . When using this method to the simply supported beam, the Eq. above is changed to the following in order to prevent stiffness matrix singular which will happen when the number of material density equal to 0:

$$E\left(x_i\right)=E_{\min}+x_i^p\left(E_0-E_{\min}\right),\quad x_i\in\left[0,1\right] \tag{2}$$

where $E_{\min}$ is the elastic modulus which is very little.

4.2 The penalization power

Material density varies continuously between 0 and 1 in SIMP method. Thus, the structure after optimization exists middle density areas which can not be manufactured in reality and are need to be suppressed as much as possible. The utility of penalization power is to penalize those areas. Supposed the elastic modulus of material $E_0 = 2.0\times10^5\,\mathrm{MPa}$, according to Eq.(1), the Figure 1 is gotten, which shows the penalization power's influence on the elastic modulus. From this picture , when the value of penalization power increases, the middle density closers to the two ends of the interval of the elastic modulus, which achieves the desired effect of penalization power.

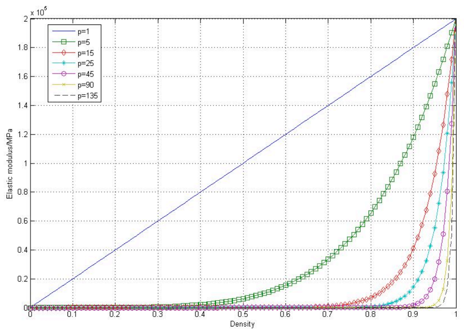

Figure 1: The penalization power's influence on the elastic modulus

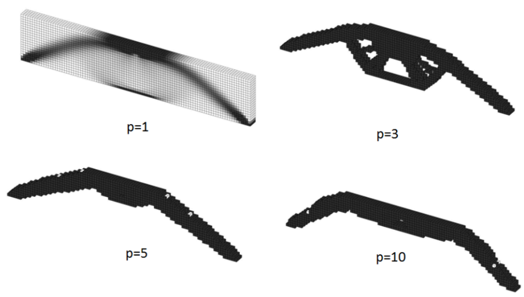

Figure 2: The impact of different penalization power on simply supported beam's optimization results

Note that the results of topology optimization change as the penalization power is changed , and once the value is bigger than a certain value, the results of topology optimization may be wrong[5] . The Figure 2 show this phenomenon. When p=1, there is no any effect of the penalization power and the result is not clear; when p=3, the structure is clear and can be used; when p=5 and p=10, the simply supported beam becomes a structure with some rods, which does not meet the engineering requirements. From this comparison, to get a right result for the simply supported beam, the penalization power's value at 3 is better ,which is used in this paper.

4.3 Optimization Model

When building an optimization model, the simply supported beam is discretized by the finite element method. And each element has a density x_i , which together forms a design space $\mathbf{X}$:

$$\mathbf{X} = \left[x_1, x_2, \cdots, x_i, \cdots, x_n\right]^{\mathrm{T}} \tag{3}$$

where n is the number of element after discretized. For the simply supported beam , we build a optimization model of which the objective function is to minimize compliance and the constraint is the volume as following:

$$\begin{aligned} &\min \quad c(\mathbf{X}) = \mathbf{F}^{\mathrm{T}}\mathbf{U}(\mathbf{X}) \\ &S.t \quad \begin{cases} v(\mathbf{X}) = \mathbf{X}^{\mathrm{T}}\mathbf{V} - \bar{v} \le 0 \\ \mathbf{0} \le \mathbf{X} \le \mathbf{1} \end{cases} \end{aligned} \tag{4}$$

where $\mathbf{F}$ is the load applied in the model , which is determined on the size of the load, direction and acting position; $\mathbf{V} = \left[v_1, \cdots, v_n\right]^{\mathrm{T}}$ is the vector of element volume; $\bar{v}$ is the volume that is wanted to reach. $\mathbf{U(X)}$ is the displacement of each node, which can be get from the Eq.(5) :

$$\mathbf{K(X)U(X) = F} \tag{5}$$

4.4 Sensitivity Analysis

Since the design variable is nothing to do with the load $\mathbf{F}$, the following Eq. can be gotten:

$$\frac{\partial \mathbf{F}}{\partial x_i} = 0 \tag{6}$$

The partial derivative of Eq. (5)is:

$$\frac{\partial \mathbf{K}}{\partial x_i}\mathbf{U} + \mathbf{K}\frac{\partial \mathbf{U}}{\partial x_i} = 0 \tag{7}$$

The partial derivative of volume constraint is：

$$\frac{\partial v(\mathbf{X})}{\partial x_i} = \frac{\partial(\mathbf{X}^{\mathrm{T}}\mathbf{V} - \bar{v})}{\partial x_i} = v_i \tag{8}$$

where v_i is the volume of the i th element.

The partial derivative of objective function is：

$$\frac{\partial c}{\partial x_i} = \frac{\partial \mathbf{U}^{\mathrm{T}}}{\partial x_i}\mathbf{KU} + \mathbf{U}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_i}\mathbf{U} + \mathbf{U}^{\mathrm{T}}\mathbf{K}\frac{\partial \mathbf{U}}{\partial x_i} \tag{9}$$

Using Eq.(7):

$$\frac{\partial c}{\partial x_i} = -\mathbf{U}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial x_i}\mathbf{U} \tag{10}$$

According to FEM theory, in each element ,there is:

$$\mathbf{D}_i(x_i) = E_i(x_i)\mathbf{D}_i^0 \tag{11}$$

where $\mathbf{D}_i^0$ is the elasticity matrix of each element, which is a constant matrix.

The element stiffness matrix is based on isoparametric element theory, the element stiffness matrix is:

$$\mathbf{k}_i(x_i) = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\mathbf{B}^{\mathrm{T}}\mathbf{D}_i\mathbf{B}\left|\mathbf{J}\right|d\xi d\eta d\zeta \tag{12}$$

where $\mathbf{B}$ is the strain-displacement matrix, $\left|\mathbf{J}\right|$ is the determinant of Jacobian matrix. Using(11):

$$\mathbf{k}_i(x_i) = E_i(x_i)\mathbf{k}_i^0 \tag{13}$$

Last, the sensitivity of the objective function is:

$$\frac{\partial c}{\partial x_i} = -\mathbf{u}_i^{\mathrm{T}}\left[px_i^{p-1}\left(E_0 - E_{\min}\right)\mathbf{k}_i^0\right]\mathbf{u}_i \tag{14}$$

4.5 Optimality criteria

The optimization model of simply supported beam is nonlinear, which can be solved by the Optimality Criteria (OC) method. This method which is based on K-T condition is used in topology optimization due to it's simply and efficient. Supposed $\mathbf{0} \le \mathbf{X} \le \mathbf{1}$ is satisfies, namely it is inactive, the design variables are updated using the following :

$$x_i^{new} = \begin{cases} \max\left(0, x_i - m\right), & when\ x_i B_i^{\eta} \le \max\left(0, x_i - m\right) \\ \min\left(1, x_i + m\right), & when\ x_i B_i^{\eta} \ge \min\left(1, x_i - m\right) \\ x_i B_i^{\eta}\ , & otherwise \end{cases} \tag{15}$$

where

$$B_i = \frac{\mathbf{u}_i^{\mathrm{T}}\left[px_i^{p-1}\left(E_0 - E_{\min}\right)\mathbf{k}_i^0\right]\mathbf{u}_i}{\lambda v_i} \tag{16}$$

To this problem, m=0.2 and η =0.5 is recommended[6]. Then, there is only one unknown number λ in the Eq.(15), which can be get by following the steps below:

1) Let $\lambda_{\min}^{(1)} = 0$, and $\lambda_{\max}^{(1)}$ is a large number, such as 1×10^9 ;
2) Calculate $\lambda^{(k)} = \left(\lambda_{\min}^{(k)} + \lambda_{\max}^{(k)}\right)/2$
3) Get $\mathbf{X}^{new}$ by Eq.(15), then ,calculate $v\left(\mathbf{X}^{new}\right)$, when $v\left(\mathbf{X}^{new}\right) < 0$, let $\lambda^{(k+1)} = \lambda_{\max}^{(k)}$; and when $v\left(\mathbf{X}^{new}\right) > 0$, let $\lambda^{(k+1)} = \lambda_{\min}^{(k)}$;
4) Repeat steps 2 and 3 until to $v\left(\mathbf{X}^{new}\right) = 0$.

4.6 Filter function

Among the optimization process of simply supported beam, there may be numerical instabilities, such as checkerboard problem and mesh-dependency. Generally , those problems appear simultaneously, thus, the method that can inhibit the checkerboard problem also can inhibit the mesh-dependence. One common approach to suppress these problems is using the density filter function[7], which is defined as:

$$x_i = \frac{\sum_{j\in N_i} H_{ij} v_j x_j}{\sum_{j\in N_i} H_{ij} v_j} \tag{17}$$

Another commonly used method is the gray scale filter[8], which is a nonlinear method. This method is an update of OC method:

$$x_i^{new} = \begin{cases} \max\left(0, x_i - m\right), & if\ x_i B_i^{\eta} \le \max\left(0, x_i - m\right) \\ \min\left(1, x_i + m\right), & if\ x_i B_i^{\eta} \ge \min\left(1, x_i - m\right) \\ \left(x_i B_i^{\eta}\right)^q\ , & otherwise \end{cases} \tag{18}$$

The value of q is a key. For this optimization model, q=2 is recommended[9]. And it is just the OC method when q=1.

Different filter functions have different impact on the topology optimization results. In Figure 3, (a) is the result without any filter function, (b) is the result using the density filter function, (c) is the result using the gray scale filter function, (d) is the result using density filter and gray scale filter. Compared with no filter function, the structure is more clear and less checkerboard phenomenon when using filter function. When using two filter functions alone, the density filter is better than gray scale filter from (b) and (c),because (c) appears checkerboard problem. That is to say the gray scale filter shouldn't be used alone. Compared (b) with (d), there are some differences in the structure. Based on this study, in order to get the most reasonable structure of simply supported beam, the density filter method and another method that uses density filter and gray scale filter together are applied to the optimization model respectively.

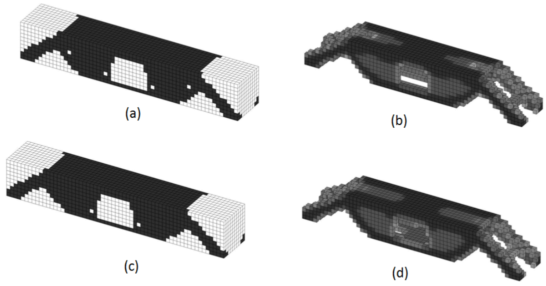

Figure 3: Different filter function's impact on optimization results

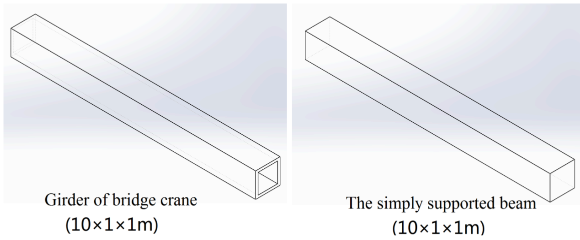

Figure 4: The girder and its Simply supported beam

5 Topologies under different conditions

Box girders are widely used in machines. In order to use the topology theory above to get a new structure of box girder, the girder of bridge crane is selected as the research object, which can be simplified as a simply supported beam.

The size of bridge crane girder is 10×1×1m, which is showed in Figure 4, and the maximum lifting capacity of it is 50 tons. The material of girder is Q345, of which the elastic modulus is 2.0e5MPa, and the Poisson's ratio is 0.3. Since the safety factor of this material is 1.4, the allowable stress is 246MPa. According to the girder's length and stiffness , the allowable deflection of grider is 12.5mm. This girder is abstracted as a simply supported beam, which is showed in Figure 4.

In terms of the model loads, they are given by the lifting trolley through four wheels , and the loads are uncertain, whose size and acting position vary from time to time. However, in the optimization model, the loads are certain, and are known constants. Thus, the uncertain loads need to be simplified to certainty in this paper. When the lifting trolley is in the middle of girder , the structure is under the most dangerous condition, and when the lifting trolley is at both sides of girder , the structural deformation is least. Considering these, there are 4 kinds of lifting conditions to the girder, which the loads are the maximum lifting capacity: (a) the lifting trolley position is at the upper middle of the simply supported beam; (b) the lifting trolley position is 1/4 away from the left side of the simply supported beam; (c) the lifting trolley position is 1/4 away from the right side of the simply supported beam; (d) the lifting trolley position is at the three position above at the same time(it does not exist in reality).

According to the filter function analysis above, different results can be gotten by using different filter function , and the results to the simply supported beam is in the Figure 5 . The results using density filter is on the left, and the results using density filter and gray scale filter is on the right. The letters in the pictures correspond to the respective lifting conditions above.

There are many common grounds in the results using two kinds of filter functions. All the structures show a trapezoidal shape , which can be divided into two parts: the main structure on the middle and the support structure on left and right sides. Middle part of the structure is wider at the top, which is hollow, and on the front and rear web position, there are big trapezoidal holes, whose position are just below the loads. The hole does not appear on the Cover on the four kinds of lifting conditions. The support structure is the two trapezoidal sides., and the upper and lower materials on each side are removed. In the results, dark black units represent the main unit , which can be enhanced by adding an appropriate amount of ribs in these place.

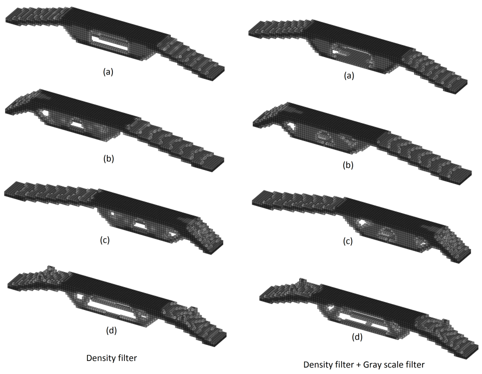

Figure 5: The results of the simply supported beam using different filters

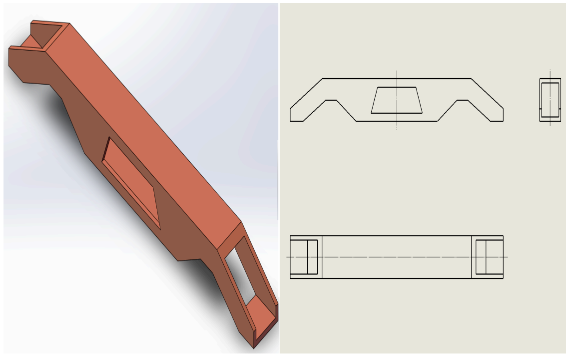

Figure 6: Bridge crane girder conceptual structure

In addition to the same points, there are differences between two results. The most obvious point is on the Under Cover. After using density filter, the Under Cover is gone, thus, the cross section is a inverted U. However, when using the other filter function, the Under Cover still exists, thus, the cross section is a rectangle. Theoretically, the loads is straight down , then these two cross-sectional structures have the same stabilities . However, the loads are uncertain in direction on actual working conditions, then the rectangle cross section is better than the other one. Therefore, the exists of Under Cover is necessary.

6 Structure synthesis and verification

According to the analysis of different results above, a conceptual structure for the bridge crane girder is gotten by removing materials on some parts of girder. The conceptual structure is showed in Figure 6.

In order to validate the conceptual structure , finite element model is built. When the lifting trolley is in the middle of girder , the structure is under the most dangerous condition. To this condition, the results is in Figure 7, Figure 8 and Table 1. In the figures, (1) represents the simply supported beam; (2) represents the bridge crane girder; (3) represents the conceptual structure. According to the results, the displacement of three models increase successively, and the conceptual structure one is 4.85mm which is the largest one, but it is still less than the allowable deflection which is 12.5mm yet. The maximum stress of three models increase successively too, and the largest one is the conceptual structure's stress which is 103MPa, but it is still less than the allowable stress which is 246MPa yet. Most importantly, stress region is more uniform in the conceptual structure , which represents material is efficiently used. In conclusion , when the trolley is in the middle of girder, the conceptual structure's displacement meet the requirements and stress is better than the others.

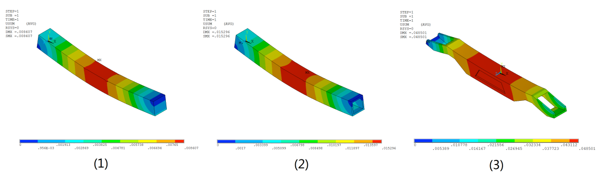

Figure 7: Displacement contour plot when trolley in the middle

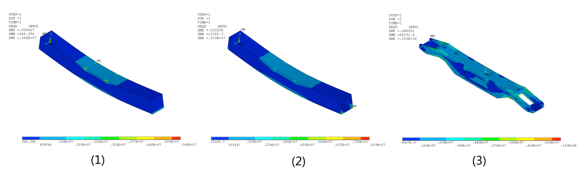

Figure 8: Von Mises stress contour plot when trolley in the middle

Table 1: Displacement and stress when trolley in the middle

	Simply supported beam	Bridge crane girder	Conceptual structure
Displacement/mm	0.86	1.53	4.85
Maximum stress/MPa	56.60	81.90	103.00

When the lifting trolley is 1/4 away from the left side , the results is in Figure 9, Figure 10 and Table 2. From the results, the displacement and stress of conceptual structure are the largest ones, but still in the allowable range, and stress region is uniform too. Due to the symmetry of the structure , the results to the condition trolley 1/4 away from the right is just the same as this results. The last condition in the four lifting conditions does not exist in reality ,thus there is no need to verify the structure in this condition.

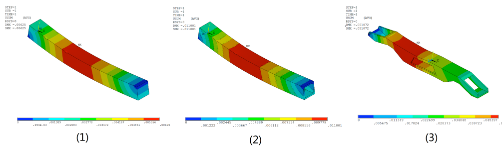

Figure 9: Displacement contour plot when trolley 1/4 away from the left

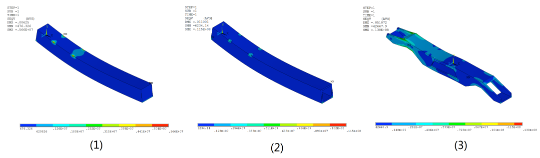

Figure 10: Von Mises stress contour plot when trolley 1/4 away from the left

Table 2: Displacement and stress when trolley 1/4 away from the left

	Simply supported beam	Bridge crane girder	Conceptual structure
Displacement /mm	0.63	1.10	5.11
Maximum stress /MPa	56.60	115	130.00

Table 3: Volumes to different models

	Volume/m^3	percentage
Simply supported beam	10.00	22.5%
Bridge crane girder	3.60	62.5%
Conceptual structure	2.25	100%

From the Table 3, compared with the other structures, the conceptual structure is more light since they all have the same density and the volume of conceptual structure is least. In conclusion , the conceptual structure meet the requirements, and is a new weight light structure for the bridge crane girder.

7 Conclusions

This paper applies topology optimization method to the box girder's concept design stage. For simply supported beam, SIMP method is briefly introduced. The filter function that density filter combined with gray scale filter is better for simply supported beam when using SIMP method. Topology optimization is used on girder of bridge crane, getting a new conceptual structure which is verified by finite element analysis in two conditions. The conceptual structure is light and efficient in material using However, the optimization model does not consider the stress, which is the future research direction.

8 References

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